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Unsteady Viscous Fluid flow of variable permeability through circular tube in the influence of slip velocity

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Abstract: The effect of slip condition at arterial wall for a viscous flow in circular tube under the influence of magnetic field filled with porous medium has been discussed, the subject of interest due to its wide applications during recent years. In this research paper we use the Adomian Fractional Modified Laplace decomposition method. We included the initial term u_0 in the solution so that the initial boundary condition met accurately and the solution tends to great accuracy.

Keywords: Slip velocity, Magnetic field, Porous medium, Adomian Fractional Modified Laplace decomposition method, MHD.

Introduction: The steady and unsteady flow have various applications in the field of chemical engineering, biotechnology, biomechanics, nano sciences, mechanical engineering and all most in all physical sciences. The pulsatile flow through a circular tube filled with porous medium have a wide applications in dialysis of blood in artificial kidney, in the recognition of cardiovascular diseas and to diagnose these problems. Also, today MHD (Magnetohydrodynamics) theory is used to control the blood flow through artery. Many mathematician and scientist have used this concept for pumping of blood through an artery both theortically and experimentally.

Methodology: To consider the body acceleration in the fluid flow found many applications in doing the sports activities and to described the position while suffering in trains and other vehicles. The fluid is taken as unsteady influenced by transverse magnetic field with constant strength B_0. Brinknan – model is applied to stimulate the problem. We consider the Newtonian character of blood. We use the Adomian Fractional modified Laplace decomposition method to find the solution.

Here, we consider a steady and unsteady (both type), viscous, axially symmetric incompressible fluid in a circular tube of radius R. A static transverse magnetic field of moderate strength B_0 is taken by considering the cylindrical polar co-ordinate (r, θ, z), z-axis being coinciding with the axis of tube. Then the equation of motion under body acceleration is given by

$$\mu_{eff}\left(\frac{d^2u}{dr^2}+\frac{1}{r}\frac{du}{dr}\right)-\frac{\mu}{K}u+A_0\cos(\omega_b t+\phi)-\sigma B_0{}^2u=\rho\frac{\partial u}{\partial t} \qquad \ldots(1)$$

The periodic body acceleration G(t) in axial direction is given by

$$G(t) = A_0\cos(\omega_b t+\phi)$$

And pressure gradient is given by

$$-\frac{\partial p}{\partial z}=P_0+P_1\cos(\omega_p t)$$

where P_0 = amplitude of steady flow due to pressure gradient

P_1 = amplitude of oscillatory flow due to pressure gradient

A_0 = amplitude of body acceleration

$\omega_p = 2\pi f_p$ in which f_p is frequency of heart pulse

$\omega_b = 2\pi f_b$ in which f_b is frequency of body acceleration

Using these values in equation , the governing equation reduces to

$$P_0+P_1\cos(\omega_p t)+\mu_{eff}\left(\frac{d^2u}{dr^2}+\frac{1}{r}\frac{du}{dr}\right)-\frac{\mu}{K}u+A_0\cos(\omega_b t+\phi)-\sigma B_0{}^2u=\rho\frac{\partial u}{\partial t} \qquad \ldots(2)$$

Under the boundary condition

r = R u = 0 no slip boundary condition

r = 0, $\frac{du}{dr}=0$ symmetry condition … (3)

Now letting non - dimensional parameters

$$u^*=\frac{\mu u}{P_0R^2},\quad t^*=\omega_p t,\quad r^*=\frac{r}{R},\quad A=\frac{P_1}{P_0},$$

$$B=\frac{A_0}{P_0},\quad \omega=\frac{\omega_b}{\omega_p},$$

Using these in equation (2), we have

$$P_0 + P_1\cos(\omega_p \frac{t^*}{\omega_p}) + \mu_{eff}\left(\frac{d^2\left(\frac{u^* P_0 R^2}{\mu}\right)}{d(r^* R)^2} + \frac{1}{r^* R}\frac{d\left(\frac{u^* P_0 R^2}{\mu}\right)}{d(r^* R)}\right)$$

