Connected and Total Edge Domination in Boolean Function Graph $B(G, L(G), NINC)$ of A Graph

S. Muthammai$^1$ and S. Dhanalakshmi$^2$
Alagappa Government Arts College, Karaikudi$^1$.
Government Arts College for Women(Autonomous), Pudukkottai.$^2$

Abstract

For any graph $G$, let $V(G)$ and $E(G)$ denote the vertex set and edge set of $G$ respectively. The Boolean function graph $B(G, L(G), NINC)$ of $G$ is a graph with vertex set $V(G) \cup E(G)$ and two vertices in $B(G, L(G), NINC)$ are adjacent if and only if they correspond to two adjacent vertices of $G$, two adjacent edges of $G$ or to a vertex and an edge not incident to it in $G$. For brevity, this graph is denoted by $B_1(G)$. In this paper, Connected edge domination and total edge domination numbers of Boolean Function Graph $B(G, L(G), NINC)$ of some standard graphs are obtained.

Keywords: Boolean Function graph, Edge Domination Number

1. INTRODUCTION

Graphs discussed in this paper are undirected and simple graphs. For a graph $G$, let $V(G)$ and $E(G)$ denote its vertex set and edge set respectively. A subset $D$ of $V$ is called a dominating set of $G$, if every vertex not in $D$ is adjacent to some vertex in $D$. The domination number $\gamma(G)$ of $G$ is the minimum cardinality taken over all dominating sets of $G$. An edge $e$ of a graph is said to be incident with the vertex $v$ if $v$ is an end vertex of $e$. In this case, it can also be said that $v$ is incident with $e$.

A subset $F \subseteq E$ is called an edge dominating set of $G$, if every edge not in $F$ is adjacent to some edge in $F$. The edge domination number $\gamma'(G)$ of $G$ is the minimum cardinality taken over all edge dominating sets of $G$. An edge dominating set $X$ of $G$ is called a total edge dominating of $G$ if the induced subgraph $\langle X \rangle$ has no isolated edges.

The total edge domination number $\gamma'_t(G)$ of $G$ is the minimum cardinality taken over all of total edge dominating sets of $G$. An edge dominating set $X$ of is called a connected edge dominating sets of $G$, if the induced subgraph $\langle X \rangle$ is connected. The connected edge domination number $\gamma'_c(G)$ of $G$ is the minimum cardinality taken over all connected edge dominating sets of $G$. The concept of edge domination was introduced by Mitchell and Hedetniemi [6]. Arumugam and Velammal [1] have discussed edge domination number and edge domatic number. Vaidya and Pandit [7] determined edge domination number of middle graphs, total graphs and shadow graphs of $P_n$ and $C_n$. For graph theoretic notations and terminology, Harary [2] is followed.
For a real x, \( \lfloor x \rfloor \) denotes the greatest integer less than or equal to x.

**Theorem 1.1.** [6] For any (p, q) graph G, \( \gamma' \leq \lfloor p/2 \rfloor \).

**Theorem 1.2.** [3] G and L(G) are induced subgraphs of \( B_3(G) \).

**Theorem 1.3.** [3] Number of vertices in \( B_1(G) \) is \( p+q \) and if \( d_i = \text{deg}_G(v_i), v_i \in V(G) \), then the number of edges in \( B_1(G) \) is \( q(p-2) + \frac{1}{2} \sum_{1 \leq i \leq p} d_i^2 \).

**Theorem 1.4.** [3] The degree of a vertex of G in \( B_2(G) \) is \( q \) and the degree of a vertex \( e' \) of L(G) in \( B_1(G) \) is \( \text{deg}_{B_1(G)}(e') + p - 2 \). Also if \( d^*(e') \) is the degree of a vertex e' of L(G) in \( B_1(G) \), then \( 0 \leq d^*(e') \leq p+q-3 \). The lower bound is attained, if \( G \cong K_2 \) and the upper bound is attained, if \( G \cong K_{1,n} \) for \( n \geq 2 \).

