(1,2)*-\(g_p\) Continuous Function in Bitopological Spaces

B. Meera Devi *
P. Subbulakshmi **
D.K. Nathan ***

Abstract

The aim of this paper is to introduce a new class of functions called (1,2)*-\(g_p\) continuous functions. We obtain several characterization and some their properties. Also we investigate its relationship with other types of functions in bitopological spaces. Further we introduce and study a new class of functions namely (1,2)*-\(g_p\) irresolute functions.

Keywords:
(1,2)*-\(g_p\) closed sets,
(1,2)*-\(g_p\) open sets,
(1,2)*-\(g_p\) Continuous and (1,2)*-\(g_p\) irresolute.

Author correspondence:
* Assistant Professor,
Department of Mathematics,
Sri S.R.N.M.College
Sattur-626 203, Tamil Nadu, India.

1. Introduction

In 1963, Kelley [3] initiated the study of bitopological spaces. A nonempty set \(X\) equipped with two topological spaces \(\tau_1\) and \(\tau_2\) is called a bitopological spaces and is denoted by \((X, \tau_1, \tau_2)\). M. Lellis Thivagar and O.Ravi [6] introduced a new type of generalized sets called (1,2)*- semi generalized closed sets and a new class of generalized functions called (1,2)*- semi generalized continuous maps in 2006. S.S. Benchalli and J.B.Toranagatti [1] introduced delta generalized pre-closed sets in topological space. The purpose of this present paper is to define a new class of generalized continuous function called (1,2)*-\(g_p\) continuous and investigate their relationships to other generalized continuous functions. We further study a new class of functions namely (1,2)*-\(g_p\) irresolute.
2. Preliminaries

Throughout this paper \((X, \tau_1, \tau_2)\) (or briefly \(X\)) represent bitopological spaces on which no separation axioms are assumed unless otherwise mentioned.

**Definition 2.1.** [8] A subset \(B\) of a bitopological space \((X, \tau_1, \tau_2)\) is called \(\tau_1 \tau_2\) - open if \(B = U_1 \cup U_2\) where \(U_1 \in \tau_1\) and \(U_2 \in \tau_2\). The complement of \(\tau_1 \tau_2\) - open is called \(\tau_1 \tau_2\) - closed.

**Remark 2.2.** [8] \(\tau_1 \tau_2\) - open subset of \(X\) need not necessarily from a topology.

**Definition 2.3.** [8] A subset \(A\) of a bitopological space \((X, \tau_1, \tau_2)\) is called

(i) The \(\tau_1 \tau_2\) - closure of \(A\), denoted by \(\tau_1 \tau_2\) - cl\((A)\) is defined by \(\tau_1 \tau_2\) - closure \((A) = \bigcap \{ F/ A \subseteq F \text{ and } F \text{ is } \tau_1 \tau_2 \text{ - closed}\}.

(ii) \(\tau_1 \tau_2\) - interior of \(A\), denoted by \(\tau_1 \tau_2\) - int\((A)\) is defined by \(\tau_1 \tau_2\) - interior \((A) = \bigcup \{ F/ A \subseteq F \text{ and } F \text{ is } \tau_1 \tau_2 \text{ - open}\}.

**Definition 2.4.** A subset \(A\) of a bitopological space \((X, \tau_1, \tau_2)\) is called

(i) \((1,2)^*\) - pre-open [8] if \(A \subseteq \tau_1 \tau_2\) - int\((\tau_1 \tau_2\) - cl\((A))\) and \((1,2)^*\) - pre-closed if \(\tau_1 \tau_2\) - cl\((\tau_1 \tau_2\) - int\((A))\) \(\subseteq A\).

