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# BOUNDARY VALUE PROBLEM AND EIGENVALUE

VISHAL

## ABSTRACT

A differential equation is that equation which involves more than one derivatives. It can be ordinary differential equation or partial differential equation depending on the dependency of variables on the equations. To find out solutions to these equations the concept of boundary value problem comes into consideration. Boundary value problem is a very wide concept and involves many aspects. In the following research paper we are going to discuss solution to boundary value problems in relation to eigenvalue and eigenfunction. Eigenvalue and eigenfunction is an advanced mathematics concept. Eigenvalues are an extraordinary arrangement of scalars related with a direct arrangement of conditions (i.e., a lattice condition) that are now and then otherwise called trademark roots, trademark esteems (Hoffman and Kunze 1971), appropriate qualities, or idle roots (Marcus and Minc 1988, p. 144).

## INTRODUCTION

The words differential and conditions surely propose explaining a condition that contains subsidiaries y', y", Comparable to a course in polynomial math and trigonometry, in which a decent measure of time is spent unraveling conditions, for example, $x^2 +5x+4=0$ for the obscure number x, right now of our assignments will be to tackle differential conditions, for example, y"+ 2y'+ y = 0 for an obscure capacity y= □ (x).

There are numerous kinds of differential conditions, and a wide assortment of arrangement methods, in any event, for conditions of a similar sort, not to mention various sorts.

- A ordinary differential condition, or ODE, is a condition that relies upon at least one subsidiaries of elements of a solitary variable. Differential conditions given in the first models are for the most part common dfiferential conditions, and we will consider these conditions solely right now course.

• An partial differential condition, or PDE, is a condition that relies upon at least one halfway subsidiaries of elements of a few factors. Much of the time, PDE are illuminated by decreasing to different ODE.

For example-The heat equation

$$\frac{dy}{dx} = k^2 \frac{d^2u}{dx^2}$$

where k is a constant, is an example of a partial differential equation, as its solution u(x, t) is a function of two independent variables, and the equation includes partial derivatives with respect to both variables.

The order of a differential equation is the order of the highest derivative of any unknown function in the equation.

Example The differential equation

$$\frac{dy}{dx} = ay - b,$$

where a and b are constants, is a first-order differential equation, as only the first derivative of the solution y(t) appears in the equation.

On the other hand, the ODE

$$y'' + 3y' + 2y = 0$$

is a second-order differential equation, whereas the PDE known as the beam equation

$$u_t = u_{xxxx}$$

is a fourth-order differential equation.

The solutionss of second order ordinary differential conditions of the structure

$$y'' = f(t, y, y') \qquad (1)$$

exist under rather broad conditions, and are novel in the event that we determine starting values $y(t_0)$, $y'(t_0)$. Let us utilize the documentation IVP for the words introductory worth issue.

In numerous applications, one needs answers for (1) in which one determines the estimations of the arrangement y(t) at two separate focuses $t_0 < t_1$ as opposed to indicating the estimation of y(t) and its subsidiary at a solitary point. This prompts the subject of Boundary Value Problems, an extremely enormous and significant territory of science. The subject is read for both normal and incomplete differential conditions. On account of halfway differential conditions, one manages arrangements which are characterized on subsets of different Euclidean spaces, what's more, thus there are many fascinating locales for which to determine limit condtions.

In this paper we will discuss about boundary value problems for scalar linear second order ordinary differential equations.In most applications, the independent variable of the differential equation represents a spatial condition along a real interval rather than time, so we use x for the independent variable of our functions instead of t. The general linear second order boundary value problem has the form

$$(x)y = h(x), \text{ BC} \qquad (2)$$

Here x is in some interval $I = (a, b) \subset R$, p(x), q(x), h(x) are continuous real valued functions on I, $\alpha < \beta$ are two fixed real numbers in I, and BC refers to specific boundary condtions. To study all solutions of such a problem. In the cases considered here, we can replace x by $x + \alpha$ if necessary and assume that $\alpha = 0$. We will denote the right boundary point by L. We will consider four types of boundary conditions, which we denote by the expressions 00, 01, 10, 11. These are defined by type 00: $y(0) = 0$, $y(L) = 0$

type 01: $y(0) = 0$, $y0 (L) = 0$

type 10: $y 0 (0) = 0$, $y(L) = 0$

type 11: $y 0 (0) = 0$, $y0 (L) = 0$

where $L > 0$.

