SOFT SUBSTRUCTURES OF SENSIBLE FUZZY SOFT RIGHT R-SUBGROUPS OF NEAR-RINGS

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Abstract: In this paper, we introduce the notion of S-anti-fuzzy soft right R-subgroups of near-rings and its basic properties are investigated. We also study the homomorphic image and pre image of S-anti-fuzzy soft right R-subgroups. Using S-norm, we introduce the notion on sensible anti-fuzzy soft right R-subgroups in near-rings and some related properties of a near-rings ‘R’ are discussed.

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Section-1: Introduction. The concept of fuzzy subset was introduced by Zadeh [29]. Fuzzy set theory is a useful tool to describe situations in which the data are imprecise or vague. Fuzzy sets handle such situation by attributing a degree to which a certain object belong to a set. Schweizar and Sklar [26] introduce the notions of Triangular norm (t-norm) and Triangular co-norm (S-norm) are the most general families of binary operations that satisfy the requirement of the conjunction and disjunction operators respectively. First, Abuosman[6] introduced the notion of fuzzy subgroup with respect to t-norm. Zaid [1] introduced the concept of R-subgroups of a near-rings and Hokim [17] introduced the concept of fuzzy R-subgroups of a near-ring. Then Zhan [30 ] introduced the properties of fuzzy hyper ideals in hyper near-rings with t-norm. Recently, Cho et. al [11] introduced the notion of fuzzy subalgebras with respect to S-norm of BCK algebras and Akram [7] introduced the notion of sensible fuzzy ideal of [2] and [7]. Soft set theory was introduced in 1999 by Molodtsov [22] for dealing with uncertainties and it has gone through remarkably rapid strides in the mean of algebraic structures as in [1, 2, 11, 14, 15, 16, 18, 25, 28]. Moreover, Atagun and Sezgin [5] defined the concepts of soft sub rings and ideals of a ring, soft subfields of a field and soft sub modules of a module and studied their related properties with respect to soft set operations. Operations of soft sets have been studied by some authors, too. Maji et. al [20] presented some definitions on soft sets and based on the analysis of several operations on soft sets Ali et.al [4] introduced several operations of soft sets and Sezgin and Atagun [25] studied on soft set operations as well. Furthermore, soft set relations and functions [8] and soft mappings [21] with many related concepts were discussed. The theory of soft set has also a wide-ranging applications especially in soft decision making as in the following studies: [6, 7, 23, 29]. In this paper, we will redefine anti-fuzzy soft right R-subgroups of a near-ring ‘R’ with respect to a S-norm and investigate it is related properties. We also study the homomorphic image and pre image of S-anti-fuzzy soft right R-subgroups. Using S-norm, we introduce the notion on sensible anti-fuzzy soft right R-subgroups in near-rings and some related properties of a near-ring ‘R’ are discussed.
2. Preliminaries

A ring ‘S’ is a system consisting of a non-empty set ‘S’ together with two binary operations on ‘S’ called addition and multiplication such that

(i) ‘S’ together with addition is a semi group.

(ii) ‘S’ together with multiplication is a semi group.

(iii) a (b+c) = ab + ac and (a+b)c = ac+bc for all a,b,c ∈ S. A semi ring ‘S’ is said to be additively commutative if a+b = b+a for all a,b ∈ S. A zero element of a semi ring ‘S’ is an element ‘o’ such that o.x = x.o = o and o+x = x+o = x for all x∈ S. By a near-ring we mean a non-empty set ‘R’ with two binary operations ‘+’ and ‘·’.

Satisfies the following axioms

(i) (R,+) is a group.

(ii) (R, ·) is a semi group.

(iii) (b+c)a =ba+ca for all a,b,c ∈ R.

Precisely speaking it is a right near-ring because it satisfies the right distribution law x·y. Note that xo= o and x(y) = -(xy) but in general ox ≠ o for some x∈ R. A two sided R- subgroups in a near- ring ‘R’ is a subset ‘N’of ‘R’ such that

(i) (N, +) is a subgroup of (R,+).