**Theorem 1.5.** [3] \( B_3(G) \) is disconnected if and only if G is one of the following graphs: \( nK_1 \), \( K_2 \), \( 2K_2 \) and \( K_2 \cup nK_1 \), for \( n \geq 1 \).

In this paper, connected edge domination numbers of Boolean Function Graph \( B(G, L(G), \text{NINC}) \) of some standard graphs are obtained.

**2. Connected edge domination in B(G, L(G), NINC) of a Graph**

In the following connected edge domination number of \( B_1(P_n) \), \( B_1(C_n) \), \( B_1(K_n) \), \( B_1(K_{1,n}) \) \( B_1(W_n) \) are found.

**Theorem 2.1.** For the Path \( P_n \) on vertices \( (n \geq 4) \), \( \gamma'_C(B_1(P_n)) = 2n-3 \)

**Proof:** Let \( v_1, v_2, \ldots , v_n \) and \( e_{12}, e_{23}, \ldots , e_{n-1,n} \) be the vertices and edges of \( P_n \) respectively. Then \( v_1, v_2, \ldots , v_n, e_{12}, e_{23}, \ldots , e_{n-1,n} \in V(B_1(P_n)) \) where \( e_{i,i+1} = \{v_i, v_{i+1}\}, i = 1, 2, \ldots , n - 1 \). \( B_1(P_n) \) has \( 2n-1 \) vertices and \( n^2 - n - 1 \) edges.

Let \( F_m = \{(v_i, v_j) / 1 \leq i \leq n, j \equiv (i+m) \mod (n-1), k \equiv i+(m+1) \mod (n-1)\} \) and \( F = (\bigcup_{m=1}^{n-1} F_m \cup \{(v_1, e_{1,n}), \{v_n, e_{n-1, n-1}\}\} \). Then \( B_1(P_n) = E(P_n) \cup E(P_{n-1}) \cup F \). If \( D' = \{(v_{i+1}, e_{i,i+1}), (e_{i,i+1}, e_{i,i+1,i+2})\} \cup \{(v_{n-1}, e_{12})\} \), then \( D' \subseteq E(B_1(P_n)) \). \( D' \) dominates edges of \( P_n, P_{n-1} \) and \( F \). \( D' \) is an edge dominating set of \( B_1(P_n) \). Also, \( <D'> \cong \mathbb{P}_{n-1}^+ \).

Therefore, \( D' \) is a connected edge dominating set of \( B_1(P_n) \) and hence \( \gamma'_C(B_1(P_n)) \leq |D'| = 2(n-2)+1 = 2n-3 \). Let \( D'' \) be a minimum edge dominating set of \( B_1(P_n) \). To dominate the edges of \( B_1(P_n) \). \( D'' \) contains at least \((n-2)\) edges of \( L(P_n) \) and hence \(|D'| \geq n-2+n-1 = 2n-3 \). Therefore, \( \gamma'_C(B_1(P_n)) = 2n-3 \).

**Remark:**[2.1] \( \gamma'_C(B_1(P_3)) = 3 \)

**Theorem 2.2.** For the Cycle \( C_n \) on \( n \) vertices \((n \geq 5)\) vertices, \( \gamma'_C(B_1(C_n)) = 2n-3 \).

**Proof:** Let \( v_1, v_2, \ldots , v_n \) be the vertices and \( e_{12}, e_{23}, \ldots , e_{n-1,n} \) be the edges of \( B_1(C_n) \) where \( e_{i,i+1} = \{v_i, v_{i+1}\}, i = 1, 2, \ldots , n - 1 \), \( e_{n,1} = \{v_n, v_1\} \). \( B_1(C_n) \) has \( 2n \) vertices and \( n^2 \) edges.

