Illustrative study on intuitionistic fuzzy hyper-graphs and dual-intuitionistic fuzzy hyper-graphs

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Abstract

Kaufmann [5] suggested the notion of a fuzzy hyper-graph. The notion of fuzzy hyper-graph is expanded by H. Lee Kwang (1995)[3]. The definitions of intuitionistic fuzzy hyper-graph (IFHG) and dual intuitionistic fuzzy hyper-graph (DIFHG) and their applications are based on this$(\alpha\ ,\ \beta)$*cuts, strength of an edge have been given and are illustrated by suitable examples.*

Keywords: *fuzzy graph, fuzzy hyper-graph, dual-Intuitionistic fuzzy hyper-graphs*

1. INTRODUCTION

In this paper, will go through some of the fundamental concepts and conclusions that are needed to create intuitionistic fuzzy hyper-graphs. With reference to dual intuitionistic fuzzy hyper-graph (DIFHG) and their applications are based on this$(\alpha\ ,\ \beta)$cuts, strength of an edge have been given and are illustrated by suitable examples.

2. INTUITIONISTIC FUZZY HYPER-GRAPHS

Definition 2.1

The support of an IFS A, denoted by supp (A), is defined as

$supp(A) = \{v : \mu_A(v) > 0 \quad and \; \nu_A(v) > 0\}$the support of the intuitionistic fuzzy set is a crisp set.

Definition 2.2

An intuitionistic fuzzy hyper-graph (IFHG), H is an ordered pair $H = (V, E)$ where

1. $V = \{v_1, \; v_2, \dots \dots .., v_n\}$, a finite set of intuitionistic fuzzy vertices,
2. $E = \{e_1, e_2, \dots .., e_m\}$, a family of intuitionistic fuzzy subsets of V ,
3. $e_j = \{\left(v_i, \mu_j(v_i), \nu_j(v_i)\right) : \mu_j(v_i), \; \nu_j(v_i) \geq 0 \quad and \quad 0 \leq \mu_j(v_i) + \nu_j(v_i) \leq 1\}$
   $j = 1, 2, \dots, m,$
4. $e_j \neq \phi, \quad j = 1,2,3, \dots \dots, m$
5. $\cup_j \; supp\left(e_j\right) = V, \quad j = 1,2, \dots .., m$

Here, the edges e_j are IFSs. $\mu_j(v_i)$and$\nu_j(v_i)$ Indicate the degrees of vertex v_i's membership and non-membership to edge e_j. As a result, the elements of the IFHG incidence matrix

Are of the form$\left(a_{ij}, \; \mu_j(v_i), \; \nu_j(v_i)\right)$. The sets V and E are crisp sets.

Example 2.3

Consider an IFHG $H = (V, E)$ such that $V = \{v_1, \; v_2, v_3, \; v_4, v_5, v_6, \; v_7\}$, $E = \{e_1, e_2, e_3\}$. Here,

$e_1 = \{(v_1, 0.3, \; 0.2), \; (v_2, \; 0.3, \; 0.4), \; (v_4, \; 0.7, \; 0.2)\},$
$e_2 = \{(v_2, 0.3, \; 0.4), \; (v_3, \; 0.6, \; 0.1), \; (v_5, \; 0.4, \; 0.5)\},$
$e_3 = \{(v_4, 0.7, 0.2), \; (v_5, 0.4, 0.5), \; (v_6, 0.1, 0.2), \; (v_7, 0.8, 0.1)\}$

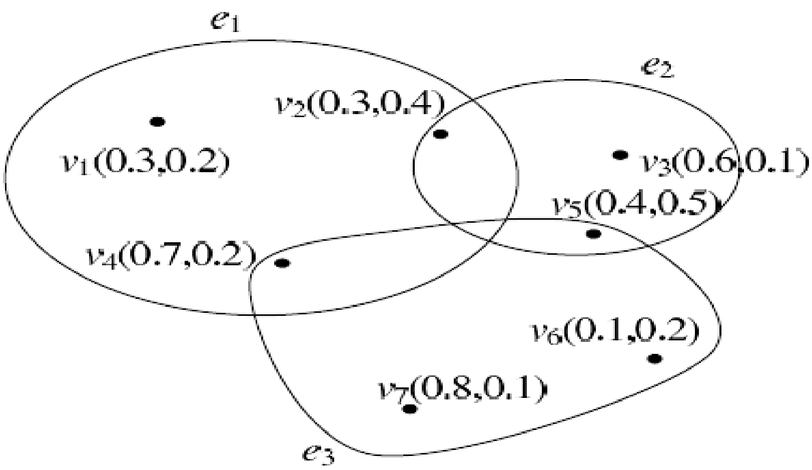

Figure 2.1: Intuitionistic fuzzy hyper-graph

The corresponding incidence matrix M_H is as follows.