(ii) RN ⊂ N

(iii) NR ⊂ N

If ‘N’ satisfies (i) and (ii) then it is called a right ‘R’ subgroup of ‘R’, we now review some fuzzy logic concepts.A fuzzy set ‘μ’ in a set ‘R’ is a function μ: R→ [0,1]. Let Im(μ) denote the image set of μ. Let ‘μ’ be a fuzzy set in ‘R’. For all x,y ∈ R, let ‘μ’ be a fuzzy set in ‘R’. For x ∈ [0,1], the set L(μ: α ) = { x ∈ R / μ(x) ≤ α } is called a lower level subset of ‘μ’.

Let ‘R’ be a near-ring and let ‘μ’ be a fuzzy set in ‘R’. we say that ‘μ’ is a fuzzy near-ring of ‘R’ if , for all x,y ∈ R,

(FS1) μ (x·y) ≥ min { μ(x) , μ(y) }

(FS2) μ (x·y) ≥ min { μ(x) , μ(y) }. If a fuzzy set ‘μ’ in a near-ring ‘R’ satisfies the property (FS1) then μ(0) ≥ μ(x) for all x ∈ R.

2.1 Definition[22]: A pair (F,A) is called a soft set over U, where F is a mapping given by  F : A→P(U).

In other words, a soft set over U is a parameterized family of subsets of the universe U.

Note that a soft set (F, A) can be denoted by F_A. In this case, when we define more than one soft set in some subsets A, B, C of parameters E, the soft sets will be denoted by F_A, F_B, F_C, respectively. On the other case, when we define more than one soft set in a subset A of the set of parameters E, the soft sets will be denoted by F_A,G_A, H_A, respectively. For more details, we refer to [11,17,18,26,29,7].

2.2 Definition[6]: The relative complement of the soft set F_A over U is denoted by F'_A, where F'_A : A → P(U) is a mapping given as F'_A(a) = U \ F_A(a), for all a ∈ A.

2.3 Definition[6]: Let F_A and G_B be two soft sets over U such that A ∩ B ≠ ∅. The restricted intersection of F_A and G_B is denoted by F_A ∩ G_B and is defined as F_A ∩ G_B =(H,C), where C = A∩B and for all c ∈ C, H(c) = F(c)∩G(c).

2.4 Definition[6]: Let F_A and G_B be two soft sets over U such that A ∩ B ≠ ∅. The restricted union of F_A and G_B is denoted by F_A U_R G_B and is defined as F_A U_R G_B = (H,C),where C = A∪B and for all c ∈ C, H(c) = F(c)∪G(c).
2.5 Definition[12]: Let $F_A$ and $G_B$ be soft sets over the common universe $U$ and $\psi$ be a function from $A$ to $B$. Then we can define the soft set $\psi(F_A)$ over $U$, where $\psi(F_A) : B \rightarrow P(U)$ is a set valued function defined by $\psi(F_A)(b) = \bigcup \{ F(a) \mid a \in A$ and $\psi(a) = b \}$,

If $\psi^{-1}(b) \neq \emptyset$ for all $b \in B$. Here, $\psi(F_A)$ is called the soft image of $F_A$ under $\psi$. Moreover we can define a soft set $\psi^{-1}(G_B)$ over $U$, where $\psi^{-1}(G_B) : A \rightarrow P(U)$ is a set-valued function defined by $\psi^{-1}(G_B)(a) = \bigcap \{ F(a) \mid a \in A$ and $\psi(a) = b \}$, if $\psi^{-1}(b) \neq \emptyset$

$= 0$ otherwise for all $b \in B$. Here, $\psi^{-1}(F_A)$ is called the soft anti image of $F_A$ under $\psi$.