The BVP

$$y'' + p(x)y' + q(x)y = 0,\ y(0) = 0,\ y(L) = 0 \qquad (3)$$

is called a homogeneous boundary value problem and will be denoted by HBVP. Any BVP which is not homogeneous will be called a non-homogeneous BVP. Given a BVP of the form (2) of type 00, 10,01, or 10, there is an associated HBVP of type 00 obtained by replacing h(x) by the zero-function and replacing the boundary conditions by y(0) = 0, y(L) = 0.

## THE DIFFRENTIAL SYSTEM CALCULUS

Let y = f(t) and assume that y fulfills the differential condition:

$$f'(t) = \lambda f(t)$$

which can likewise be composed

$$\frac{dy}{dt} = \lambda y,$$

where λ is a steady. This condition says that the pace of development of y is corresponding to y, and λ is the proportionality steady. You may recall that such conditions are related to populace development (the rate at which a microscopic organisms culture recreates is relative to the measure of microorganisms effectively present) and radioactive rot (the rate at which a substance rots is corresponding to themeasure of substance there). We have decided to call our consistent λ to portend the way that it will be seen as an eigenvalue. The above condition is known as a "differential condition" since it is a condition that includes a "subsidiary." It is called a "customary differential condition" in light of the fact that it just includes "conventional" subsidiaries, and not "halfway subordinates" from multivariable math.

This condition is a "first request" condition since it just includes a first subsidiary. It is moreover called a "straight" condition, since y and dy/dt just show up as direct terms, and subsequently the inevitable association with direct variable based math. At long last, the condition is designated "independent", since it doesn't change with time t (despite the fact that its answer does).

Comprehending a differential condition implies discovering all potential capacities f(t) that fulfill the condition. In contrast to most differential conditions, it occurs that the above differential condition is anything but difficult to illuminate, and you may have figured out how to unravel it in your first year recruit analytics class. To be specific, the factors can be "isolated" with the goal that the left side contains just y and the correct side contains just :

$$\frac{dy}{y} = \lambda dt.$$

We would then be able to incorporate the two sides to get

$$\int \frac{dy}{y} = \int \lambda dt$$

what's more, by really finding the antiderivatives

ln

$$y = \lambda t + c,$$

where c is a steady of joining. We needn't bother with a consistent of mix on each side, in light of the fact that the steady on the left can be joined with the consistent on the right. Presently, exponentiating both sides of the condition, we find

$$y = e^{\lambda t+c} = e^{c} \cdot e^{\lambda t}$$

Since c is likewise a consistent, we can mean it basically as C, and we have

$$y = Ce^{\lambda t}$$

This is known as the "general arrangement" of the differential condition. Note that when t = 0, at that point y = C, thus it is additionally regular to mean C by y0 and compose

$$y = y_0 e^{\lambda t}$$

On the off chance that we determine an estimation of y0, at that point we have what is known as a "specific arrangement" to the condition, and it is the arrangement of the differential condition that fulfills the "underlying condition" y(0) = y0.

## SYSTEMS OF DIFFERENTIAL EQUATIONS

The most straightforward approach to make an association with direct polynomial math is to think about frameworks of differential conditions. The reading material examines models originating from basic electric circuits, yet I think that its more enjoyable to consider some "Romeo and Juliet" models. I took in these models as a procedure for showing the job of eigenvalues in differential conditions from the book:

Steven H. Strogatz, Nonlinear Dynamics and Chaos, Addison Wesley, 1994.