M_H	e_1	e_2	e_3
v_1	$(0.3, 0.2)$	$(0, 0)$	$(0, 0)$
v_2	$(0.3, 0.4)$	$(0.3, 0.4)$	$(0, 0)$
v_3	$(0, 0)$	$(0.6, 0.1)$	$(0, 0)$
v_4	$(0.7, 0.2)$	$(0, 0)$	$(0.7, 0.2)$
v_5	$(0, 0)$	$(0.4, 0.5)$	$(0.4, 0.5)$
v_6	$(0, 0)$	$(0, 0)$	$(0.1, 0.2)$
v_7	$(0, 0)$	$(0, 0)$	$(0.8, 0.1)$

Definition 2.4

In an IFHG, the adjacent level between two vertices v_r and v_s, denoted by, $\gamma(v_r, v_s)$, is defined as

$$\gamma(v_r,\ v_s) = max_j\left(min\left(\mu_j(v_r), \mu_j(v_s)\right)\right),\ min_j\left(max\left(v_j(v_r), v_j(v_s)\right)\right), j\ =\ 1, 2, \ldots, m.$$

Definition 2.5

In an IFHG, the adjacent level between the edges e_j and e_k , denoted by $\sigma(e_j,\ e_k)$, is defined by $\sigma(e_j,\ e_k) = max_j\left(min\left(\mu_j(v), \mu_k(v)\right)\right),\ min_j\left(max\left(v_j(v), v_k(v)\right)\right).$

The neighbouring level between vertices v_5 and v_7 in Figure 2.1is $(0.4, 0.5)$, whereas the adjacent level between edges e_2 and e_3 is $(0.4, 0.5). (0.3, 0.5)$.

Definition 2.6

The $(\alpha,\ \beta) -$ cut of an IFHG, H denoted by $H_{(\alpha,\beta)}$, is an ordered pair $H_{(\alpha,\beta)} = \left(V_{(\alpha,\beta)},\ e_{(\alpha,\beta)}\right)$ where

1. $V_{(\alpha,\beta)} = \{v_1, \dots \dots, v_n\}$
2. $e_{(\alpha,\beta)} = \{v_i : \mu_j(v_i) \geq \alpha \quad or \quad v_j(v_i) \leq \beta, \quad j = 1,2, \dots \dots, m\}$
3. $e_{m+1,(\alpha,\beta)} = \{v_i : \mu_j(v_i) < \alpha \quad or \quad v_j(v_i) > \beta \quad for\ every\ j\}$

Note

(i) The edge $e_{m+1,(\alpha,\beta)}$is added to group the elements which are not contained in any edge $e_{j,(\alpha,\beta)}$ of $H_{(\alpha,\beta)}$.

(ii) The edges in the $(\alpha,\ \beta) -$ cut hyper-graph are now crisp sets.

Example 2.7

Take a look at the IFHG H in Figure 2.1. Figure 2.2 depicts the $(0.4, 0.2)$ - cut of IFHG H. The additional edges $e_4, (0.4,0.2), e_5, (0.4,0.2), e_6, (0.4,0.2), e_7, (0.4,0.2)$ are added to all edges, and they include the vertices v_1, v_2, v_5, v_6 with membership values of 0.4 or non-membership values of > 0.2.

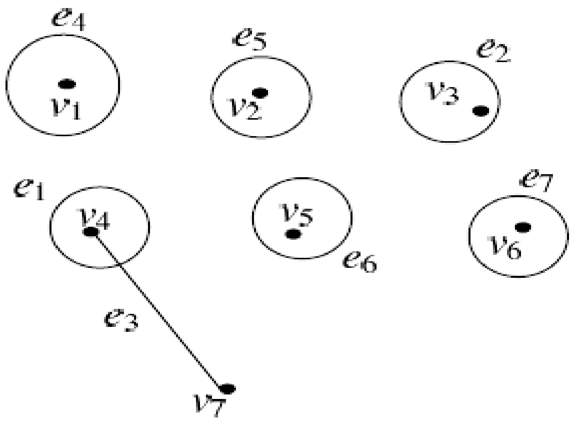

Figure 2.2: (0.4, 0.2) - cut

The incidence matrix of $H_{(0.4,0.2)}$ is as follows.