2.6 Definition[13]: Let $F_A$ and $G_B$ be soft sets over the common universe $U$ and $\psi$ be a function from $A$ to $B$. Then we can define the soft set $\psi^{-1}(F_A)$ over $U$, where $\psi^{-1}(F_A) : B \rightarrow P(U)$ is a set-valued function defined by $\psi^{-1}(F_A)(b) = \{ F(a) \mid a \in A$ and $\psi(a) = b \}$, if $\psi^{-1}(b) \neq \emptyset$

Replacing 0 by 1 in condition ‘S1’ we obtain the concept of t- norm ‘T’.

2.7 Definition: By a s- norm ‘S’, we mean a function $S: [0,1] \rightarrow [0,1]$ satisfying the following conditions :

(S1) $S(x,0) = x$

(S2) $S(x,y) \leq S(x,z) \leq S(y,z)$ for all $x,y,z \in [0,1]$.

(S3) $S(x,y) = S(y,x)$

(S4) $S(x,S(y,z)) = S(S(x,y),z)$, for all $x,y,z \in [0,1]$.

2.8 Definition: Let $\alpha$ be a soft set over $U$ and $F$ be a function from $A$ to $B$. Then, upper $\alpha$-inclusion of a soft set $f_A$, denoted by $f^\alpha A$, is defined as $f^\alpha A = \{ x \in A : f_A(x) \supseteq \alpha \}$

2.9 Proposition: For a S-norm $S$, then the following statement holds $S(x,y) \geq \max\{x,y\}$, for all $x,y \in [0,1]$.

2.10 Definition: Let ‘S’ be a s-norm. A fuzzy soft set ‘$\mu$’ in ‘R’ is said to be sensible with respect to ‘S’ if $\text{Im}(\mu) \subseteq \Delta_S$, where $\Delta_S = \{ s(\alpha, \alpha) = \alpha / \alpha \in [0,1]\}$. 

2.11 Definition: Let $(R,+)$ be a near-ring. A fuzzy soft set ‘$\mu$’ in ‘$R$’ is called an anti fuzzy right (resp. left) $R$- subgroup of ‘$R$’ if

(AF1) $\mu(x-y) \leq \max\{ \mu(x), \mu(y) \}$, for all $x,y \in R$.

(AF2) $\mu(xr) \leq \mu(x)$ for all $r,x \in R$.

2.12 Definition: Let $(R,+)$ be a near-ring. A fuzzy soft set ‘$\mu$’ in $R$ is called a fuzzy soft right (resp. left) $R$- subgroup of ‘$R$’ if

(FR1) ‘$\mu$’ is a fuzzy subgroup of $(R,+)$.

(FR2) $\mu(xr) \geq \mu(x)$ (resp. $\mu(rx) \geq \mu(x)$), for all $r,x \in R$.

2.13 Definition: Let ‘S’ be a s-norm. A function $\mu : R \rightarrow [0,1]$ is called a fuzzy soft right (resp. left) $R$- subgroup of ‘R’ with respect to ‘S’ if

(C1) $\mu(x-y) \leq S(\mu(x), \mu(y))$

(C2) $\mu(xr) \leq \mu(x)$ (resp. $\mu(rx) \leq \mu(x)$) for all $r,x \in R$. If a fuzzy soft $R$-subgroup ‘$\mu$’ of R with respect to ‘S’ is sensible, we say that ‘$\mu$’ is a sensible fuzzy soft $R$-subgroup of R with respect to ‘S’.

2.14 Example: Let ‘$K$’ be the set of natural numbers including ‘0’ and ‘$K$’ is a $R$-subgroup with usual addition and multiplication.
2.15 Proposition: Define a fuzzy subset $\mu: \mathbb{R} \rightarrow [0,1]$ by

$$
\mu(x) = \begin{cases} 
0 & \text{if } x \text{ is even} \\
1 & \text{otherwise}.
\end{cases}
$$

And let $S_m: [0,1] \rightarrow [0,1]$ be a function defined by $S_m(\alpha, \beta) = \min \{x+y, 1\}$ for all $x,y \in [0,1]$. Then $S_m$ is a t-norm. By routine calculation, we know that $\mu$ is sensible $R$-fuzzy soft subgroup of $R$.

