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# PLASMA PHYSICS RELATED TODAY VLASOV EQUATION IS NEEDFUL

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## ABSTRACT

Five action principles for the Vlasov-Poisson and Vlasov- Maxwell equations, which differ by the variables incorporated to describe the distribution of particles in phase space, are presented. Three action principles previously known for the Vlasov-Maxwell equations are altered so as to produce the Vlasov-Poisson equation upon variation with respect to particle and field variables independently. Also, a new action principle for both systems, which is called the leaf action, is presented. This new action has the desirable features of using only a single generating function as the dynamical variable for describing the particle distribution, and manifestly preserving invariants of the system known as Casimir invariants. The relationships between the various actions is a link between actions written in terms of Lagrangian and Eulerian variables.

## Introduction

Referring to the system of equations that now goes under the name of the Vlasov-Poisson system, in his celebrated 1946 paper 11 - On the vibrations of an electronicplasma" Lev Landau wrote a very sharp remark: theseequations were used by A.A. Vlasov for investigation ofthe vibrations of plasma. However most of his results areincorrect"Strange indeed is the fate of the name of this fundamental equation for plasma physics that was neitherdiscovered by, nor correctly solved by the man whosename it bears, although Vlasoy correctly recognized 12that for a system of charged particles the kinetic equationmethod which considers only hinary interactions - interactions through collisions - is an approximation which isstrictly speaking inadequate, so that in the theory of suchsystems an essential role must be played by the interaction forces, particularly at large distances and hence, asystem of charged particles is, in essence, not a gas but adistinctive system coupled by long-range forces. In factthis equation was tased in the context of star dynamics, wewould now say of galactie dynamics, one hundred yearsago by Jeans 31 and is still generally known in the astrophysics community as the Collisionless Boltzmann equation.

## DESCRIPTION

The Vlasov equation is the master equation whichprovides a statistical description for the collectivebehavior of large numbers of charged particles inmutual, long-range interaction. In other words, alow collision (or "Vlasov') plasma. Plasma physics isitself a relatively young discipline, whose “birth" canbe ascribed to the 1920s. The origin of the Vlasovmodel, however, is even more recent, dating back tothe late 1940s. This "young age" is due to the rareoccurrence of Vlasov plasma on Earth, despite thefact it characterizes most of the visible matter in theuniverse.

## The Vlasov-Maxwell system of equations (gaussian units)

Instead of collision-based kinetic description for interaction of charged particles in plasma, Vlasov utilizes a self-consistent collective field created by the charged plasma particles. Such a description uses distribution functions $f_e(\mathbf{r},\mathbf{p},\ \mathbf{t})$ and $f_i(\mathbf{r},\ \mathbf{p},\mathbf{t})$ for electrons and (positive) plasma ions. The distribution function $f_\alpha($ $(\mathbf{r},\ \mathbf{p},\ \mathbf{t})$ for species a describes the number of particles of the species a having approximately the momentum

**p**near the position **r** at time t. Instead of the Boltzmann equation, the following system of equations was proposed for description of charged components of plasma (electrons and positive ions):

$$\frac{\partial f_e}{\partial t} + v_e \,.\nabla f_e - e\,(\mathbf{E} + \frac{\boldsymbol{v_e}}{\boldsymbol{c}} \times \mathbf{B})$$

$$\frac{\partial f_i}{\partial t} + v_i \,.\nabla f_i - Z_{ie}(\mathbf{E} + \frac{\boldsymbol{v_i}}{\boldsymbol{c}} \times \mathbf{B})$$

$$\nabla \times B = \frac{4\pi\mathbf{j}}{c} + \frac{1}{c}\frac{\partial B}{\partial t}$$

$$\nabla \times E = -\frac{1}{c}\frac{\partial B}{\partial t}$$

$$\nabla \,.\, E = 4\pi p$$

$$\nabla \,.\, B = 0$$

$$\mathrm{p} = \mathrm{e}\int (Z_i f_i - f_e) d^3 p, \qquad j = e\int Z_i$$

Here e is the elementary charge (e > 0), c is the speed of light, mi is the mass of the ion, E (r,t) and B(r, t) represent collective self- consistent electromagnetic field created in the point **r** at time moment t by all plasma particles. The essential difference of this system of equations from equations for particles in an external electromagnetic field is that the self-consistent electromagnetic field depends in a complex way on the distribution functions of electrons and ions $f_e$(**r,p, t**) and $f_i$ (**r, p,t**).