(ii) \((1,2)^*\) - b open [4] if \(A \subseteq \bigcup (\tau_1 \tau_2\) - cl\((\tau_1 \tau_2\) - int\((A))\) \(\cup (\tau_1 \tau_2\) - int\((\tau_1 \tau_2\) - cl\((A))\)) and \((1,2)^*\) - b closed if \(\bigcap (\tau_1 \tau_2\) - cl\((\tau_1 \tau_2\) - int\((A))\) \(\cap (\tau_1 \tau_2\) - int\((\tau_1 \tau_2\) - cl\((A))\)) \(\subseteq A\).

(iii) \((1,2)^*\) - regular-open [12] if \(A \subseteq \tau_1 \tau_2\) - int\((\tau_1 \tau_2\) - cl\((A))\) and \((1,2)^*\) - regular closed if \(A = \tau_1 \tau_2\) - cl\((\tau_1 \tau_2\) - int\((A))\).

The \((1,2)^*\) - pre-closure of a subset \(A\) of \(X\), denoted by \((1,2)^*\) - pcl\((A)\) is the intersection of all \((1,2)^*\) - pre-closed sets containing \(A\). The \((1,2)^*\) - pre-interior of a subset \(A\) of \(X\), denoted by \((1,2)^*\) - pint\((A)\) is the union of \((1,2)^*\) - pre-open sets contained in \(A\).

**Definition 2.5.** [9] The \((1,2)^*\) - \(\delta\) interior of a subset \(A\) of \(X\) is the union of all \((1,2)^*\) - regular open set of \(X\) contained in \(A\) and is denoted by \((1,2)^*\) - \(\delta\) int\((A)\). The subset \(A\) is called \((1,2)^*\) - \(\delta\) open if \(A = (1,2)^*\) - \(\delta\) int\((A)\), ie.a set is \((1,2)^*\) - \(\delta\) open if it is the union of \((1,2)^*\) - regular open sets. The complement of a \((1,2)^*\) - \(\delta\) open is called \((1,2)^*\) - \(\delta\) closed.

Alternatively, a set \(A \subseteq (X, \tau_1, \tau_2)\) is called \((1,2)^*\) - \(\delta\) closed if \(A = (1,2)^*\) - \(\delta\) cl\((A)\), where \((1,2)^*\) - \(\delta\) cl\((A)\) = \(\{x \in X: \tau_1 \tau_2\) - int\((\tau_1 \tau_2\) - cl\((A))\) \(\cap A = \emptyset\} U \in \tau_1 \tau_2\) and \(x \in U\).

**Definition 2.6.** A subset \(A\) of a bitopological space \((X, \tau_1, \tau_2)\) is called

(i) \((1,2)^*\) - generalized closed set (briefly \((1,2)^*\) - gc closed) [11] if \(\tau_1 \tau_2\) - cl\((A)\) \(\subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(\tau_1 \tau_2\) - open in \(X\).

(ii) \((1,2)^*\) - generalized b-closed set (briefly \((1,2)^*\) - gb closed) [13] if \(\tau_1 \tau_2\) - bcl\((A)\) \(\subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(\tau_1 \tau_2\) - open in \(X\).

(iii) \((1,2)^*\) - generalized pre-closed set (briefly \((1,2)^*\) - gp closed) [14] if \(\tau_1 \tau_2\) - pcl\((A)\) \(\subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(\tau_1 \tau_2\) - open in \(X\).

(iv) \((1,2)^*\) - generalized pre regular closed set (briefly \((1,2)^*\) - gpr closed) [10] if \(\tau_1 \tau_2\) - pcl\((A)\) \(\subseteq U\) whenever \(A \subseteq U\) and \(U\) is \((1,2)^*\) - regular open in \(X\).
(v) $(1,2)^* - \delta$ generalized closed set (briefly $(1,2)^* - \delta_gp$ closed) [9] if $\tau_1 \tau_2 - \delta_cl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $(1,2)^* - open$ in $X$.

(vi) $(1,2)^* - \delta$ generalized pre-closed (briefly, $(1,2)^* - \delta_gp$ closed) [9] if $\tau_1 \tau_2 - pcl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $(1,2)^* - \delta$ open in $X$.

