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CUBIC SPLINE APPROXIMATION FOR TWO DIMENSIONAL NON-LINEAR ELLIPTIC BOUNDARY VALUE PROBLEMS

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ABSTRACT

This section exhibits a nine point reduced discretization of order two in y-and three in x-bearings for the solution of two dimensional nonlinear elliptic boundary value problems on a non-uniform mesh using cubic spline approximations. We examine the total deduction strategy of the method in details and furthermore talk about how our discretization can handle Poisson's equation in polar coordinates. Convergence of the method has been set up. Some physical examples and their numerical outcomes are given to legitimize the convenience of the proposed method. The second order elliptic equations are gotten as the consistent state solutions $(as\ t \longrightarrow \infty)$ of the illustrative and wave equations. Solutions of these equations are of incredible significance in numerous fields of science, for example, electromagnetics, astronomy, heat transfer, fluid mechanics and so on the grounds that they may speak to a temperature, electric or attractive potential, and relocation for an elastic membrane.

1. INTRODUCTION

Reenactments of relentless heat streams or irrotational streams of an inviscid, incompressible fluid, pressure computations for either the move through a permeable medium or that associated with the stream of a thick, incompressible fluid, and numerous others all include comprehending elliptic equations. Fourth order minimal finite difference scheme for the solution of linear elliptic equations on a constant mesh have been talked about by a few creators. Jain et al have first inferred fourth order difference methods for the solution of the system of nonlinear elliptic equations on a constant mesh in Cartesian coordinates and got concurrent solution for some, physical models like Navier-Stokes equations of movement. Afterward, Mohanty et al have stretched out their technique to get fourth order approximation for non linear elliptic equations in polar coordinates. As a result of the flimsiness very few numerical methods for the solution of elliptic equations on a geometric mesh (or, variable mesh) have been created. Further, the utilization of cubic spline approximations for the solution of non linear differential equations assumes an essential job in numerous physical models, particularly on a non-uniform mesh. Amid most recent three decades numerous scientists have grown high order numerical methods with (or without) cubic spline approximations for the solution of nonlinear two point boundary value problems. Much as of late, using cubic spline approximations Mohanty et al have inferred high order stable numerical methods on both uniform and non-uniform mesh for the solution of non linear illustrative and hyperbolic partial differential equations and got united outcomes. In our insight, no high order nine point reduced numerical scheme of order two in y-bearing and order three in x-heading on a non-uniform mesh for the solution of second order nonlinear elliptic partial differential equation has been talked about in the writing up until now.

In this part, using nine-point minimized stencil, we examine another steady method of order two in y-and order three in x-bearings on a variable mesh dependent on cubic spline approximations for the solution of two-dimensional nonlinear elliptic boundary value problems. It has been knowledgeable about the past that for problems in polar coordinates the solution for high order methods typically breaks down in the region of the singularities. We beat this trouble by changing our method so that the solution holds its order and accuracy wherever in the region of the singularity.

We start by considering a two dimensional nonlinear elliptic partial differential equation of the form

$$\nabla^2 u = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) u = f\left(x, y, u, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}\right), \qquad (x, y) \in R \tag{1}$$

Where region R is the unit square, $R = (0,1) \times (0,1)$

The relating Dirichlet boundary conditions are recommended by

$$u(x,y) = \psi(x,y) \quad , \qquad (x,y) \in \partial R \tag{2}$$

Where ∂R is the boundary of region? R.

We assume that for $0 < x, y < 1$.

i) $f(x, y, u, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}$ is continuous (3)

ii) The partial derivatives $\frac{\partial f}{\partial u}, \frac{\partial f}{\partial u_x}, \frac{\partial f}{\partial u_y}$ exist and are continuous.

iii) $\frac{\partial f}{\partial u} \geq 0, \left|\frac{\partial f}{\partial u_x}\right| \leq G$ and $\left|\frac{\partial f}{\partial u_y}\right| \leq H$

Where G and H are positive constants. What's more we expect that $u(x,y)$ is differentiable as high order as conceivable under consideration?

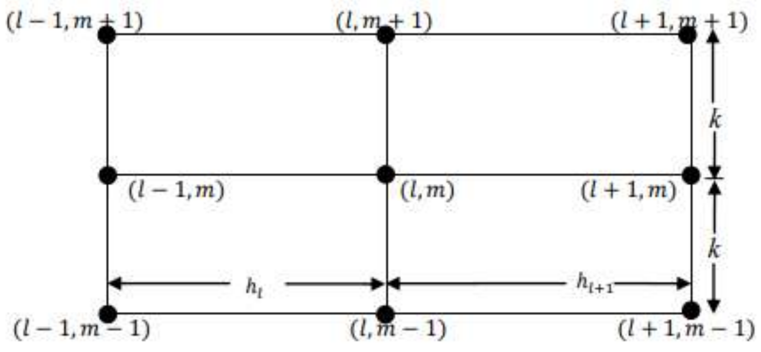

Fig. 1: Schematic representation of a single computational cell

2. LINE ITERATIVE ANALYSIS

In this area, we examine the convergence and performance of some line iterative methods with the geometric mesh cubic spline numerical scheme for the two dimensional linear elliptic partial differential equation.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = D(x)\frac{\partial u}{\partial x} + g(x,y), \qquad 0 < x, y < 1 \tag{3}$$