## Difficulties of the standard kinetic approach

First, Vlasov argues that the standard kinetic approach based on the Boltzmann equation has difficulties when applied to a description of the plasma with long-range Coulomb interaction. He mentions the following problems arising when applying the kinetic theory based on pair collisions to plasma dynamics:

1. Theory of pair collisions disagrees with the discovery by Rayleigh, Irving Langmuir and LewiTonks of natural vibrations in electron plasma.

2. Theory of pair collisions is formally not applicable to Coulomb interaction due to the divergence of the kinetic terms.

3. Theory of pair collisions cannot explain experiments by Harrison Merrill and Harold Webb on anomalous electron scattering in gaseous plasma.

## The Vlasov-Poisson equation -

The Vlasov-Poisson equations are an approximation of the Vlasov-Maxwell equations in the nonrelativistic zero-magnetic field limit:

$$\frac{\partial f_\alpha}{\partial t} + v_\alpha \ . \frac{\partial f_\alpha}{\partial x} + \frac{q_\alpha E}{m_\alpha} . \frac{\partial f_\alpha}{\partial v} = 0,$$

and Poisson's equation for self-consistent electric field:

$\nabla^2 \emptyset + p = 0.$

Here $q_a$ is the particle's electric charge, mq is the particle's mass, E(x, t) is the self-consistent electric field, Ø(x, t) the self- consistent electric potential and p is the electric charge density.

Vlasov-Poisson equations are used to describe various phenomena in plasma, in particular Landau damping and the distributions in a double layer plasma, where they are necessarily strongly non-Maxwellian, and therefore inaccessible to fluid models.

Landau damping and the distributions in a double layer plasma, where they are necessarily strongly non-Maxwellian, and therefore inaccessible to fluid models.

Vlasov suggests that these difficulties originate from the long-range character of Coulomb interaction. He starts with the collisionless Boltzmann equation (sometimes called the Vlasov equation, anachronistically in this context), in generalized coordinates:

$$\frac{d\,f(\boldsymbol{r},\boldsymbol{p},t)}{dt} = 0,$$

explicitly a PDE:

$$\frac{\partial f}{\partial t} + \frac{dr}{dt}\cdot\frac{\partial f}{\partial r} + \frac{dp}{dt}\cdot\frac{\partial f}{\partial p} = 0,$$

and adapted it to the case of a plasma, leadingto the systems of equations shown below.Here f is a general distribution function ofparticles with momentum **p** at coordinates**r**and given time t.

**Continuity equation**

The continuity equation describes how the density changes with time. It can be found by integration of the Vlasov equation over the entire velocity space.

$$\int \frac{d}{dt} f\, d^3 v = \int \frac{\partial}{\partial t} f + (v.\nabla_r) f +$$

After some calculations, one ends up with

$$\frac{\partial}{\partial t} n + \nabla.(nu) = 0.$$

The number density n, and the momentum density nu, are zeroth and first order moments:

$$n = \int f d^3 v$$

$$nu = \int v f d^3 v$$

**The frozen-in approximation**

As for ideal MHD, the plasma can be considered as tied to the magnetic field lines when certain conditions are fulfilled. One often says that the magnetic field lines are frozen into the plasma. The frozen-in conditions can be derived from Vlasov equation.

We introduce the scales T, L and V for time, distance and speed respectively. They represent magnitudes of the different parameters which give large changes in *f*. By large we mean that

$$\frac{\partial f}{\partial t} T \sim f \ \left|\frac{\partial f}{\partial r}\right| L \sim f \ \left|\frac{\partial f}{\partial v}\right| V \sim f$$

We then write

$t' = \frac{t}{T} r' = \frac{r}{L} v' = \frac{v}{V}$.