The circumstance is as per the following. Let R or R(t) signify Romeo's love for Juliet at time t. We will say R > 0 compares to positive friendship i.e., love, for Juliet, and R < 0 relates to negative warmth, i.e., despise. Essentially, J will mean Juliet's friendship for Romeo at time t. At that point, dR/dt and dJ/dt speak to how Romeo and Juliet's expressions of love for one another are changing at an moment in time. To start, how about we consider a situation where Romeo and Juliet's affections for one another depend as it were on themselves and not on how different feels about them. All the more decisively, assume the change in Romeo's fondness for Juliet is corresponding to how a lot of love he as of now has for her, and likewise for Juliet. In conditions, this could be composed:

$$\frac{dR}{dt} = \lambda_1 R$$

$$\frac{dJ}{dt} = \lambda_2 J$$

In spite of the fact that this is an arrangement of conditions, the two conditions are totally autonomous of each other.

In this way, they can be explained as above to get

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$R = R_0 e^{\lambda 1t}$

$J = J_0 e^{\lambda 2t}$

Be that as it may, to stress the association with direct polynomial math, we should compose the first framework in grid structure:

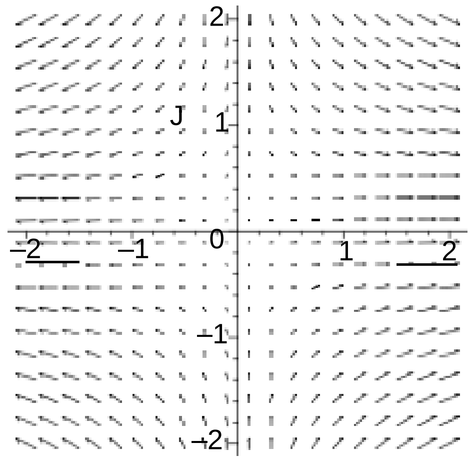
$$\begin{bmatrix} dR/dt \\ dJ/dt \end{bmatrix} = \begin{bmatrix} \lambda 1 & 0 \\ 0 & \lambda 2 \end{bmatrix} \left[\begin{bmatrix} R \\ J \end{bmatrix}\right] . x$$

The way that the lattice is corner to corner is the thing that makes the conditions so natural to explain. In the event that $\lambda_1 > 0$ andRomeo begins with some adoration for Juliet ($R_0 > 0$), at that point Romeo's affection for Juliet nourishes off itself and develops exponentially. In the event that $R_0 < 0$, at that point Romeo's detest for Juliet bolsters off itself and develops exponentially. Then again, in the event that $\lambda_1 < 0$, at that point R rots exponentially, so Romeo's energy (either positive or negative) for Juliet progressively whithers away. Juliet's emotions are likewise controlled by $\lambda_2$.

In spite of the fact that right now, was not hard to locate a precise equation for Romeo and Juliet's sentiments as a component of time, regularly we are less inspired by such careful recipes and just need to realize what occurs, i.e., do Romeo and Juliet fall and in the long run stay in adoration with one another? This sort of inquiry can regularly be replied by drawing what is known as a "stage graph" or "stage picture." To plot a stage chart for our Romeo and Juliet conditions, at each point (R, J) in the plane, we plot the vector

$$\begin{bmatrix} dR/dt \\ dJ/dt \end{bmatrix}$$

By way of example, let's take $\lambda_1 = 2$ and $\lambda_2 = -1$. The corresponding phase plane would then



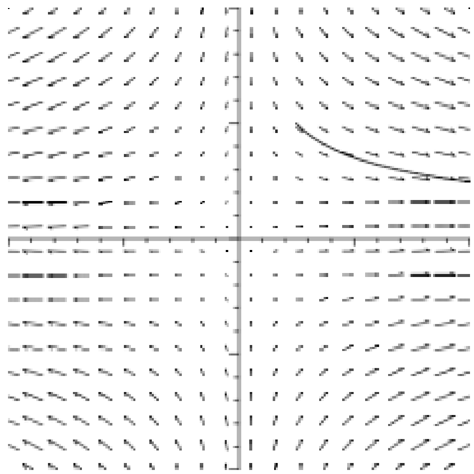

look like:

Notice, for example, that at the point (1, 1), the vector

$$\begin{bmatrix} dR/dt \\ dJ/dt \end{bmatrix} = \begin{bmatrix} 2R \\ -J \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

has been plotted.