Let \( F_m = \{(v_i, v_j) / 1 \leq i \leq n, j \equiv (i+m) \mod n, k \equiv (i+(m+1)) \mod n\}, e_{01} = e_{n1} \) and \( F = \bigcup_{m=1}^{n-1} F_m \). \( B_1(C_n) = E(2C_n) \cup F \cup E(B_1(C_n)) \). Let \( D' = \bigcup_{i=1}^{n-2} (\{v_i, e_{i,i+1}, e_{i,i+1,i+2}\}) \cup \{(v_1, e_{12})\} \). Then \( D' \) is a connected edge dominating set of \( B_1(C_n) \). Also, \( <D'> \cong \mathbb{P}_{n-1}^+ \). Therefore, \( D' \) is a connected edge dominating set of \( B_1(C_n) \).
\[ \gamma_c'(B_2(C_n)) \leq |D'| = 2(n-2) + 1 = 2n - 3. \]

Let \( D'' \) be a minimum connected edge dominating set of \( B_1(C_n) \). \( D'' \) contains at least \( (n-1) \) edges of \( F \) and \( (n-2) \) edges of \( L(C_n) \). \( |D''| \geq 2n-3 \). Therefore, \( \gamma_c'(B_2(C_n)) = 2n-3 \).

**Remark: 2.2**

(i) \( \gamma_c'(B_1(C_3)) = 5 \)

(ii) \( \gamma_c'(B_1(C_4)) = 6 \)

**Theorem 2.3.** For the complete graph \( K_n \) on \( n \geq 5 \) vertices, \( \gamma_c'(B_2(K_n)) = (n+3)(n-2)/2 \).

**Proof:** Let \( v_1, v_2, \ldots, v_n \) be the vertices of \( K_n \) and \( E(K_n) = \{e_{ij} = (v_i,v_j) / 1 \leq i \leq n, 1 \leq j \leq n, i \neq j \} \). \( B_1(K_n) \) has \( n+1 \) vertices.

\( E(B_1(K_n)) = \{|E(K_n)| + |E(L(K_n))| + n(n-1)\} \)

\( (n-2)/2 = n(n-1) (2n-3)/2 \). Let \( F = \bigcup_{i=1}^{n-1} (e_1,e_{i+1}) \), \( F = \bigcup_{i=1}^{n-2} (v_i,v_{i+1}) \)

... \( F_{n-3} = \bigcup_{i=1}^{n-3} (v_{n-3},v_{n-2}) \), \( F_{n-2} = \bigcup_{i=1}^{n-2} (v_{n-2},v_{n-1}) \), \( F_{n-1} = \bigcup_{i=1}^{n-1} (v_1,v_n) \)

and let \( F = \bigcup_{i=1}^{n-1} (v_i,v_{i+1}) \). Then \( F \subseteq E(B_1(K_n)) \), \( F \) is a dominating set of \( B_1(K_n) \). Let \( P_n \) be the path induced by the vertices \( v_1, v_2, \ldots, v_n \). \( F \) is a graph obtained by attaching \( n-2, n-3, n-4, \ldots, 2 \) and \( n-2 \) pendant edges at \( v_1, v_2, \ldots, v_{n-3}, v_{n-2} \) of \( P_n \), respectively. Therefore, \( F \) is a connected edge dominating set of \( B_1(K_n) \) and hence,

\( \gamma_c'(B_2(K_n)) \leq |F| = |F_{n-1}| - F = (n-2) + (n-3) + \ldots + 2 + (n-2) + (n-1) n / 2 - 1 + n = (n^2 - n - 2 + 2n - 4) / 2 = (n^2 + n - 6) / 2 = (n+3)(n-2) / 2 \). \( F \) is also a minimum connected edge dominating set of \( B_1(K_n) \) and hence \( \gamma_c'(B_1(K_n)) = (n+3)(n-2) / 2 \).