The complement of the above mentioned closed sets are their respective open sets.

**Definition 2.7.** Recall that function $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called

(i) $(1,2)^* - continuous$ [7] if $f^{-1}(V)$ is $(1,2)^* - closed$ in $(X, \tau_1, \tau_2)$ for every closed $V$ of $(Y, \sigma_1, \sigma_2)$.

(ii) $(1,2)^* - generalized continuous$ (briefly $(1,2)^* - g$ continuous) [6] if $f^{-1}(V)$ is $(1,2)^* - g$ closed in $(X, \tau_1, \tau_2)$ for every closed $V$ of $(Y, \sigma_1, \sigma_2)$.

(iii) $(1,2)^* - generalized b continuous$ (briefly $(1,2)^* - gb$ continuous) [13] if $f^{-1}(V)$ is $(1,2)^* - gb$ continuous in $(X, \tau_1, \tau_2)$ for every closed $V$ of $(Y, \sigma_1, \sigma_2)$.

(iv) $(1,2)^* - generalized pre continuous$ (briefly $(1,2)^* - gp$ continuous) [5] if $f^{-1}(V)$ is $(1,2)^* - gp$ closed in $(X, \tau_1, \tau_2)$ for every closed $V$ of $(Y, \sigma_1, \sigma_2)$.

(v) $(1,2)^* - generalized pre regular continuous$ (briefly $(1,2)^* - gpr$ continuous) [11] if $f^{-1}(V)$ is $(1,2)^* - gpr$ closed in $(X, \tau_1, \tau_2)$ for every closed $V$ of $(Y, \sigma_1, \sigma_2)$.

3. $(1,2)^* - \delta_gp$ Continuous function

We introduce the following definitions.

**Definition 3.1.** A function $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is said to be $(1,2)^* - \delta_gp$ continuous, if $f^{-1}(V)$ is $(1,2)^* - \delta_gp$ closed set in $X$ for every $\sigma_1, \sigma_2$ - closed set $V$ in $Y$.

**Example 3.2.** Let $X = Y = \{a, b, c\}$, $\tau_1 = \{\phi, \{a\}, X\}$, $\tau_2 = \{\phi, \{b\}, \{a, b\}, X\}$, $\sigma_1 = \{\phi, \{b\}, Y\}$ and $\sigma_2 = \{\phi, \{a\}, Y\}$.

Let a map $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be defined by $f(a) = b$, $f(b) = a$, $f(c) = c$. Then $f$ is $(1,2)^* - \delta_gp$ continuous.

**Theorem 3.3.** Every $(1,2)^* - continuous$ map is $(1,2)^* - \delta_gp$ continuous.

**Proof.** Let $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be $(1,2)^* - continuous$. Let $V$ be $\sigma_1, \sigma_2$ - closed set in $Y$.

Since $f$ is $(1,2)^* - continuous$, $f^{-1}(V)$ is $\tau_1, \tau_2$ - closed. But every $\tau_1, \tau_2$ - closed set is $(1,2)^* - \delta_gp$ closed. Therefore $f^{-1}(V)$ is $(1,2)^* - \delta_gp$ closed. Hence $f$ is $(1,2)^* - \delta_gp$ continuous.

**Remark 3.4.** The converse of the above theorem is not true in general as shown in the following example.

**Example 3.5.** Let $X = Y = \{a, b, c\}$, $\tau_1 = \{\phi, \{a\}, X\}$, $\tau_2 = \{\phi, \{b\}, \{a, c\}, X\}$, $\sigma_1 = \{\phi, \{a\}, Y\}$ and $\sigma_2 = \{\phi, \{a, c\}, \{a, c\}, Y\}$.

Let a map $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be defined by $f(a) = c$, $f(b) = a$, $f(c) = b$. Clearly $f$ is $(1,2)^* - \delta_gp$ continuous but not $(1,2)^* - continuous$ because $f^{-1}(\{b\}) = \{c\}$ is not $\tau_1, \tau_2$ closed.
Theorem 3.6. Every $(1,2)^{-}\text{pre}$ continuous map is $(1,2)^{-}\text{gp}$ continuous.