Truth be told, the vectors have been abbreviated to make the plot increasingly comprehensible in light of the fact that it is principally the bearing of the vector we are keen on and the overall size of the vector contrasted and its neighboring vectors. The real size of the vector isn't significant. We get an answer direction by beginning at a given introductory point and afterward following the bolts. For instance, assume that we start at the point (R, J) = (0.5, 1), relating to Romeo having a slight enthusiasm for Juliet and Juliet having a moderate enthusiasm for Romeo. Following the bolts, we discover the arrangement direction drawn here:



As the grid comparing to this framework was corner to corner, the two eigendirections are the arrange tomahawks. In this manner, we see that regardless of where we start, in light of the fact that Juliet's warmth rots, that we wind up moving toward the eigendirection comparing to the R-hub as an asymptote. Obviously on the off chance that we start on the R-hub, we remain there. The J - hub is additionally an eigenspace, and in the event that we start on the J - pivot, we remain there, however moving toward the birthplace. In the event that we start close to the J - hub, we in the end move away from it. At the point when this happens the J - hub is known as a "repeller." Also, we see that in the long run we approach the R-pivot, so we call the R-hub an "attractor."

CONCEPT OF EIGENVALUE

An eigenvector is essentially a vector that is unaffected (to inside a scalar worth) by a change. Officially, an eigenvector is any vector x with the end goal that for an administrator $\Omega$, $\Omega x=\lambda x$ for some scalar steady $\lambda$. All administrators of measurement n have precisely n eigenvectors/eigenvalues (however these are just all unmistakable if $\Omega$ is diagonalizable).

Eigenvectors (or truly, eigen-things, as material science appears to love to slap the expression "eigen" before any word it needs) show up all over the place.

In math, the arrangement of exponential capacities (for example $n^x$) are the eigenfunctions of the separation administrator, and $e^x$ is the eigenfunction with eigenvalue . You can utilize eigenvector/esteem culmination in the separation administrator to demonstrate Euler's character, that $e^{i\theta}=\cos\theta+i\sin\theta$, and to demonstrate that $e^x$ relates to its Taylor arrangement.

For fundamental Fourier arrangement hypothesis we will require the
accompanying three eigenvalue issues.

$$x''+\lambda x=0, x(a)=0, x(b)=0, \quad (4.1)$$

$$x''+\lambda x=0, x'(a)=0, x'(b)=0, \quad (4.2)$$

and

$$x''+\lambda x=0, x(a)=x(b), x'(a)=x'(b). \quad (4.3)$$

A number λ is called an eigenvalue of 4.1 (resp.4.2 or4.3 ) if and just if there exists a nonzero (not indistinguishably zero) answer for 4.1 (4.2 or 4.3) given that particular. λA nonzero arrangement is known as a corresponding eigenfunction. Note-the comparability to eigenvalues and eigenvectors of matrices. The similitude isn't simply adventitious. On the off chance that we think about the conditions as differential administrators, at that point we are doing likewise correct thing. Think about a capacity x(t)as a vector with unendingly numerous parts (one for each t). Let

$$L = -\frac{d^2}{dt^2}$$

be the direct administrator. At that point the eigenvalue eigenfunction pair ought to be λ what's more, nonzero x with the end goal that

$$L\,x = \lambda x$$

At the end of the day, we are searching for nonzero capacities x fulfilling certain endpoint conditions that understand $(L-\lambda)x=0$. A great deal of the formalism from direct variable based math despite everything applies here, however we won't seek after this line of thinking excessively far