Subject to fitting Dirichlet boundary conditions endorsed. Expect that the functions $D(x)$and $g(x,y)$have a place with $C^2(R)$ that is, they have two derivatives that are continuous in R.

We take note of that, for the nine diagonal matrix that emerges because of discretization of two dimensional elliptic equation the line iterative method is equal to block iterative method.

On applying formula (2) to the elliptic equation (3) and disregarding the local truncation error, we acquire the accompanying difference scheme.

$$u_{l+1,m} - (1+\sigma_l)u_{l,m} + \sigma_l u_{l-1,m}$$

$$+\frac{h_l^2}{12}[P_l\, \bar{u}_{yyl+1,m} + Q_l\, \bar{u}_{yyl,m} + R_l\, \bar{u}_{yyl-1,m}]$$

$$= \frac{h_l^2}{12}[\, P_l D_{l+1}\bar{\bar{u}}_{xl+1,m} + Q_l D_l \hat{u}_{xl,m} + R_l D_{l-1}\, \bar{\bar{u}}_{xl-1,m}\,]$$

$$+\frac{h_l^2}{12}[P_l g_{l+1,m} + Q_l g_{l,m} + R_l g_{l-1,m}], \quad l = 1,2,..,N, m = 1,2..,M \tag{4}$$

Note that, the scheme (4) is of $(k^2 + k^2 h_l + h_l^3)$. Nonetheless, an important observation is that the scheme neglects to acknowledge at $= 1$, when the functions $D(x)$and/or $g(x,y)$ involve singular terms like $\frac{1}{x}, \frac{1}{x^2}, \frac{1}{xy^3}$, and so forth. Consider for instance,when $D(x) = \frac{1}{x}$, then $D_{l-1} = \frac{1}{xl-1}$which winds up infinite at $l = 1$ as $x_0 = 0$. We beat this trouble by refining our scheme so that the solution holds its order and accuracy wherever in the solution region.

We consider the following approximation

$$D_{l+1} = D_{00} + \sigma_l h_l D_{10} + \frac{\sigma_l^2 h_l^2}{2} D_{20} + O(h_l^3) \equiv D_l^* + O(h_l^3) \qquad (5)$$

$$D_{l-1} = D_{00} - h_l D_{10} + \frac{h_l^2}{2} D_{20} - O(h_l^3) \equiv D_l^{**} - O(h_l^3) \qquad (6)$$

$$g_{l+1,m} = g_{00} + \sigma_l h_l g_{10} + \frac{\sigma_l^2 h_l^2}{2} g_{20} + O(h_l^3) \equiv g_{l,m}^* + O(h_l^3) \qquad (7)$$

$$g_{l-1,m} = g_{00} - h_l g_{10} + \frac{h_l^2}{2} g_{20} - O(h_l^3) \equiv g_{l,m}^{**} - O(h_l^3) \qquad (8)$$

Where $g_{l,m} = g_{00} = g(x_l, y_m)\ etc$ (9)

And $D_l^* = D_{00} + \sigma_l h_l D_{10} + \frac{\sigma_l^2 h_l^2}{2} D_{20}$, $D_l^{**} = D_{00} - h_l D_{10} + \frac{h_l^2}{2} D_{20}$, ... etc.

Presently, substituting the approximations (8) in the difference scheme (9) and blending the higher order terms in local truncation error, we acquire the altered scheme.

3. COMPUTATIONAL IMPLEMENTATION

We consider a lower order method, for correlation. Substituting the approximations in the differential equation (1), we get a focal difference scheme of $(k^2 h_l)$ of the frame

$$\bar{U}_{xxl,m} + \bar{U}_{yyl,m} = f(x_l, y_m, U_{l,m}, \bar{U}_{xl,m}, \bar{U}_{yl,m}) + O(k^2 + h_l) \qquad (10)$$