Proof. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be $(1,2)^{-}\text{pre}$ continuous. Let $V$ be $\sigma_1 \sigma_2^{-}\text{closed}$ set in $Y$. Since $f$ is $(1,2)^{-}\text{pre}$ continuous, $f^{-1}(V)$ is $(1,2)^{-}\text{pre}$ closed. But every $(1,2)^{-}\text{pre}$ closed set is $(1,2)^{-}\text{gp}$ closed. Therefore $f^{-1}(V)$ is $(1,2)^{-}\text{gp}$ closed. Hence $f$ is $(1,2)^{-}\text{gp}$ continuous.

Remark 3.7. The converse of the above theorem is not true in general as shown in the following example.

Example 3.8. Let $X = Y = \{a, b, c, d\}$, $\tau_1 = \{\phi, \{a\}, \{a, b\}, X\}$, $\tau_2 = \{\phi, \{b\}, \{a, b\}, X\}$, $\sigma_1 = \{\phi, \{a, d\}, \{a, c, d\}, Y\}$ and $\sigma_2 = \{\phi, \{b\}, \{b, c, d\}, Y\}$. Let a map $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be defined by $f(a) = d, f(b) = c, f(c) = a$ and $f(d) = b$. Clearly $f$ is $(1,2)^{-}\text{pre}$ continuous but not $(1,2)^{-}\text{pre}$ continuous because $f^{-1}(\{b, c\}) = \{b, d\}$ is not $(1,2)^{-}\text{pre}$ closed.

Theorem 3.9. Every $(1,2)^{-}\text{gp}$ continuous map is $(1,2)^{-}\text{gp}$ continuous.

Proof. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be $(1,2)^{-}\text{gp}$ continuous. Let $V$ be $\sigma_1 \sigma_2^{-}\text{closed}$ set in $Y$. Since $f$ is $(1,2)^{-}\text{gp}$ continuous, $f^{-1}(V)$ is $(1,2)^{-}\text{gp}$ closed. But every $(1,2)^{-}\text{gp}$ closed set is $(1,2)^{-}\text{gp}$ closed. Therefore $f^{-1}(V)$ is $(1,2)^{-}\text{gp}$ closed. Hence $f$ is $(1,2)^{-}\text{gp}$ continuous.

Remark 3.10. The converse of the above theorem is not true in general as shown in the following example.

Example 3.11. Let $X = Y = \{a, b, c\}$, $\tau_1 = \{\phi, \{b\}, \{a, b\}, X\}$, $\tau_2 = \{\phi, \{a\}, \{a, c\}, X\}$, $\sigma_1 = \{\phi, \{b\}, \{b, c\}, Y\}$ and $\sigma_2 = \{\phi, \{a\}, \{a, c\}, Y\}$. Let a map $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be defined by $f(a) = b, f(b) = c, f(c) = a$. Clearly $f$ is $(1,2)^{-}\text{gp}$ continuous but not $(1,2)^{-}\text{gp}$ continuous because $f^{-1}(\{b\}) = \{a\}$ is not $(1,2)^{-}\text{gp}$ closed.

Theorem 3.12. Every $(1,2)^{-}\text{gp}$ continuous is $(1,2)^{-}\text{gpr}$ continuous.

Proof. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be $(1,2)^{-}\text{gp}$ continuous. Let $V$ be $\sigma_1 \sigma_2^{-}\text{closed}$ set in $Y$. Since $f$ is $(1,2)^{-}\text{gp}$ continuous, $f^{-1}(V)$ is $(1,2)^{-}\text{gpr}$ closed. But every $(1,2)^{-}\text{gpr}$ closed set is $(1,2)^{-}\text{gpr}$ closed. Therefore $f^{-1}(V)$ is $(1,2)^{-}\text{gpr}$ closed. Hence $f$ is $(1,2)^{-}\text{gpr}$ continuous.