$$-\frac{\mu}{K}\frac{u^* P_0 R^2}{\mu} + A_0\cos(\omega_b \frac{t^*}{\omega_p} + \phi) - \sigma B_0{}^2 \frac{u^* P_0 R^2}{\mu} = \rho \frac{\partial\left(\frac{u^* P_0 R^2}{\mu}\right)}{\partial\left(\frac{t^*}{\omega_p}\right)}$$

$$\Rightarrow P_0 + P_1\cos(t^*) + \mu_{eff}\left(\frac{P_0 R^2}{\mu R^2}\frac{d^2 u^*}{dr^{*2}} + \frac{P_0 R^2}{r^* \mu R^2}\frac{du^*}{dr^*}\right) - \frac{\mu}{K}\frac{P_0 R^2}{\mu}u^* + A_0\cos(\omega t^* + \phi)$$

$$-\frac{\sigma B_0{}^2 P_0 R^2}{\mu}u^* = \frac{\rho P_0 R^2}{\mu}\omega_p \frac{\partial u^*}{\partial t^*}$$

$$\Rightarrow \mu_{eff}\frac{P_0 R^2}{\mu R^2}\left(\frac{d^2 u^*}{dr^{*2}} + \frac{1}{r^*}\frac{du^*}{dr^*}\right) - \frac{P_0 R^2}{K}u^* + P_0 + P_1\cos t^* + A_0\cos(\omega t^* + \phi)$$

$$-\frac{\sigma B_0{}^2 P_0 R^2}{\mu}u^* = \frac{\rho P_0 R^2}{\mu}\omega_p \frac{\partial u^*}{\partial t^*}$$

Dividing through out by P_0

$$\frac{\mu_{eff}}{\mu}\left(\frac{d^2 u^*}{dr^{*2}} + \frac{1}{r^*}\frac{du^*}{dr^*}\right) - \frac{R^2}{K}u^* + 1 + \frac{P_1}{P_0}\cos t^* + \frac{A_0}{P_0}\cos(\omega t^* + \phi) - \frac{\sigma B_0{}^2 R^2}{\mu}u^*$$

$$= \frac{\rho \omega_p R^2}{\mu}\frac{\partial u^*}{\partial t^*}$$

$$M\left(\frac{d^2 u^*}{dr^{*2}} + \frac{1}{r^*}\frac{du^*}{dr^*}\right) - \frac{1}{Da^2}u^* + 1 + A\cos t^* + B\cos(\omega t^* + \phi) - H^2 u^* = \alpha^2 \frac{\partial u^*}{\partial t^*}$$

where

$\frac{\mu_{eff}}{\mu} = M$ = relative permeability

$Da^2 = \frac{K}{R^2}$ = Darcy number

$H^2 = \frac{\sigma B_0^2 R^2}{\mu}$ = Hartmann number

$$\alpha^2 = \frac{\rho R^2 \omega_p}{\mu} \quad = \text{Womersely number}$$

with boundary condition :-

r = 1   u = 0   no slip condition

r = 0, $\frac{du}{dr} = 0$ symmetry condition

$u(r,0) = \frac{1-r^2}{2M}$ initial condition

Particular Case: when K = constant

Dividing throughout by M

$$\frac{d^2u^*}{dr^{*2}} + \frac{1}{r^*}\frac{du^*}{dr^*} - \frac{1}{Da^2 M}u^* + \frac{1}{M} + \frac{A}{M}\cos t^* + \frac{B}{M}\cos(\omega t^* + \phi) - \frac{H^2}{M}u^* = \frac{\alpha^2}{M}\frac{\partial u^*}{\partial t^*}$$