Numerical experiments are done to delineate the feasibility of proposed method and to exhibit computationally its assembly. We tackle the accompanying two dimensional elliptic boundary esteem issues on a geometric mesh both on rectangular and tube shaped polar directions whose correct arrangements are known to us. The Dirichlet boundary conditions can be gotten utilizing the correct arrangements as a test technique. We supplant the arrangement area R by a rectangular grid $G_R$where $h_l = x_l - x_{l-1}$ and $h_{l+1} = x_{l+1} - x_l$ is the variable mesh estimate in the $x - direction$and $\sigma_1 = \frac{h_{l+1}}{h_1} > 0$, $l = 1,2 \dots N$, the mesh ratio parameter $y_m = mk$, $m = 0,1, \dots M+1, k > 0$ and Since, we have

$$1 = x_{N+1} - x_0 = (x_{N+1} - x_N) + (x_N - x_{N-1}) + .. + (x_1 - x_0)$$

$$= h_{N+1} + h_N + .. + h_1$$

$$= h_1(1 + \sigma_1 + \sigma_1\sigma_2 + \sigma_1\sigma_2\sigma_3 + .. + \sigma_1\sigma_2 .. \sigma_N) \qquad (11)$$

Thus,

$$h_1 = \frac{1}{1+\sigma_1+\sigma_1\sigma_2+..+\sigma_1\sigma_2..\sigma_N} \qquad (12)$$

This decides the beginning estimation of the initial step length in x – heading and the consequent advance lengths are determined by $h_2 = \sigma_1 h_1 h_3 = \sigma_2 h_{2...}$etc. Consequently we decide every grid point (x_l , y_m) of the rectangular grid.

With the end goal of effortlessness, we may consider $\sigma_1 = \sigma$ (a consistent), for all $l = 1,2, ... N$ at that point $h_1$diminishes to

$$h_1 = \frac{(1-\sigma)}{(1-\sigma^{N+1})}, \quad \sigma \neq 1 \qquad (13)$$

Subsequently, by endorsing the complete number of grid points in the x –direction, state to be, $N + 2$ and the estimation of σ, we can figure h_1 from the above connection and the rest of the mesh points in x –direction is dictated by $h_{l+1} = \sigma h_{1,l} = 1(1)N$. We have contrasted our method and the relating lower order variable mesh difference scheme of as far as arrangement precision. In all cases, we have speculated $u(x_1, y_m) = 0$.

Stopping criteria: The iterations were ended when the greatest standard of the progressions to the arrangement was not exactly the resilience of 10^{-10} . All calculations were done in double precision arithmetic utilizing MATLAB. Graphs delineating precise and numerical solutions for chose parameters for each of the problems discussed have been included.

Example1 (Convection-diffusion equation)

The issue is to explain (4) in the arrangement region $0 < x, y < 1$whose correct arrangement is , where.

$$u(x,y) = e^{\frac{\beta x}{2}} sin\pi y \left(2e^{\frac{\beta}{2}} sinh\sigma x + sinh\sigma(1-x)\right)/sinh\sigma \text{ , where } \sigma^2 = \pi^2 + \frac{\beta^2}{4}$$

Table 1 presentations the maximum absolute errors for u for $\sigma = 0.9$. The graphs of correct and numerical solutions are plotted in Fig. 1 (i) and 1 (ii) for $\sigma = 0.9$ and $\beta = 100$.

Table 1

Example 1: The maximum absolute errors ($\sigma = 0.9$)

	Proposed Method (6.2.12)				Method (6.5.1)		
β →(N+1,M+1)	20	100	500	1000	20	100	500,1000
(30,30)	.2808(-03)	.4515(-03)	.3364(-01)	.1693(+00)	.7447(-02)	.2337(-01)	unstable
cpu-BGS*	0.424358s	0.128150 s	0.702202 s	2.048380 s	0.208971 s	0.113939 s	
(40,40)	.1611(-03)	.1029(-03)	.1300(-02)	.1100(-01)	.5357(-02)	.8504(-02)	unstable
cpu-BGS	1.297527s	0.364116 s	0.566823 s	3.389510 s	0.525688 s	0.219316 s	
(50,50)	.1094(-03)	.5529(-04)	.1320(-03)	.5223(-03)	.4754(-02)	.5188(-02)	unstable
cpu-BGS	2.730789s	1.024950 s	1.418493 s	3.551660 s	1.004496 s	0.604247 s	
(60,60)	.8233(-04)	.4028(-04)	.4152(-04)	.7567(-04)	.4557(-02)	.4253(-02)	unstable
cpu-BGS	3.652260s	2.140070 s	1.998329 s	5.026471 s	1.986801 s	1.459012 s	
(70,70)	.6635(-04)	.3335(-04)	.2605(-04)	.3131(-04)	.4497(-02)	.3945(-02)	unstable
cpu-BGS	7.254613s	3.948505 s	2.828473 s	5.911080 s	3.003325 s	2.822480 s	
(80,80)	.5617(-04)	.2953(-04)	.2165(-04)	.2227(-04)	.4481(-02)	.3842(-02)	unstable
cpu-BGS	9.439466s	6.499839 s	5.524024 s	9.2007633 s	3.753915 s	3.335627 s	
*cpu-BGS : CPU time using Block-Gauss Seidel method							