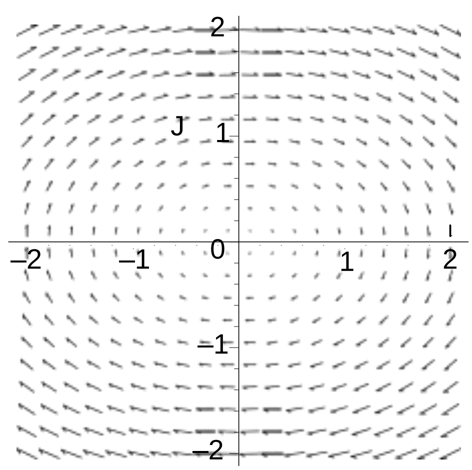
*For example if we take imaginary eigenvalues then*

Presently think about the arrangement of conditions

$$\frac{dR}{dT} = aJ$$

$$\frac{dJ}{dT} = -bR$$

with an and b positive. Right now, reacts emphatically to Juliet's love for him, yet Juliet likes Romeo more when Romeo hates her, and then again, she loves Romeo less at the point when Romeo enjoys her more. The phase diagram is



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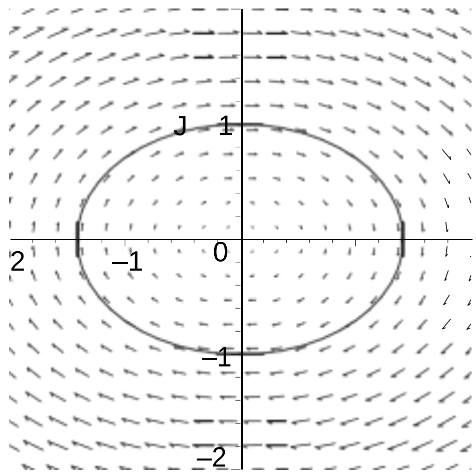
In this case, we see that a typical solution trajectory looks like

R




Assume that things began with Juliet inspired by Romeo, yet Romeo conflicted to Juliet. At that point Juliet's enthusiasm for Romeo makes Romeo increasingly attached to Juliet. Be that as it may, this at that point turns Juliet off, and she turns out to be less captivated with Romeo, until she starts to despise him. As Juliet's aversion for Romeo develops, Romeo loses his friendship for Juliet, until at last he likewise develops to loathe her. However, this causes Juliet to turn out to be more pulled in to Romeo, until at long last Juliet gets enamored with Romeo once more, and the cycle rehashes itself. Composing the framework that prompted this stage representation in grid structure, we get

$$\begin{bmatrix} dR/dt \\ dJ/dt \end{bmatrix} = \begin{bmatrix} 0 & a \\ -b & 0 \end{bmatrix} \begin{bmatrix} R \\ J \end{bmatrix}$$

This time we find that the trademark condition is

$\lambda^2 = \sqrt{-ab}$, thus the eigenvalues are unadulterated nonexistent:

$\lambda = \pm i\sqrt{ab}$

$$e^{i\sqrt{abt}} e^{-i\sqrt{abt}}$$

abdominal muscle. Since the eigenvalues are fanciful, we have no genuine eigenvectors, thus we don't perceive any asymptotic headings related to eigendirections in our stage chart. In the event that we permit ourselves to utilize complex eigenvectors to diagonalize our network, we will discover the express recipes for the answer for this arrangement of condition will include

$e^{i\sqrt{abt}} = \cos\sqrt{abt} + isin\sqrt{abt}$ and $e^{-i\sqrt{abt}} = \cos(\sqrt{abt}) - isin(\sqrt{abt})$

we can re-compose our answer regarding $\cos(\sqrt{abt})$ and $\sin(\sqrt{abt})$. Notice that the time of the cycle is legitimately identified with the eigenvalues.

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