Remark 3.13. The converse of the above theorem is not true in general as shown in the following example.

Example 3.14. Let $X = Y = \{a, b, c\}$, $\tau_1 = \{\phi, \{b\}, \{a, c\}, X\}$, $\tau_2 = \{\phi, \{a\}, \{a, b\}, X\}$, $\sigma_1 = \{\phi, \{a\}, Y\}$ and $\sigma_2 = \{\phi, \{a, b\}, Y\}$. Let a map $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be defined by $f(a) = b, f(b) = c, f(c) = a$. Clearly $f$ is $(1,2)^{-}\text{gpr}$ continuous but not $(1,2)^{-}\text{gpr}$ continuous because $f^{-1}(\{b, c\}) = \{a\}$ is not $(1,2)^{-}\text{gpr}$ closed.

Remark 3.15. The following example shows that $(1,2)^{-}\text{gp}$ continuous function is independent of $(1,2)^{-}\text{b}$ continuous function and $(1,2)^{-}\text{gb}$ continuous function.
Example 3.16. Let $X = Y = \{a, b, c, d\}$, $\tau_1 = \{\phi, \{b\}, \{a, b\}, X\}$, $\tau_2 = \{\phi, \{a\}, \{a, b, c\}, X\}$, $\sigma_1 = \{\phi, \{b, c\}, \{a, b, c\}, Y\}$ and $\sigma_2 = \{\phi, \{c\}, \{a, c\}, Y\}$. Let a map $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be defined by $f(a) = c$, $f(b) = a$, $f(c) = d$, $f(d) = b$. Clearly $f$ is $(1, 2)^* - \delta_{gp}$ continuous but not $(1, 2)^* - \delta_{gp}$ continuous and also it is not $(1, 2)^* - gb$ continuous because $f^{-1}\{(a, b, d)\} = \{a, b, c\}$ is not $(1, 2)^* - gb$ closed and also it is not $(1, 2)^* - b$ closed.

Example 3.17. Let $X = Y = \{a, b, c\}$, $\tau_1 = \{\phi, \{a\}, X\}$, $\tau_2 = \{\phi, \{c\}, \{a, c\}, X\}$, $\sigma_1 = \{\phi, \{b\}, \{b, c\}, Y\}$ and $\sigma_2 = \{\phi, \{a\}, \{a, b\}, Y\}$. Let a map $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be defined by $f(a) = c$, $f(b) = a$, $f(c) = b$. Clearly $f$ is $(1, 2)^* - gb$ and $(1, 2)^* - b$ continuous but not $(1, 2)^* - \delta_{gp}$ continuous because $f^{-1}\{(c)\} = \{a\}$ is not $(1, 2)^* - \delta_{gp}$ closed.

Remark 3.18. If $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is $(1, 2)^* - \delta_{gp}$ continuous and $g : (Y, \sigma_1, \sigma_2) \rightarrow (Z, \eta_1, \eta_2)$ is $(1, 2)^* - \delta_{gp}$ continuous, then $g \circ f : (X, \tau_1, \tau_2) \rightarrow (Z, \eta_1, \eta_2)$ need not be $(1, 2)^* - \delta_{gp}$ continuous. The following example supports our claim.

Example 3.19. Let $X = Y = Z = \{a, b, c\}$, $\tau_1 = \{\phi, \{a\}, X\}$, $\tau_2 = \{\phi, \{c\}, \{a, c\}, X\}$, $\sigma_1 = \{\phi, \{a\}, Y\}$, $\sigma_2 = \{\phi, \{c\}, Y\}$, $\eta_1 = \{\phi, \{a\}, Z\}$ and $\eta_2 = \{\phi, \{b\}, \{a, c\}, Z\}$. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ and $g : (Y, \sigma_1, \sigma_2) \rightarrow (Z, \eta_1, \eta_2)$ be the identity maps. Then $f$ and $g$ are $(1, 2)^* - \delta_{gp}$ continuous functions, but $g \circ f$ is not $(1, 2)^* - \delta_{gp}$ continuous because $(g \circ f)^{-1}\{(a, c)\} = f^{-1}(g^{-1}\{(a, c)\}) = f^{-1}\{(c)\} = \{a\}$ is not $(1, 2)^* - \delta_{gp}$ closed.