$$\frac{d^2u^*}{dr^{*2}} + \frac{1}{r^*}\frac{du^*}{dr^*} - \frac{1}{M}\left(\frac{1}{Da^2} + H^2\right)u^* + \frac{1}{M} + \frac{A}{M}\cos t^* + \frac{B}{M}\cos(\omega t^* + \phi) = \frac{\alpha^2}{M}\frac{\partial u^*}{\partial t^*}$$

$$\frac{d^2u^*}{dr^{*2}} + \frac{1}{r^*}\frac{du^*}{dr^*} - \frac{N^2}{M}u^* + \frac{1}{M} + \frac{A}{M}\cos t^* + \frac{B}{M}\cos(\omega t^* + \phi) = \frac{\alpha^2}{M}\frac{\partial u^*}{\partial t^*}$$

where $\frac{1}{Da^2} + H^2 = N^2$

$$\frac{\partial u^*}{\partial t^*} = \frac{M}{\alpha^2}\left[\frac{d^2u^*}{dr^{*2}} + \frac{1}{r^*}\frac{du^*}{dr^*} - \frac{N^2}{M}u^* + \frac{1}{M} + \frac{A}{M}\cos t^* + \frac{B}{M}\cos(\omega t^* + \phi)\right]$$

$$= \frac{M}{\alpha^2}\left[\frac{d^2u^*}{dr^{*2}} + \frac{1}{r^*}\frac{du^*}{dr^*}\right] - \frac{N^2}{\alpha^2}u^* + \frac{1}{\alpha^2} + \frac{A}{\alpha^2}\cos t^* + \frac{B}{\alpha^2}\cos(\omega t^* + \phi)$$

…(4)

For the solution of above equation, we adopt fractional modified Laplace decomposition method. It is the combination of Adomain decomposition method and Laplace decomposition method.

Basic definitions and concepts related to this method are discussed in the last chapter. Now equation(4) in the symbolic form can be written as :-

$$\frac{\partial u}{\partial t}=\frac{M}{\alpha^2}\left[\frac{d^2u}{dr^2}+\frac{1}{r}\frac{du}{dr}\right]-\frac{N^2}{\alpha^2}u+\frac{1}{\alpha^2}+\frac{A}{\alpha^2}\cos t+\frac{B}{\alpha^2}\cos(\omega t+\phi)$$

(by removing astrick for simplicity)

$$L_t(u)=\frac{M}{\alpha^2}\left[L_{rr}(u)+\frac{1}{r}L_r(u)\right]-\frac{N^2}{\alpha^2}u+\frac{1}{\alpha^2}+\frac{A}{\alpha^2}\cos t+\frac{B}{\alpha^2}\cos(\omega t+\phi)$$

and

$$u(r,t)=\frac{1}{\alpha^2}\left\{\begin{matrix}\left[\sum_{k=0}^{m-1}\frac{\partial^k u(r,0)}{\partial t^k}\frac{t^k}{k!}\right]+MJ\left[L_{rr}(u)+\frac{1}{r}L_r(u)\right]-J\left(N^2u\right)+J\left(A\cos t\right)+J(1)\\ +J\left(B\cos(\omega t+\phi)\right)\end{matrix}\right\}$$

or

$$u(r,t)=\frac{1}{\alpha^2}\left\{\begin{matrix}\left[\sum_{k=0}^{m-1}\frac{\partial^k u(r,0)}{\partial t^k}\frac{t^k}{k!}\right]+MJ\left[L_{rr}(u)+\frac{1}{r}L_r(u)\right]-N^2J\left(u\right)+AJ\left(\cos t\right)+J(1)\\ +BJ\left(\cos(\omega t+\phi)\right)\end{matrix}\right\}$$

and the solution of flow field is the summation of this series.