Vlasov equation can now be written

$$\frac{1}{T}\frac{\partial f}{\partial t'} + \frac{V}{L} v' . \frac{\partial f}{\partial r'} + \frac{q}{mV} \left(E + V v'\right.$$

So far no approximation have been done. To be able to proceed we set V = R$w_{g.}$ where $w_g = \frac{qB}{m}$ is the gyro frequency and R is the gyroradius. By dividing by $w_g$. we get

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$$\frac{1}{w_g T}\frac{\partial f}{\partial t'} + \frac{R}{L}v' \cdot \frac{\partial f}{\partial r'}\left(\frac{E}{VB} + v^i\right)$$

if $1/w_g \ll T$ and $R \ll L,$ the two first terms will be much less than *f*, since

$\partial f/\partial t' \sim f, v' \lesssim 1$ and $\partial f/\partial r' \sim f$ due to the definitions of T, L and V above. Since the last term is of the order of *f*, we can neglect the two first terms and write

$$\left(\frac{E}{VB} + v^i \times \frac{B}{B}\right) \cdot \frac{\partial f}{\partial v'} \approx 0 \Rightarrow (E +$$

This equation can be decomposed into a field aligned and a perpendicular part:

$$E_{||}\frac{\partial f}{\partial v_{||}} + (E_{\perp} + v \times B) \cdot \frac{\partial f}{\partial \mathrm{v}_{\perp}} \approx 0$$

The next step is to write v = $v_0 + \Delta v,$ where

$v_0 \times B = -E_{\perp}$

it will soon be clear why this is done. with this substitution, we get

$$E_{||}\frac{\partial f}{\partial v_{||}} + (\nabla v_{\perp} \times B) \cdot \frac{\partial f}{\partial \mathrm{v}_{\perp}} \approx 0$$

if the parallel electric field is small,

$$(\nabla v_{\perp} \times B).\frac{\partial f}{\partial v_{\perp}} \approx 0$$

This equation means that the distribution is gyrotropic.The mean velocity of a gyrotropic distribution is zero. Hence, Vo is identical with the mean velocity, u, and we have

$$E + u \times B \approx 0$$

To summarize, the gyro period and the gyro radius must be much smaller than the typical times and lengths which give large changes in the distribution function. The gyro radius is often estimated by replacing V with the thermal velocity or the Alfvén velocity. In the latter case R is often called the inertial length. The frozen- in conditions must be evaluated for each particle species separately. Because electrons have much smaller gyro period and gyro radius than ions, the frozen-in conditions will more often be satisfied.

## **Moment equations**

In fluid descriptions of plasmas (see plasma modeling and magnetohydrodynamics (MHD)) one does not consider the velocity distribution. This is achieved by replacing *f*(r, v,t) with plasma moments such as number density n, flow velocity u and pressure p. They are named plasma moments because the n-th moment of f can be found by integrating $v^n f$ over velocity. These variables are only functions of position and time, which means that some information is lost. In multifluid theory, the different particle species are treated as different fluids with different pressures, densities and flow velocities. The equations governing the plasma moments are called the moment or fluid equations.

Below the two most used moment equations are presented (in SI units). Deriving the moment equations from the Vlasov equation requires no assumptions about the distribution function.

## Momentum equation

The rate of change of momentum of a particle is given by the Lorentz equaton:

$$m\frac{dv}{dt} = q(E + v \times B)$$

By using this equation and the Vlasov Equation, the momentum equation for each fluid becomes

$$mn\frac{D}{Dt}u = -\nabla.p + qnE + qnu \times B$$

where p is pressure tensor. The material derivative is

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u.\nabla.$$

The pressure tensor is defined as the particle mass times the covariance matrix of the velocity:

$$p_{ij} = m\int (v_i - v_i)(v_j - v_j) f d^3 v.$$

## Conclusions and perspectives

Two major points are evident from reading the articlescollected in this topical issue.

(1) The level of theoretical and numerical maturity thathas been reached by the investigation of the collectivedynamics of nearly dissipationless electromagnetic orgravitational multi-particle systems it is now becom-ing possible to simulate directly within a well definedmathematical framework and without unrealistic perameter restrictions, fully nonlinear kinetic regimes in6+1-dimensions (six dimensions in phase space plustime). This has enormous implications from costol-ogy to fusion science, also

at the purely conceptuallevel where the use of simplified models can easily endup playing a conservative role.

(2) The extended scope that the Vlasceequation canhurvein fields beyond its standard range of applications,including e.s. the novel field of collective Quantum

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