**Remark: 2.3**

(i) \( \gamma_c'(B_1(K_5)) = 5 \)

(ii) \( \gamma_c'(B_1(K_4)) = 6 \)

**Theorem 2.4.** For the star \( K_{1,n} \) on \( n+1 \) vertices \( n \geq 4 \), \( \gamma_c'(B_2(K_{1,n})) = n+1 \).

**Proof:** Let \( v_1, v_2, \ldots, v_n \) be the vertices of \( K_{1,n} \) with \( v \) as the central vertex. \( E(K_{1,n}) = \{e_{i+1} = (v,v_{i+1}) / 1 \leq i \leq n \} \).

\( B_1(K_{1,n}) \) has \( 2n+1 \) vertices and \( 3(n-1)/2 \) edges.

Let \( D' = \{ \bigcup_{i=1}^{n-1} (e_i,e_{i+1}) \} \cup \{ (v,v_i), (v_1,v_i) \} \). Then \( |D'| = n+1 \). The edge \( (v,v_1) \) in \( D' \) dominates all the edges of \( G \) and the edges \( U^{n-1}_{i=1} (e_i,e_{i+1}) \), \( (v_1,v_n) \) dominate remaining edges of \( K_{1,n} \) and \( D' \) contains \( n+1 \) edges of \( K_{1,n} \) and hence \( \gamma_c'(B_2(K_{1,n})) \leq |D'| = n+1 \). Let \( D'' \) be a connected edge dominating set of \( K_{1,n} \). To dominate \( K_{1,n} \), \( D'' \) contains one edge of \( K_{1,n} \) and to dominate \( n(n-1) \) edges of the form \( (v_i,e_j) \) \( (e_i \neq (v_i,v_{i+1}) \). \( D'' \) contains at least \( (n-1) \) edges. Since \( D'' \) is connected, \( D'' \) contains one more edge and hence \( |D'| \geq n+1 \). Therefore, \( \gamma_c'(B_2(K_{1,n})) = n+1 \).

**Theorem 2.5:** For the Wheel \( W_n \) on \( n \) vertices \( n \geq 5 \), \( \gamma_c'(B_1(W_n)) = 3n-5 \).

**Proof:** Let \( v_1, v_2, \ldots, v_n \) be the vertices of \( W_n \) with \( v_1 \) as the central vertex and \( e_{i+1} = (v_{i+1}, v_i) / 1 \leq i \leq n \). Then \( e_{i+1} = (v_{i+1}, v_i) / 2, 3, \ldots, n \). \( B_1(W_n) \) has \( 2n-1 \) vertices and \( (n-1)(3n-4)/2 \) edges.

Let \( F_1 = \bigcup_{i=1}^{n-1} (v_i,v_{i+1}), F_2 = \bigcup_{i=1}^{n-2} (v_i,e_{i+1,i+2}) \)
\[
F_3 = \bigcup_{i=2}^{n-2} \{ (e_{i+1}, e_{i}) \} \cup \{ e_{n-1}, e_{2n} \}
\]

Let \( D' = F_3 \cup F_3 \cup F_3 \) and \( D' \) dominates all the edges of \( W_n \) and edges of the form \((v_i, e_{i+1})\) where \( e_{i+1} \) is not incident with \( v_i \). \( F_2 \cup F_3 \) dominates all the edges of \( L(W_n) \). Therefore, \( D' \) is a edge dominating set of \( B_1(W_n) \). \(|D'| \leq n-1+n-2+n-2 = 3n-5\). <D'> is a graph obtained from \( P_{n-2} \) by subdividing each pendant edge and then attaching a path of length 2 at a pendant vertex of \( P_{n-2} \). \( D' \) is a connected edge dominating set of \( B_1(W_n) \).

Let \( D'' \) be a minimum connected edge dominating set of \( B_1(W_n) \). To dominate edges of \( W_n \) and edges of the form \((v_i, e_{i+1})\) and to maintain connectedness of <D’>, \( D'' \) contains at least \((n-1)\) edges of \( W_{n-2} \) and \((n-2)\) edges of the form \((v_i, e_{i+1})\) and \((n-2)\) edges of \( L(W_n) \).