Now, $u_0(r,t)=\frac{1}{\alpha^2}\left\{\left[\sum_{k=0}^{1}\frac{\partial^k u(r,0)}{\partial t^k}\frac{1}{1!}\right]+AJ\left(\cos t\right)+J(1)+BJ\left(\cos(\omega t+\phi)\right)\right\}$

$$=\frac{1}{\alpha^2}\left\{\left(\frac{1-r^2}{2M}\right)+t+A\int_0^t\cos z\,dz+B\int_0^t\cos(\omega z+\phi)dz\right\}$$

$$=\frac{1}{\alpha^2}\left\{\left(\frac{1-r^2}{2M}\right)+t+A(\sin t)+B\frac{\sin(\omega t+\phi)}{\omega}\right\}$$

$$=\frac{1}{\alpha^2}\left(\frac{1-r^2}{2M}\right)+\frac{1}{\alpha^2}t+\frac{A}{\alpha^2}(\sin t)+\frac{B}{\alpha^2\omega}\sin(\omega t+\phi)$$

$$=A_{01}\left(1-r^2\right)+A_{02}t+A_{03}\sin t+A_{04}\sin(\omega t+\phi) \qquad \dots (5)$$

where

$$A_{01}=\frac{1}{2M\alpha^2}$$

$$A_{02}=\frac{1}{\alpha^2}$$

$$A_{03}=\frac{A}{\alpha^2}$$

$$A_{04}=\frac{B}{\alpha^2\omega}$$

Also $u_{n+1}(r,t)=\frac{M}{\alpha^2}J\left[L_{rr}(u_n)+\frac{1}{r}L_r(u_n)\right]-\frac{N^2}{\alpha^2}J(u_n)$ …(6)

Letting n = 0

$$u_1(r,t)=\frac{M}{\alpha^2}J\left[L_{rr}(u_0)+\frac{1}{r}L_r(u_0)\right]-\frac{N^2}{\alpha^2}J(u_0)$$

$$=\frac{M}{\alpha^2}\left[J\left\{\begin{array}{l}\frac{\partial^2}{\partial r^2}\left[A_{01}\left(1-r^2\right)+A_{02}t+A_{03}\sin t+A_{04}\sin(\omega t+\phi)\right]+\\ \frac{1}{r}\frac{\partial}{\partial r}\left[A_{01}\left(1-r^2\right)+A_{02}t+A_{03}\sin t+A_{04}\sin(\omega t+\phi)\right]\end{array}\right\}\right]$$

$$-\frac{N^2}{\alpha^2}J\left[A_{01}\left(1-r^2\right)+A_{02}t+A_{03}\sin t+A_{04}\sin(\omega t+\phi)\right]$$

$$=\frac{M}{\alpha^2}J\left[A_{01}(-2)+\frac{1}{r}A_{01}(-2r)\right]-\frac{N^2}{\alpha^2}J\left[\begin{array}{l}A_{01}\left(1-r^2\right)+A_{02}t+A_{03}\sin t\\ +A_{04}\sin(\omega t+\phi)\end{array}\right]$$

$$=\frac{M}{\alpha^2}J\left[-4A_{01}\right]-\frac{N^2}{\alpha^2}J\left[A_{01}\left(1-r^2\right)+A_{02}t+A_{03}\sin t+A_{04}\sin(\omega t+\phi)\right]$$

$$=\frac{-4M}{\alpha^2}A_{01}t-\frac{N^2}{\alpha^2}A_{01}\left(1-r^2\right)t-\frac{N^2}{\alpha^2}A_{02}\frac{t^2}{2!}+\frac{N^2}{\alpha^2}A_{03}\cos t-\frac{N^2}{\alpha^2}A_{03}$$

$$+\frac{N^2}{\alpha^2}A_{04}\frac{\cos(\omega t+\phi)}{\omega}+\frac{N^2}{\alpha^2}A_{04}\cos\phi$$

$$=\left(\frac{-4M}{\alpha^2}A_{01}\right)t+\left(-\frac{N^2}{\alpha^2}A_{01}\right)\left(1-r^2\right)t+\left(-\frac{N^2}{2\alpha^2}A_{02}\right)t^2+\left(\frac{N^2}{\alpha^2}A_{03}\right)\cos t$$