Proposition 3.20. A map $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is $(1, 2)^* - \delta_{gp}$ continuous if and only if $f^{-1}(U)$ is $(1, 2)^* - \delta_{gp}$ open in $X$ for every $\sigma_1 \sigma_2$ - open set $U$ in $Y$.

Proof. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be $(1, 2)^* - \delta_{gp}$ continuous and $\sigma_1 \sigma_2$ - open set $U$ in $Y$. Then $U^c$ is $\sigma_1 \sigma_2$ - closed in $Y$ and since $f$ is $(1, 2)^* - \delta_{gp}$ continuous, $f^{-1}(U^c)$ is $(1, 2)^* - \delta_{gp}$ closed in $X$. But $f^{-1}(U^c) = f^{-1}(f^{-1}(U))^c$ and so $f^{-1}(U)$ is $(1, 2)^* - \delta_{gp}$ open in $X$.

Conversely, assume that $f^{-1}(U)$ is $(1, 2)^* - \delta_{gp}$ open in $X$ for every $\sigma_1 \sigma_2$ - open set $U$ in $Y$. Let $F$ be a $\sigma_1 \sigma_2$ - closed set in $Y$. Then $F^c$ is $\sigma_1 \sigma_2$ - open in $Y$ and by assumption $f^{-1}(F^c)$ is $(1, 2)^* - \delta_{gp}$ open in $X$. Since $f^{-1}(F^c) = f^{-1}(f^{-1}(F))^c$, $f^{-1}(F)$ is $(1, 2)^* - \delta_{gp}$ closed in $X$ and so $f$ is $(1, 2)^* - \delta_{gp}$ continuous.

Definition 3.21. A function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called $(1, 2)^* - \delta_{gp}$ irresolute if $f^{-1}(V)$ is $(1, 2)^* - \delta_{gp}$ closed in $(X, \tau_1, \tau_2)$ for each $(1, 2)^* - \delta_{gp}$ closed set $V$ in $(Y, \sigma_1, \sigma_2)$.

Example 3.22. Let $X = Y = \{a, b, c\}$, $\tau_1 = \{\phi, \{a\}, X\}$, $\tau_2 = \{\phi, \{b\}, \{a, c\}, X\}$, $\sigma_1 = \{\phi, \{a\}, \{a, c\}, Y\}$ and $\sigma_2 = \{\phi, \{c\}, Y\}$. Let a map $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be defined by $f(a) = c$, $f(b) = a$, $f(c) = b$. Hence $f$ is $(1, 2)^* - \delta_{gp}$ irresolute.

Theorem 3.23. If a map $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ called $(1, 2)^* - \delta_{gp}$ irresolute then it is $(1, 2)^* - \delta_{gp}$ continuous.

Proof. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be $(1, 2)^* - \delta_{gp}$ irresolute. Let $V$ be $\sigma_1 \sigma_2$ - closed set in $(Y, \sigma_1, \sigma_2)$. Since every $\sigma_1 \sigma_2$ - closed set is $(1, 2)^* - \delta_{gp}$ closed, $V$ is $(1, 2)^* - \delta_{gp}$ closed in $Y$. 

International Journal of Engineering, Science and Mathematics
http://www.ijesm.co.in, Email: ijesmj@gmail.com
Since \( f \) is \((1,2)^* - \partial \) irresolute, \( f^{-1}(V) \) is \((1,2)^* - \partial \) closed in \( X \). Hence \( f \) is \((1,2)^* - \partial \) continuous.