Therefore, \(|D'| \geq 3n-5\). Hence, \( \gamma_c(B_1(W_n)) = 3n-5 \).

**Remark:** Since every connected edge dominating set is also an edge dominating set of a graph \( G, \gamma'(B_1(G)) \leq \gamma_c(B_1(G)) \)

**Remark:** Any connected edge dominating set is also a total edge dominating set and hence \( \gamma'_t(B_1(G)) \leq \gamma_c(B_1(G)) \).

### 3. Total edge domination in \( B(G, L(G), \text{NINC}) \) of a Graph

In the following total edge domination number of \( B_1(P_n), B_1(C_n), B_1(K_{1,n})B_1(W_n) \) are found.

**Theorem:** For the path \( P_n \) on \( n \geq 4 \) vertices, \( \gamma'_t(B_1(P_n)) \leq n \).

Proof: Let \( v_1, v_2, \ldots, v_n \) be the vertices and \( e_{i+1} = (v_i, v_{i+1}) \) \((i = 1, 2, \ldots, n-1)\) be the edges of \( P_n \). Then \( v_1, v_2, \ldots, v_n, e_{12}, e_{23}, \ldots, e_{n-1n} \in V(B_1(P_n)). B_1(P_n) \) has \( 2n-1 \) vertices and \( n^2 - n - 1 \) edges.

Case (i): \( n \) is even

Let \( D' = \bigcup_{i=1}^{n/2} \{ (v_{2i-1}, v_{2i}) \} \) and \( D'' = \bigcup_{i=1}^{n/2} \{ (v_{2i+1}, e_{2i-1,2i}) \} \) and \( D = D' \cup D'' \) \((v_i, e_{n-2n-1})\)}

Then \( D \subseteq E(B_1(P_n)) \) and \(|D| = \frac{n}{2} + \frac{n-2}{2} + 1 = n \). \( D \) is an edge dominating set of \( B_1(P_n) \) and \( \gamma'(B_1(P_n)) \leq n \).

Therefore, \( D \) is a total edge dominating set of \( B_1(P_n) \) and hence \( \gamma'_t(B_1(P_n)) \leq |D| = n \).

Case (ii): \( n \) is odd

Let \( D' = \bigcup_{i=1}^{n-1/2} \{ (v_{2i-1}, v_{2i}) \} \) and \( D'' = \bigcup_{i=1}^{n-3/2} \{ (v_{2i+1}, e_{2i-1,2i}) \} \)

and let \( F = F' \cup F'' \cup \{ (v_{n+1}, v_n) \} \) then \( F \subseteq E(B_1(P_n)) \) and \(|F| = \frac{n-1}{2} + \frac{n-3}{2} + 2 = n \). \( F \) is an edge dominating set of \( B_1(P_n) \) and \( \gamma'(B_1(P_n)) \leq |F| = n \).

**International Journal of Engineering, Science and Mathematics**

http://www.ijesm.co.in, Email: ijesmj@gmail.com
(1) Let \( V(P_8) = \{v_1, v_2, ..., v_8\} \) and \( E(P_8) = \{(v_i, v_{i+1}) \mid i = 1, 2, ..., 7\} \).

Then \( D = \{(v_1, v_2) \cup (v_3, v_4) \cup (v_5, v_6) \cup (v_7, v_8) \cup (v_1, e_{67}) \cup (v_3, e_{12}) \cup (v_5, e_{34}) \cup (v_7, e_{56})\} \) is an edge dominating set of \( B_n \) and \( D \subseteq E(B_1(P_8)) \) and \( <D> \geq 4 \). \( D \) is a total edge dominating set of \( B_1(P_8) \). Therefore, \( \gamma'_t(B_1(P_8)) \leq 8 \).

(2) Let \( V(P_7) = \{v_1, v_2, ..., v_7\} \) and \( E(P_7) = \{(v_i, v_{i+1}) \mid i = 1, 2, ..., 6\} \).