$$+\left(\frac{N^2}{\alpha^2\omega}A_{04}\right)\cos(\omega t+\phi)+\left(-\frac{N^2}{\alpha^2}A_{03}+\frac{N^2}{\alpha^2}A_{04}\cos\phi\right)$$

$$u_1(r,t)=A_{11}t+A_{12}\left(1-r^2\right)t+A_{13}t^2+A_{14}\cos t+A_{15}\cos(\omega t+\phi)+A_{16} \qquad \ldots(7)$$

where

$$A_{11} = -\frac{4M}{\alpha^2} A_{01}$$

$$A_{12} = -\frac{N^2}{\alpha^2} A_{01}$$

$$A_{13} = -\frac{N^2}{2\alpha^2} A_{03}$$

$$A_{14} = \frac{N^2}{\alpha^2} A_{03}$$

$$A_{15} = \frac{N^2}{\alpha^2 \omega} A_{04}$$

$$A_{16} = -\frac{N^2}{\alpha^2} A_{03} + \frac{N^2}{\alpha^2} A_{04} \cos\phi$$

Letting n = 1

$$u_2(r,t) = \frac{M}{\alpha^2} J\left[L_{rr}(u_1) + \frac{1}{r} L_r(u_1) \right] - \frac{N^2}{\alpha^2} J(u_1)$$

Now $J(L_{rr}(u_1)) = J\left[\frac{\partial^2}{\partial r^2} \left\{ \begin{array}{l} A_{11}t + A_{12}(1-r^2)t + A_{13}t^2 + A_{14}\cos t + A_{15}\cos(\omega t + \phi) \\ + A_{16} \end{array} \right\} \right]$

$$= J[0 + A_{12}(-2)t + 0 + 0 + 0 + 0]$$

$$= -2A_{12}\frac{t^2}{2!} = -A_{12}t^2$$

$$J\left(\frac{1}{r} L_r(u_1) \right) = J\left[\frac{1}{r}\frac{\partial}{\partial r} \{ A_{11}t + A_{12}(1-r^2)t + A_{13}t^2 + A_{14}\cos t + A_{15}\cos(\omega t + \phi) + A_{16} \} \right]$$

$$= J\left[\frac{1}{r} \{ 0 + A_{12}(-2r)t + 0 + 0 + 0 + 0 \} \right]$$

$$= J(-2A_{12}t) = -2A_{12}\frac{t^2}{2!} = -A_{12}t^2$$

Also $J(u_1) = J\left[\begin{array}{l} A_{11}t + A_{12}(1-r^2)t + A_{13}t^2 + A_{14}\cos t + A_{15}\cos(\omega t + \phi) \\ + A_{16} \end{array} \right]$

$$= A_{11}\frac{t^2}{2} + A_{12}\left(1-r^2\right)\frac{t^2}{2} + A_{13}\frac{t^3}{3} + A_{14}\sin t + A_{15}\frac{\sin(\omega t+\phi)}{\omega}$$
$$+ A_{16}t$$
$$= A_{16}t + \frac{A_{11}}{2}t^2 + \frac{A_{12}}{2}\left(1-r^2\right)t^2 + \frac{A_{13}}{3}t^3 + A_{14}\sin t$$
$$+ \frac{A_{15}}{\omega}\sin(\omega t+\phi)$$