**SECTION-3: PROPERTIES OF ANTI-FUZZY SOFT R SUBGROUPS.**

3.1 Proposition: Let ‘$S$’ be a s-norm. Then every sensible $S$-anti fuzzy soft right $R$-subgroups ‘$\mu$’ of $R$ is an anti-fuzzy soft $R$-subgroups of $R$.

Proof: Assume that ‘$\mu$’ is a sensible $S$-anti fuzzy soft right $R$-subgroups of $R$, then we have (AF1) $\mu(x-y) \leq S(\mu(x), \mu(y))$ and (AF2) $\mu(xr) \leq \mu(x)$ for all $x,y \in S$.

Since ‘$\mu$’ is sensible, we have

$$
\begin{align*}
\max \{ \mu(x) , \mu(y) \} &= S( \min \{ \mu(x), \mu(y) \} , \min \{ \mu(x), \mu(y) \} ) \\
&\geq S(\mu(x), \mu(y)) \\
&\geq \max \{ \mu(x) , \mu(y) \}
\end{align*}
$$

and so $S(\mu(x), \mu(y)) = \max \{ \mu(x) , \mu(y) \}$. It follows that

$$
\mu(x-y) \leq S(\mu(x), \mu(y)) = \max \{ \mu(x), \mu(y) \} \text{ for all } x,y \in R.
$$

clearly $\mu(xr) \leq \mu(x)$ for all $r,x \in R$. so ‘$\mu$’ is an anti-fuzzy soft $R$-subgroups of $R$.

3.2 Proposition: If ‘$\mu$’ is a $S$-anti fuzzy soft right $R$-subgroups of a near ring $R$ and ‘$\theta$’ is an endomorphism of $R$, then $\mu[\theta]$ is a $S$-anti fuzzy soft right $R$-subgroups of $R$.

Proof: For any $x,y \in R$, we have

$$
\begin{align*}
(i) \quad \mu[\theta](x-y) &= \mu(\theta)(x-y) \\
&= \mu(\theta)(x - \theta(y)) \\
&\leq S(\mu[\theta](x) , \mu[\theta](y))
\end{align*}
$$

$$
\begin{align*}
(ii) \quad \mu[\theta](xr) &= \mu(\theta)(xr) \\
&= \mu(\theta(x) r) \\
&\leq \mu(\theta(x))
\end{align*}
$$

3.3 Definition: Let ‘$f$’ be a mapping defined on $R$. If ‘$\psi$’ is a fuzzy soft subset in $f(R)$, then the fuzzy soft subset $\mu = \psi$ in $R$ (ie) $\mu(x) = \psi(f(x))$ for all $x \in R$ is called the pre-image of ‘$\psi$’ under ‘$f$’.

3.4 Proposition: If ‘$\mu$’ is a $S$-anti fuzzy soft right $R$-subgroups of a near ring $R$, then ‘$\mu$’ is a $S$-anti fuzzy soft right $R$-subgroups of $R$.

Proof: Let $f: R \rightarrow R'$ be an onto homomorphism of near-ring and let ‘$\psi$’ be an $S$-anti fuzzy soft right $R$-subgroups of $R$ and ‘$\mu$’ the pre-image of ‘$\psi$’ under ‘$f$’. Then we have
(i) $\mu(x - y) = \psi(f(x) - f(y))$
\[\leq S(\psi(f(x)), \psi(f(y))) = S(\mu(x), \mu(y))\]

(ii) $\mu(xr) = \psi(f(xr)) = \psi(f(x)r) \leq \psi(f(x)) = \mu(x)$. Hence ‘$\mu$’ is a $S$-anti fuzzy soft right $R$-subgroups of $R$.

### 3.5 Proposition

An onto homomorphic image of a anti fuzzy soft right $R$-subgroups with the inf property is a anti-fuzzy soft right $R$- subgroups of $R$.