Then \( D = \{(v_1, v_2) \cup (v_3, v_4) \cup (v_5, v_6) \cup (v_7, v_8) \cup (v_1, e_{67}) \cup (v_3, e_{12}) \cup (v_5, e_{34}) \cup (v_7, e_{56})\} \) is an edge domination set of \( B_1(P_7) \) and \( D \subseteq E(B_1(P_7)) \) and \( <D> \geq 2 \). \( D \) is a total edge dominating set of \( B_1(P_7) \). Therefore, \( \gamma'_t(B_1(P_7)) \leq 7 \).

**Theorem 3.2** For the cycle \( C_n \) on \( n (n \geq 3) \) vertices, \( \gamma'_t(B_1(C_n)) \leq n \), if \( n \) is even
\[ \leq n + 1, \text{if } n \text{ is odd} \]

Proof: Let \( v_1, v_2, ..., v_n \) be the vertices and \( e_{i} = (v_i, v_{i+1}) \) \( (i = 1, 2, ..., n-1) \) and \( e_n = (v_n, v_1) \) be the edges of \( C_n \). Then \( v_1, v_2, ..., v_n, e_{12}, e_{23}, ..., e_{n-1,n} \in V(B_1(C_n)) \). \( B_1(C_n) \) has \( 2n \) vertices and \( n^2 \) edges.

Case (i): \( n \) is even

Let \( D' = \bigcup_{i=1}^{n/2} \{(v_{2i-1}, v_{2i})\} \) and \( D'' = \bigcup_{i=1}^{n-2} \{(v_{2i-1} + 1, v_{2i} - 1)\} \) and \( D = D' \cup D'' \). Then \( D \subseteq E(B_1(C_n)) \) and \( |D| = \frac{n}{2} + \frac{n-2}{2} + 1 = n \). \( D \) is an edge dominating set of \( B_1(C_n) \) and with central vertices \( v_1, v_2, ..., v_{n/2} \) respectively.

Therefore, \( D \) is a total edge dominating set of \( B_1(C_n) \) and hence \( \gamma'_t(B_1(C_n)) \leq |D| = n \).

Case (ii): \( n \) is odd

Let \( F' = \bigcup_{i=1}^{n/2} \{(v_{2i-1}, v_{2i})\} \)

\[ F'' = \bigcup_{i=1}^{(n-2)/2} \{(v_{2i-1} + 1, v_{2i} - 1, 2)\} \] and let \( F = F' \cup F'' \cup \{(v_i, e_{n-1,n})\} \) then \( F \subseteq E(B_1(C_n)) \) and \( |F| = \frac{n}{2} + \frac{n-1}{2} + 1 = n + 1 \). \( D \) is an edge dominating set of \( B_1(C_n) \) and \( <D> \geq \frac{n-3}{2} + P_3 \cup P_4 \) where the central vertices of \( P_3 \) and \( P_5 \) are \( v_1, v_2, ..., v_{n-4} \) and \( v_3, v_4 \). \( D \) is induced by the edges \( (v_{n-2}, v_{n-1}) \), \( (v_{n-1}, v_n) \) and \( (v_n, e_{n-2,n-1}) \) and \( (v_{n-2}, e_{n-4,n-3}) \). Therefore, \( D \) is a total edge dominating set of \( B_1(C_n) \) and hence \( \gamma'_t(B_1(C_n)) \leq |D| = n + 1 \).

**Theorem 3.3** For the star \( K_{1,n} \) on \( (n+1) \) vertices (\( n \geq 3 \)), \( \gamma'_t(B_1(K_{1,n})) \leq n + 1 \)

Proof: Let \( v_1, v_2, v_3, ..., v_{n+1} \) be the vertices of \( K_{1,n} \) with \( v_1 \) as the central vertex. Let \( e_{i+1} = (v_i, v_i), i = 2, 3, ..., n+1 \) be the edges of \( K_{1,n} \). Then \( v_1, v_2, ..., v_{n+1}, e_{12}, e_{13}, ..., e_{n+1} \in V(B_1(K_{1,n})) \). \( B_1(K_{1,n}) \) has \( 2n + 1 \) vertices and \( 2n + 1 \) and \( (n(3n-1))/2 \) edges.