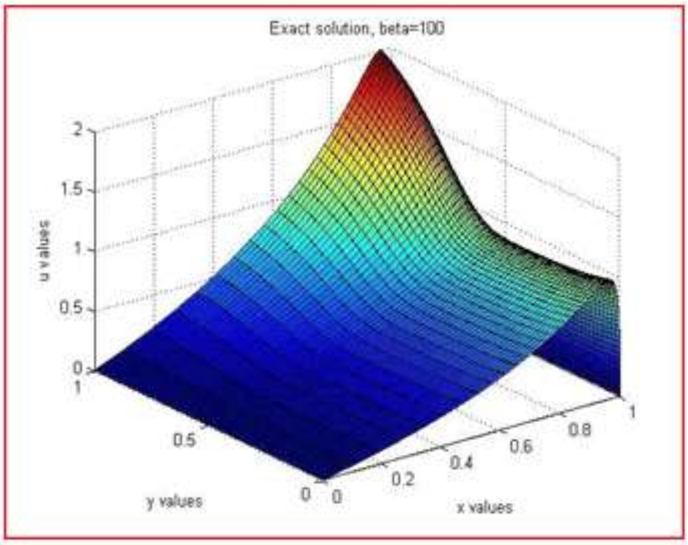

Fig 1: Convection Diffusion Equation

Exact Solution ($\sigma = 0.9\beta = 100$)

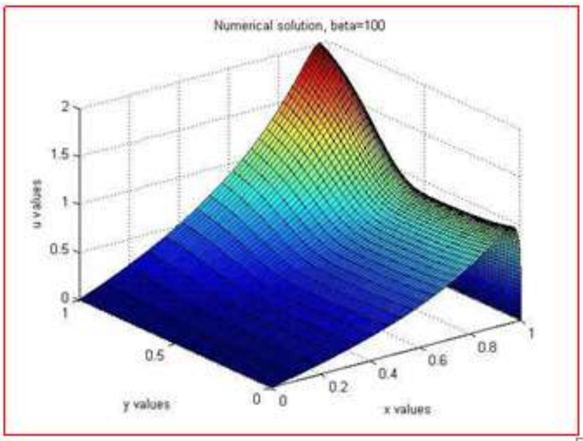

Fig 2: Convection Diffusion Equation

4. CONCLUSION

The accessible numerical methods for the solution of two dimensional nonlinear elliptic boundary value problems on a non-uniform mesh are of first order exact just and from application perspective as a rule the method is temperamental. In this section, we have built up another steady high order nine point minimal plan of $O(k^2, k^2h_l + h_l^3)$dependent on cubic spline approximations for the solution of two dimensional nonlinear elliptic boundary value problems. The proposed method is effectively connected to Poisson‟s equation in round and hollow polar coordinates and two-dimensional Burgers‟ equation with high Reynolds number.

REFERENCES

[1]. Pathak, A.K.and, Doctor, H.D. (2014),Comparative Study of Collocation Method-Spline, Finite Difference and Finite Element Method For Solving Flow of Electricity in a Cable of Transmission Lines, International Journal of Modern Science and Engineering Technology,Vol-1,Issue-3,pp-36-43.

[2]. Prasad, H.S., and Reddy, Y.N. (2014), A Fitted Second Order Finite Difference Method for Singular Perturbation Problems Exhibiting Dual Layers, American Journal of Numerical Analysis, Vol-2, Issue-6, pp-184-189

[3]. Raffaele, D. and Beatrice, P.(2014), Numerical Solution of a diffusion problem by exponentially fitted finite difference methods,Springer,3,425,pp-1-7.

[4]. Fukuchi, T. (2014), Finite Difference Method and algebraic polynomial interpolation for numerically solving Poisson's equation over arbitrary domains, AIP Advances, pp-1-30.

[5]. Sharma, P.S. and Datta,D.(2015),Fuzzy Finite Difference Scheme to simulate Radon Transport from Subsurface Soil to Buildings, International Journal of Energy, Information and Communications,Vol-6,Issue-4,pp-1-16

[6]. Riaz,S.,Muhammad, R.,et.al.(2015),Non standard finite difference method for Quadratic Riccati Differential Equation, Journal of Mathematics,Vol.47(2) Vol.47, Issue 2, pp.1-8.

[7]. Jater, S. (2015), Block Unification Scheme for Elliptic, Telegraph, and Sine-Gordon Partial Differential Equations, American Journal of Computational Mathematics, 5, pp-175-185.

[8]. Chaudhari, T.U.and Patel, T.M. (2015), Finite Element Solution of Poissons Equation in a Homogeneous Medium, International Research Journal of Engineering and Technology, Vol-2, Issue-9, pp-674-679