**Proof:** Let $f: R \to R^1$ be an onto homomorphism of near-ring and let ‘$\mu$’ be an $S$-anti fuzzy soft right $R$-subgroup of $R$ with inf property. Given $x, y \in R$, we let $x_o \in f^{-1}(x^1)$ and $y_o \in f^{-1}(y^1)$ be such that $\mu(x_o) = \inf \mu(h)$ and $\mu(y_o) = \inf \mu(h)$ respectively. Then we can deduce that

\[
\mu(f(x^1 - y^1)) = \inf \mu(z) \leq \max \{ \mu(x_o), \mu(y_o) \} = \max \{ \inf \mu(h), \inf \mu(h) \}
\]

\[
\mu(f(x) r) = \inf \mu(z) \leq \mu(y_o)
\]

\[
\mu(f(x) r) = \inf \mu(h) = \mu(y^1)
\]

Hence $\mu'$ is anti fuzzy soft right $R$-subgroups of $R$.

The above proposition can be further strengthened, we first give the following definition.

### 3.6 Definition

A $S$-norm $S$ on $[0,1]$ is called a continuous function from $[0,1] \times [0,1] \to [0,1]$ with respect to the usual topology. We observe that the function ‘max’ is always a continuous $S$-norm.

### 3.7 Proposition

Let $f: R \to R^1$ be a homomorphism of near-rings. If ‘$\mu$’ is a $S$-anti fuzzy soft right $R$-subgroups of $R^1$, then $\mu'$ is $S$-anti fuzzy soft right $R$-subgroup of $R$.

**Proof:** suppose ‘$\mu$’ is a $S$-anti fuzzy soft right $R$-subgroups of $R^1$, then
(i) for all $x, y \in R$, we have

$$
\mu^f(x - y) = \mu(f(x) - f(y))
\leq S(\mu(f(x)), \mu(f(y)))
\leq S(\mu(f(x)), \mu^f(y))
$$

(ii) for all $x, y \in R$, we have

$$
\mu^i(xr) = \mu(f(xr)) = \mu(\mu^i(f(x)), r)
\leq \mu(f(x))
\leq \mu(f(x))
$$

Hence $\mu^i$ is a $S$-anti fuzzy soft right $R$-subgroup of $R$.

### 3.8 Proposition

Let $f : R \to R^1$ be a homomorphism of near-rings. If $\mu^i$ is a $S$-anti fuzzy soft right $R$-subgroups of $R$, then $\mu$ is a $S$-anti fuzzy soft right $R$-subgroup of $R^1$.

**Proof:** Let $x^1, y^1 \in R^1$. There exist $x, y \in R$, such that $f(x) = x^1$ and $f(y) = y^1$.

We have (i) $\mu(x^1 - y^1) = \mu(f(x) - f(y))$

$$
= \mu(f(x) - f(y))
= \mu^i(x - y)
\leq S(\mu(f(x)), \mu(f(y)))
= S(\mu(f(x)), \mu(f(y)))
\leq \mu^i(x)
$$

Hence $\mu$ is a $S$-anti fuzzy soft right $R$-subgroup of $R^1$.

### 3.9 Proposition

Let $S'$ be a continuous $S$-norm and let $f$ be a homomorphism on a near-ring $R$. If $\mu^i$ is a $S$-anti fuzzy soft right $R$-subgroups of $R$, then $\mu$ is a $S$-anti fuzzy soft right $R$-subgroups of $f(R)$.

**Proof:** Let $A_1 = f^{-1}(y_1)$, $A_2 = f^{-1}(y_2)$ and $A_{12} = f^{-1}(y_1 - y_2)$, where $y_1, y_2 \in f(R)$. Consider the set

$$
A_1 - A_2 = \{ x \in R / x = a_1 - a_2 \text{ for some } a_1 \in A_1, a_2 \in A_2 \}.
$$

If $x \in A_1 - A_2$, then $x = x_1 - x_2$ for some $x_1 \in A_1$ and $x_2 \in A_2$, so that we have $f(x) = f(x_1 - x_2) = f(x_1) - f(x_2) = y_1 - y_2$ (ie) $x \in f^{-1}(y_1 - y_2) = A_{12}$. We have $A_1 - A_2 \subseteq A_{12}$.