Case (i): \( n \) is odd
Let \( D' = \bigcup_{i=3}^{n+3/2} \{(v_i, e_{2i-2}), (e_{2i-2}, e_{2i-1})\} \) where \( e_{1,n+2} = e_{12} \) and let \( D = D' \cup \{(v_1, v_2), (v_2, e_{13})\} \). Then \( D \subseteq E(B_1(K_{n,n}) \) and \( |D| = 2 + \frac{n+2}{2} = n+1 \). \( D \) is an edge dominating set of \( B_1(K_{n,n}) \) and \( <D>D_{\frac{n+1}{2}-2} \) with central vertices \( v_2, e_{14}, e_{16}, ..., e_{1,n} \). Therefore, \( D \) is a total edge dominating set of \( B_1(K_{n,n}) \) and hence \( \gamma'(B_1(K_{n,n})) \leq |D| = n+1 \).

case(ii): \( n \) is even

Let \( F' = \bigcup_{i=1}^{n+2/2} \{(v_1, e_{1,2i-2}), (v_2, e_{1,2i-1})\} \) and \( F = F' \cup \{(v_1, v_2), (v_2, e_{13})\} \) where \( e_{1,n+2} = e_{12} \) and let \( D = D' \cup \{(v_1, v_2), (v_2, e_{13})\} \). Then \( D \subseteq E(B_1(K_{n,n}) \) and \( |D| = 2 + \frac{n+3}{2} = n+1 \). \( D \) is an edge dominating set of \( B_1(K_{n,n}) \) and \( <D>D_{\frac{n+2}{2}-2} \) with central vertices \( v_2, e_{14}, e_{16}, ..., e_{1,n} \). Therefore, \( D \) is a total edge dominating set of \( B_1(K_{n,n}) \) and hence \( \gamma'(B_1(K_{n,n})) \leq |D| = n+1 \).

**Theorem 3.4** For the Wheel \( W_n \) \((n \geq 5)\) on \( n \) vertices, \( \gamma'(B_1(W_n)) \leq 2n - 2 \).

**Proof:** Let \( v_1, v_2, v_3, ..., v_n \) be the vertices of \( W_n \) with \( v_1 \) as the central vertex. Let \( e_{1,i} = (v_i, v_i) \) \((i = 2, 3, ..., n)\) and \( e_{i,i+1} = (v_i, v_{i+1}) \) \((i = 2, 3, ..., n-1)\) be the edges of \( W_n \). Then \( v_1, v_2, v_3, ..., v_n \) are \( e_{12}, e_{13}, e_{14}, ..., e_{n-1,n} \). Therefore, \( D \) is a total edge dominating set of \( B_1(W_n) \) and hence \( \gamma'(B_1(W_n)) \leq |D| = n+1 \).

Case(i): \( n \) is even

Let \( D' = \bigcup_{i=3}^{n+3/2} \{(v_i, e_{2i-2}), (e_{2i-2}, e_{2i-1})\} \) and \( D'' = \bigcup_{i=3}^{n+3/2} \{(v_i, e_{2i-2}), (e_{2i-2}, e_{2i-1})\} \) where \( e_{1,n+2} = e_{12} \) and let \( D = D' \cup D'' \cup \{(v_1, v_2), (v_2, e_{13})\} \). Then \( D \subseteq E(B_1(W_n)) \) and \( |D| = 2 + \frac{n+2}{2} = n+1 \). \( D \) is an edge dominating set of \( B_1(W_n) \) and hence \( \gamma'(B_1(W_n)) \leq |D| = n+1 \).