$H_{(0.4,0.2)}$	$e_{1,(0.4,0.2)}$	$e_{2,(0.4,0.2)}$	$e_{3,(0.4,0.2)}$	$e_{4,(0.4,0.2)}$	$e_{5,(0.4,0.2)}$	$e_{6,(0.4,0.2)}$	$e_{7,(0.4,0.2)}$
v_1	0	0	0	1	0	0	0
v_2	0	0	0	0	1	0	0
v_3	0	1	0	0	0	0	0
v_4	1	0	1	0	0	0	0
v_5	0	0	0	0	0	1	0
v_6	0	0	0	0	0	0	1
v_7	0	0	1	0	0	0	0

Definition 2.8

The strength δ of an edge e_j is the minimum membership value $\mu_j(v)$and maximum non-membership value$\nu_j(v)$of vertices in the edge e_j. That is,

$\delta(e_j) = \left(min_v\left(\mu_j(v)\right), max_v\left(\nu_j(v)\right)\right)$for every $\mu_j(v) > 0, \nu_j(v) > 0$.

Example 2.9

In Figure 2.12, the strength of the edges are

$$\delta(e_1) = (0.3, 0.4), \delta(e_2) = (0.3, 0.5), \qquad \delta(e_3) = (0.1, 0.5)$$

Note: The edge with the most membership and the least non-membership is considered to be the strongest among the edges. As a result, in Figure 2.2, the edge e_1 is stronger than the edges $e_2$and e_3.

3. DUAL INTUITIONISTIC FUZZY HYPER-GRAPHS

Definition 3.1

If an IFHG $H = (V; e_1, e_2, \dots \dots, e_m)$, $V = (v_1, v_2, \dots \dots, v_n)$ is given, its dual intuitionistic fuzzy hyper-graph (DIFHG) $H^* = (E;\ V_1, V_2, \dots . . V_n)$ where

(i) $E = (E_1, E_2, \dots \dots, E_n)$, set of vertices corresponding to $e_1, e_2, \dots \dots, e_m$ respectively.

(ii) $V_j = \left\{\left(E_j, \mu_j(E_j), v_j(E_j)\right) : \mu_i(E_j) = \mu_j(v_i),\ v_i(E_j) = v_j(v_i)\right\}$.

Example 3.2

The DIFHG $H^* = \{E, V_1, V_2, V_3, V_4, V_5, V_6, V_7\}$of IFHG H in Figure 2.12 is given below. Here, $E = \{E_1, E_2, E_3\}$, $V_1 = \{(E_1, 0.3, 0.4)\}$, $V_2 = \{(E_1, 0.3, 0.2), (E_2, 0.3, 0.4)\}$,

$$V_3 = \{(E_2, 0.6, 0.1)\},\ V_4 = \{(E_1, 0.7, 0.2), (E_3, 0.7, 0.2)\},$$

$V_5 = \{(E_2, 0.4, 0.5), (E_3, 0.4, 0.5)\}$, $V_6 = \{(E_3, 0.1, 0.2)\}$, $V_7 = \{(E_3, 0.8, 0.1)\}$.

The diagrammatic representation is shown in Figure 3.1

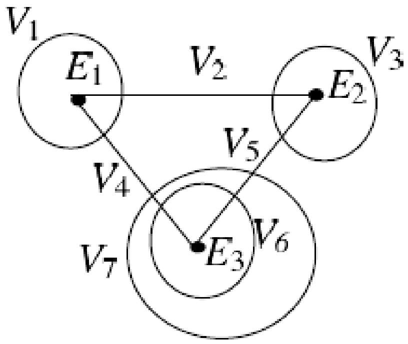

Figure 3.1:$\boldsymbol{H}^*$ - Dual of H

The corresponding incidence matrix is

M_H^*	V_1	V_2	V_3	V_4	V_5	V_6	V_7
E_1	(0.3, 0.2)	(0.3, 0.4)	(0, 0)	(0.7, 0.2)	(0, 0)	(0, 0)	(0, 0)
E_2	(0, 0)	(0.3, 0.4)	(0.6, 0.1)	(0, 0)	(0.4, 0.5)	(0, 0)	(0, 0)
E_3	(0, 0)	(0, 0)	(0, 0)	(0.7, 0.2)	(0.4, 0.5)	(0.1, 0.2)	(0.8, 0.1)

The (0.4, 0.2) - cut IFHG $H^*_{(0.4,0.2)}$is shown in Figure.3.2.