Using all these values in equation (B), we have

$$u_2(r,t) = \frac{M}{\alpha^2}\left[-A_{12}t^2 - A_{12}t^2\right] - \frac{N^2}{\alpha^2}\left[\begin{array}{l} A_{16}t + \frac{A_{11}}{2}t^2 + \frac{A_{12}}{2}\left(1-r^2\right)t^2 + \frac{A_{13}}{3}t^3 + A_{14}\sin t \\ + \frac{A_{15}}{\omega}\sin(\omega t+\phi) \end{array}\right]$$
$$= \left(-\frac{2M}{\alpha^2}A_{12}\right)t^2 + \left(-\frac{N^2}{\alpha^2}A_{16}\right)t - \frac{N^2}{\alpha^2}\frac{A_{11}}{2}t^2 - \frac{N^2}{\alpha^2}\frac{A_{12}}{2}\left(1-r^2\right)t^2 - \frac{N^2}{\alpha^2}\frac{A_{13}}{3}t^3$$
$$- \frac{N^2}{\alpha^2}A_{14}\sin t - \frac{N^2}{\alpha^2}\frac{A_{15}}{\omega}\sin(\omega t+\phi)$$
$$= \left(-\frac{N^2}{\alpha^2}A_{16}\right)t + \left(-\frac{2M}{\alpha^2}A_{12} - \frac{N^2}{2\alpha^2}A_{11}\right)t^2 - \frac{N^2}{2\alpha^2}A_{12}\left(1-r^2\right)t^2 - \frac{N^2}{3\alpha^2}A_{13}t^3$$
$$- \frac{N^2}{\alpha^2}A_{14}\sin t - \frac{N^2}{\alpha^2\omega}A_{15}\sin(\omega t+\phi)$$
$$= A_{21}t + A_{22}t^2 + A_{23}\left(1-r^2\right)t^2 + A_{24}t^3 + A_{25}\sin t + A_{26}\sin(\omega t+\phi) \qquad \ldots(8)$$

where

$$A_{21} = -\frac{N^2}{\alpha^2}A_{16}$$

$$A_{22} = -\frac{2M}{\alpha^2}A_{12} - \frac{N^2}{2\alpha^2}A_{11}$$

$$A_{23} = -\frac{N^2}{2\alpha^2}A_{13}$$

$$A_{24} = -\frac{N^2}{3\alpha^2}A_{13}$$

$$A_{25} = -\frac{N^2}{\alpha^2}A_{14}$$

$$A_{26} = -\frac{N^2}{\alpha^2\omega}A_{15}$$

Similarly we can calculate the values of u_3, u_4, ---------

$$\therefore u(r,t) = u_0(r,t) + u_1(r,t) + u_2(r,t) + u_3(r,t) + ------------$$
$$= A_{01}(1-r^2) + A_{02}t + A_{03}\sin t + A_{04}\sin(\omega t+\phi) + A_{12}(1-r^2) + A_{11}t + A_{13}t^2$$
$$+ A_{14}\cos t + A_{15}\cos(\omega t+\phi) + A_{16} + A_{21}t + A_{22}t^2 + A_{23}\left(1-r^2\right)t^2 + A_{24}t^3$$
$$+ A_{25}\sin t + A_{26}\sin(\omega t+\phi) + -----$$

$$\therefore u(r,t) = A_{16} + (1-r^2)\left[A_{01} + A_{12}t + A_{23}t^2\right] + (A_{02} + A_{11} + A_{21})t + (A_{13} + A_{22})t^2 + A_{24}t^3 + (A_{03} + A_{25})\sin t + (A_{04} + A_{26})\sin(\omega t + \phi) + A_{14}\cos t + A_{15}\cos(\omega t + \phi) + - - - - - - -$$

$$\therefore u(r,t) = A_{16} + (1-r^2)\left[A_{01} + A_{12}\, t + A_{23}\, t^2\right] + A\, t + B\ \ t^2 + A_{24}t^3 + C\sin t + D\sin\,(\omega\, t + \phi) + A_{14}\cos\, t + A_{15}\cos\,(\omega\, t + \phi) + - - - - - - - - - -$$

…(9)

where

$$A = A_{01} + A_{11} + A_{21}$$

$$B = A_{13} + A_{22}$$

$$C = A_{03} + A_{25}$$

$$D = A_{04} + A_{26}$$

which gives the expression of velocity flow field.

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