Case(ii): \( n \) is odd

Let \( F' = \bigcup_{i=3}^{n+3/2} \{(v_i, e_{2i-2}), (e_{2i-2}, e_{2i-1})\} \) and \( F'' = \bigcup_{i=3}^{n+3/2} \{(v_i, e_{2i-2}), (e_{2i-2}, e_{2i-1})\} \) where \( e_{1,n+2} = e_{12} \) and let \( F = F' \cup F'' \cup \{(v_1, v_2), (v_2, e_{13})\} \). Then \( F \) is a total edge dominating set of \( B_1(W_n) \) and hence \( \gamma'(B_1(W_n)) \leq |F| = 2n - 2 \).

**Theorem 3.5** If \( \gamma'(B_1(G)) \leq 2 \beta_1(G) + 2 \alpha_0(L(G)) \).

**Proof:** Let \( K \subseteq E(G) \) be a perfect matching such that \( |K| = k = \beta_1(G) \). Then \( K \subseteq E(B_1(G)) \).
Let \( K = \{(v_1, u_1) \ (v_2, u_2), \ldots, (v_k, u_k)\} \). Let \( M \) be a point cover of \( L(G) \) and let \( |M| = \alpha_0(L(G)) = m = \{e_1, e_2, \ldots, e_m\} \). Case: (i) \( k \geq m \ (\beta_1(G) > \alpha_0) \).

Choose one of \( u_i \) and \( v_i \). Let it be \( v_i \) \((i = 1, 2, \ldots, k)\). Choose a distinct vertex \( e_i \) in \( M \) such that the corresponding edge in \( G \) is not incident with \( v_i \). Then the edge \( (v_i, u_i) \in E(B_1(G)) \). Let \( L \) be the set of all these edges. \( |L| = k \). Then \( L \subseteq E(B_1(G)) \). Let \( D = K \cup L \subseteq E(B_1(G)) \). \( K \) dominates all the edges of \( G \) in \( B_1(G) \) and edges of the form \((w, e)\) where \( e \in E(G) \) is not incident with \( w \in V(G) \). \( L \) dominates all the edges of \( L(G) \). Therefore, \( D \) is an edge dominating set of \( B_1(G) \). Also \( <D> \) contains no isolated edges. Therefore, \( D \) is a total edge dominating set of \( B_1(G) \) and hence \( \gamma'_t(B_1(G)) \leq |D| = |K \cup L| = 2K = 2\beta_1(G) \). 

Case(ii): \( k \leq m \), that is \( \beta_1(G) > \alpha_0(L(G)) \). For each vertex \( e_i \in M \), choose a vertex \( u_i \) (or) \( v_i \), which is not incident with \( e_i \). Then the edge \( (v_i, e_i) \in E(B_1(G)) \). Let \( N \) be the set of all these edges. \( |N| = m \), \( N \subseteq E(B_1(G)) \). Then the set \( D' = K \cup N \) is a total edge dominating set of \( B_1(G) \) as in case(i). Therefore, \( \gamma'_t(B_1(G)) \leq |D'| = |K \cup N| = \beta_1(G) + m = \beta_1(G) + \alpha_0(L(G)) \leq \alpha_0(L(G)) \).

Therefore, \( \gamma'_t(B_1(G)) \leq 2\beta_1(G) \) if \( \beta_1(G) > \alpha_0(L(G)) \).

\[
\leq 2\alpha_0(L(G)) \text{ if } \beta_1(G) \leq \alpha_0(L(G)).
\]

4. CONCLUSION

In this paper, connected edge and total edge domination numbers of Boolean Function Graph \( B(G, L(G), N) \) of paths, cycles, complete graphs, stars, wheels are obtained.

REFERENCE:

[3]. T. N. Janakiraman, S.Muthammai, M.Bhanumathi, On the Boolean Function Graph of a Graph and on its Complement, Mathematica Bohemica, 130(2005), No.2, pp. 113-134.
[4]. T. N. Janakiraman, S. Muthammai, M. Bhanumathi, Domination Numbers on the Boolean Function Graph of a Graph, Mathematica Bohemica, 130(2005), No.2, 135-151.