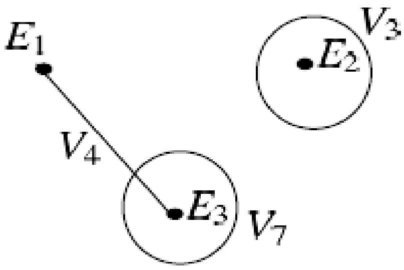

Figure 3.2:$\boldsymbol{M^*_{H(0.4,0.2)}}$

The corresponding incidence matrix $M^*_{H(0.4,0.2)}$is

$M^*_{H_{0.4,0.2}}$	V_1	V_2	V_3	V_4	V_5	V_6	V_7
E_1	0	0	0	1	0	0	1
E_2	0	0	1	0	0	0	0
E_3	0	0	0	1	0	0	1

4. REVIEW OF LITERATURE

It is important to identify the applications of intuitionistic fuzzy set theory in many domains such as decision making, quantitative analysis, and information processing, among others. The necessity for knowledge-handling systems capable of dealing with and discriminating between distinct facts of imprecision necessitates a clear and formal characterisation of the mathematical models that perform such analyses, which is done using intuitionistic fuzzy hyper-graphs and dual intuitionistic fuzzy hyper-graphs(DIFHG).

5. OBJECTIVES OF THE THESIS

The objective of this research work is

1. To learn more about intuitionistic fuzzy hyper-graphs and dual intuitive fuzzy hyper-graphs.

2. Investigate the characteristics of the generalisedintuitionistic fuzzy hyper-graph and the generaliseddual intuitionistic fuzzy hyper-graph.

3. For intuitionistic fuzzy hyper-graphs and dual intuitionistic fuzzy hyper-graphs, get line graph construction to use the knowledge in real-world applications.

6.CONCLUSION

On the inspiration of intuitionistic fuzzy graphs (IFGs) and fuzzy hyper-graphs (undirected and directed), this work extends to an intuitionistic fuzzy hyper-graphs (undirected and directed)(IFHGs). Based at the definition of IFHGs, twin of an IFHG, Strength, (α, β) - reduce are defined. The (α, β) reduce used to generate the sub-hyper-graphs may bevarious with distinct values of (α, β). The 2delementis dedicated to intuitionistic fuzzy directed graphs (IFDGs), intuitionistic fuzzy directed hyper-graphs (IFDHGs), dual intuitionistic fuzzy hyper-graph(DIFHG) and their index matrix representation.

REFERENCE

[1] Akram M and Dudek W.D, “Intuitionistic fuzzy hyper-graphs with applications”, Information Sciences, 2012.

[2] Bershtein L.S and Bozhenyuk, “Fuzzy Graphs and Fuzzy Hyper-graphs”, Encycl.Artif.Intell (2009)704-709.

[3] H. Lee-Kwang and K. M. Lee, Fuzzy hyper-graph and fuzzy partition, IEEE Transactions on Systems, Man and Cybernetics, 25 (1), 1995, 196 - 201.

[4] J. N. Mordeson and P. S. Nair, Fuzzy graphs and fuzzy hyper-graphs, Physicaverlag, Heidelberg 1998, Second Edition, 2001.

[5] Kaufmann A,”Introduction to the Theory of Fuzzy Subsets”, Vol. 1, Academic Press, New York, 1975.

[6] M. Akram and B. Davvaz, Strong intuitionistic fuzzy graphs, Filomat, 26 (1), 2012, 177 - 195.

[7] Moderson N. John, Nair S. Premchand, ”Fuzzy Graphs and Fuzzy Hyper-graphs”, New York, Physica-Verlog,2000.

[8] ParvathiR, Thilagavathi.S and Karunambigai.M.G, ”Intuitionistic Fuzzy Hyper-graphs”, Cybernetics and Information Technologies, Vol 9,No.2,2009,46-53.

[9] R.Parvathi and S.Thilagavathi, Intuitionistic fuzzy directed hyper-graphs, Advances in Fuzzy Sets and Systems, 14 (1), 2012, 39 - 52.

[10] Shanon and K. T. Atanassov, Intuitionistic fuzzy graphs from α, β and (α, β) - levels, Notes on Intuitionistic Fuzzy Sets, 1 (1), 1995, 32 - 35.

[11] Thilagavathi S, Parvathi R. and Karunambigai M.G, ” Intuitionistic fuzzy hyper-graphs”, J. Cybernetics and Information Technologies, 9(2), (2009), p. 46-53 Sofia.

[12] Tyshkevich,R.I,Zverovich,V.E,Line,"Hyper-graphs",DiscreteMathematics,Vol 161,1996,265-283.