It follows that

$$
\mu(f(y_1 - y_2)) = \inf \{ \mu(x) / x \in f^{-1}(x_1 - x_2) \}
= \inf \{ \mu(x) / x \in A_{12} \}
\leq \inf \{ \mu(x) / x \in A_1 - A_2 \}
\leq \inf \{ \mu(x_1 - x_2) / x_1 \in A_1, x_2 \in A_2 \}
\leq \inf \{ S(\mu(x_1), \mu(x_2)) / x_1 \in A_1, x_2 \in A_2 \}
$$
Since ‘S’ is continuous for every $\varepsilon > 0$, we see that if $\inf \{ (\mu(x_1)/ x_1 \in A_1) - x_1^* \leq \delta \}$ then $S(\inf \{ (\mu(x_1)/ x_1 \in A_1) \}, \inf \{ (\mu(x_2)/ x_2 \in A_2) \}) - S(x_1^*, x_2^*) \leq \varepsilon$. Choose $a_1 \in A_1$, and $a_2 \in A_2$ such that

$$\inf \{ (\mu(x_1)/ x_1 \in A_1) \} - \mu(a_1) \leq \delta \quad \text{and}$$

$$\inf \{ (\mu(x_2)/ x_2 \in A_2) \} - \mu(a_2) \leq \delta \quad \text{then}$$

$$S(\inf \{ (\mu(x_1)/ x_1 \in A_1) \}, \inf \{ (\mu(x_2)/ x_2 \in A_2) \}) - S(\mu(a_1), \mu(a_2)) \leq \varepsilon.$$

Thus we have

(i) $\mu^*(y_1-y_2) \leq \inf \{ S(\mu(x_1), \mu(x_2)/ x_1 \in A_1, \ x_2 \in A_2) \}$

$$= S(\inf \{ (\mu(x_1)/ x_1 \in A_1) \} \}, \inf \{ (\mu(x_2)/ x_2 \in A_2) \})$$

$$= S(\mu^*(y_1), \mu^*(y_2)).$$

(ii) Similarly, we can prove that $\mu^*(x) \leq \mu^*(x)$. Hence $\mu^*$ is a S- anti fuzzy right R-subgroups of f(R).

3.10 Lemma: Let ‘T’ be a t-norm. Then t-co-norm ‘S’ can be defined as $S(x,y) = 1 - T(1-x, 1-y)$.

Proof: Straight forward.

3.11 Proposition: If a fuzzy soft subset ‘$\mu$’ of R is a T-anti fuzzy soft right R-subgroup of R, then ‘$\mu^*$’ is ‘S’ anti fuzzy soft right R-subgroup of R.

Proof: Let ‘$\mu$’ be a ‘T’-anti fuzzy soft right R-subgroup of R. For all $x,y \in R$, we have

(i) $\mu^*(x-y) = 1 - \mu(x-y)$

$$\leq 1 - T(\mu(x), \mu(y))$$

$$= 1 - T(1 - \mu^*(x), 1 - \mu^*(y))$$

$$= S(\mu^*(x), \mu^*(y)).$$

(ii) $\mu^*(x) = 1 - \mu(x)$

$$\leq 1 - \mu(x) = \mu^*(x).$$

Hence $\mu^*$ is ‘S’ anti fuzzy soft right R-subgroup of R.

SECTION-4 SOFT STRUCTURES OF ANTI FUZZY RIGHT R-SUBGROUPS.

4.1 Definition: A fuzzy soft relation on any set ‘X’ is a fuzzy soft set $\mu: X \times X \to [0,1]$.

4.2 Definition: Let ‘S’ be a s-norm. If ‘$\mu$’ is a fuzzy soft relation on a set ‘R’ and ‘$\chi$’ be fuzzy soft set in R, Then ‘$\mu$’ is a S-fuzzy soft relation on ‘$\chi$’ if $\mu(x,y) \geq S(\chi(x), \chi(y))$ for all $x,y \in R$.