**Remark 3.24.** The converse of the above theorem is not true in general as shown in the following example.

**Example 3.25.** Let \( X = Y = \{a, b, c\} \), \( \tau_1 = \{\phi, \{a\}, X\} \), \( \tau_2 = \{\phi, \{c\}, X\} \), \( \sigma_1 = \{\phi, \{c\}, Y\} \) and \( \sigma_2 = \{\phi, \{a\}, \{a, c\}, Y\} \). Let a map \( f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2) \) be the identity map. Then \( f \) is \((1,2)^* - \partial \) continuous but it is not \((1,2)^* - \partial \) irresolute because \( f^{-1}\{\{a, c\}\} = \{a, c\} \) is not \((1,2)^* - \partial \) closed in \( X \).

**Theorem 3.26.** Let \( f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2) \) and \( g : (Y, \sigma_1, \sigma_2) \rightarrow (Z, \eta_1, \eta_2) \) be two functions. Then

(i) \( g \circ f : (X, \tau_1, \tau_2) \rightarrow (Z, \sigma_1, \sigma_2) \) is \((1,2)^* - \partial \) continuous, if \( g \) is \((1,2)^* - \partial \) continuous and \( f \) is \((1,2)^* - \partial \) continuous.

(ii) \( g \circ f : (X, \tau_1, \tau_2) \rightarrow (Z, \sigma_1, \sigma_2) \) is \((1,2)^* - \partial \) irresolute, if \( g \) is \((1,2)^* - \partial \) irresolute and \( f \) is \((1,2)^* - \partial \) irresolute.

(i) \( g \circ f : (X, \tau_1, \tau_2) \rightarrow (Z, \sigma_1, \sigma_2) \) is \((1,2)^* - \partial \) continuous, if \( g \) is \((1,2)^* - \partial \) continuous and \( f \) is \((1,2)^* - \partial \) continuous.

**Proof.** (i) Let \( A \) be any \( \eta_1 \eta_2 \) closed set in \( (Z, \eta_1, \eta_2) \). Since \( g \) is \((1,2)^* - \partial \) continuous, \( g^{-1}(A) \) is closed in \( (Y, \sigma_1, \sigma_2) \). Also \( f \) is \((1,2)^* - \partial \) continuous, \( f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A) \) is \((1,2)^* - \partial \) closed in \( (X, \tau_1, \tau_2) \). Hence \( g \circ f \) is \((1,2)^* - \partial \) continuous function.

(ii) Let \( A \) be any \( \eta_1 \eta_2 \) closed set in \( (Z, \eta_1, \eta_2) \). Then \( A \) is \((1,2)^* - \partial \) closed in \( (Z, \eta_1, \eta_2) \). Since \( g \) is \((1,2)^* - \partial \) irresolute, \( g^{-1}(A) \) is \((1,2)^* - \partial \) closed in \( (Y, \sigma_1, \sigma_2) \). Also \( f \) is \((1,2)^* - \partial \) irresolute, \( f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A) \) is \((1,2)^* - \partial \) closed in \( (X, \tau_1, \tau_2) \). Hence \( g \circ f \) is \((1,2)^* - \partial \) irresolute function.

(iii) Let \( A \) be any \( \eta_1 \eta_2 \) closed set in \( (Z, \eta_1, \eta_2) \). Since \( g \) is \((1,2)^* - \partial \) continuous, \( g^{-1}(A) \) is \((1,2)^* - \partial \) closed in \( (Y, \sigma_1, \sigma_2) \). Also \( f \) is \((1,2)^* - \partial \) irresolute, \( f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A) \) is \((1,2)^* - \partial \) closed in \( (X, \tau_1, \tau_2) \). Hence \( g \circ f \) is \((1,2)^* - \partial \) irresolute function.

**4. Conclusion**

In this paper we define a new class of generalized continuous function called \((1,2)^* - \partial \) continuous and investigate their relationships to other generalized continuous functions. We further study a new class of functions namely \((1,2)^* - \partial \) irresolute. Also discussed some of their properties.
References