4.3 Definition: Let ‘S’ be a s-norm. Let ‘$\mu$’ and ‘$\chi$’ be a fuzzy soft subset of R. Then direct S-product of ‘$\mu$’ and ‘$\chi$’ is defined as

$$(\mu \times \chi)(x,y) = S(\mu(x), \chi(y)), \text{ for all } x,y \in R.$$
4.4 Lemma: Let ‘S’ be a s- norm. let ‘μ’ and ‘χ’ be a fuzzy soft set of R, then

(i) \( μ \times χ \) is a S-fuzzy soft relation on S.
(ii) \( L(μ \times χ; t) = L(μ; t) \times L(χ; t) \) for all \( t \in [0,1] \).

Proof: obvious.

4.5 Definition: Let ‘S’ be a s- norm . let ‘μ’ be a fuzzy soft subset of R , then μ is called strongest S- fuzzy soft relation on R if

\[ μ(x,y) ≥ S( χ(x), χ(y)) \] for all \( x,y \) in R.

4.6 Proposition : Let ‘S’ be a s- norm and let ‘μ’and ‘χ’ be a S- anti fuzzy soft right R- subgroup of R. Then \( μ \times χ \) is also anti fuzzy soft right R- subgroup of R.

Proof:

(i) \( (μ \times χ)(x-y) = (μ \times χ) ((x_1,x_2) - (y_1,y_2)) \)

\[ = (μ \times χ) ( (x_1-y_1), (x_2-y_2) ) \]

\[ = S(μ(x_1-y_1), χ(x_2-y_2)) \]

\[ ≤ S(S(μ(x_1), μ(y_1)), S(χ(x_2), χ(y_2)) \]

\[ = S(S(μ(x_1), χ(x_2)), S(μ(y_1), χ(y_2)) \]

\[ = S((μ \times χ)(x_1,x_2), (μ \times χ)(y_1,y_2)) \]

\[ = S( μ \times χ)(x), (μ \times χ)(y) \]

(ii) \( (μ \times χ)(x_r) = (μ \times χ) ((x_1,x_2)(r_1,r_2)) \)

\[ = (μ \times χ) ( (x_1r_1, x_2r_2) ) \]

\[ = S(μ(x_1), χ(x_2)) \]

\[ = (μ \times χ)(x) \]

Hence \( μ \times χ \) is also anti fuzzy soft right R- subgroup of R.

4.7 Proposition : Let ‘ μ’ and ‘χ’ be sensible S- anti fuzzy soft right R- subgroup of a near- ring R. Then \( μ \times χ \) is a sensible S- anti fuzzy soft right R- subgroup of R×R.

Proof: By proposition 4.6, we have \( μ \times χ \) is S- anti fuzzy soft right R- subgroup of R×R.

let x = \((x_1,x_2)\) be any element of S×S, then

\[ S((μ \times χ)(x), (μ \times χ)(x)) = S((μ \times χ)(x_1,x_2), (μ \times χ)(y_1,y_2)) \]

\[ = S(S(μ(x_1), χ(x_2)), S(μ(x_1), χ(x_2)) \]

\[ = S(S(μ(x_1), μ(x_1)), S(χ(x_2), χ(x_2)) \]

\[ = S(μ(x_1), χ(x_2)) \]

\[ = (μ \times χ)(x). \]
Hence $\mu \times \chi$ is a sensible $\mathbf{S}$-anti fuzzy soft right $\mathbf{R}$-subgroup of $\mathbf{R} \times \mathbf{R}$

**4.8 Remark :** If $\mu \times \chi$ is a sensible $\mathbf{S}$-anti fuzzy soft right $\mathbf{R}$-subgroup of $\mathbf{R} \times \mathbf{R}$, Then $\mu \times \chi$ need not be sensible $\mathbf{S}$-anti fuzzy soft right $\mathbf{R}$-subgroup of $\mathbf{R}$.

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