SOME ZWEIER I-CONVERGENT DIFFERENCE & LACUNARY STRONG ZWEIER DOUBLE CONVERGENT SEQUENCE SPACES

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Abstract

This article discusses some new Zweier sequence spaces. The detailed explanation is divided into three different sections. The first section deals with the study of some Zweier I-convergent difference sequence spaces in which we introduce the sequence spaces Z I (M, Δr , p, u), Z I 0 (M, Δr , p, u) and Z I ∞(M, Δr , p, u). We have also made an attempt to study topological properties and algebraic properties. A study of new class of generalized lacunary strong Zweier Convergent Sequence spaces $[N\theta_{rs}$, Z,M, Δn , p, u, k·, ..., ·k]0, $[N\theta_{r,s}$, Z,M, Δr , p, u, k·, ..., ·k] and $[N\theta_{r,s}$, Z,M, Δr , p, u, k·, ..., ·k]∞ is introduced in the second section. We have also studied the different properties of these spaces like linearity, paranorm, solid, monotone etc. Some interesting inclusion relations between these sequence spaces are also established with counter examples. At the end of this section we give the concept of double lacunary statistical Zweier convergence over n-normed spaces. Here we prove that the spaces are neither monotone nor solid, not convergence free in general.

Keywords: Spaces, Sequence, Zweier etc.

1. INTRODUCTION

The sequence spaces played an important role in the study of pure and applied Mathematics. In several branches of analysis, for instance, the structural theory of topological vector spaces, Schauder basis theory, summability theory and the theory of functions, the study of sequence spaces occupies a very prominent position [1]. The impact and importance of this study can be appreciated when one sees the construction of numerous examples of locally convex spaces obtained as a consequence of the dual structure displayed by several pairs of distinct sequence spaces, thus reflecting in depth the distinguishing structural features of the spaces in question. Besides these distinct sequence spaces endowed with different polar topologies provide an excellent source to vector spaces pathologists for the introduction on locally convex spaces to several new and penetrating notions implicit in the theory of Banach spaces [2]. Apart from this, the theory of sequence spaces is a powerful tool for obtaining positive results concerning Schauder bases and their associated types. Moreover, the theory of sequence spaces has made remarkable advances in enveloping summability theory via unified techniques effecting matrix transformation from one sequence space into another [3].

2. ON SOME ZWEIER I-CONVERGENT DIFFERENCE SEQUENCE SPACES

The main implication of this section is to introduce the sequence spaces Z^I (M, Δ^r, p, u), z_0^I(M, Δ^r, p, u) and z_∞^1(M, Δ^r, p, u). An endeavor has been made to study some topological properties and inclusion relations between these sequence spaces [4].

Throughout this section Z^I , z_0^I, z_∞^I, m_{z0}^I and mI Z0 represents Zweier Iconvergent, Zweier I-null, Zweier bounded I-convergent and Zweier bounded I-null sequence spaces, respectively.

Let M = (M_k) be a sequence of Orlicz functions, u = (u_k) be a sequence of strictly positive real numbers and p = (p_k) be a bounded sequence of positive real numbers. We define the following sequence spaces [5]:

$$Z^I(\mathcal{M},\Delta^r,p,u) =$$

$$\left\{x=(x_n):\left\{k\in\mathbb{N}:\sum_{k=1}^{\infty}M_k\left[\frac{|u_k(Z^p(\Delta^r x))_k-L|}{\rho}\right]^{p_k}\geq\epsilon\right\}\in I\right\}$$

for some L ∈ C.

$$Z_0^I(\mathcal{M},\Delta^r,p,u) =$$

$$\left\{x=(x_n):\left\{k\in\mathbb{N}:\sum_{k=1}^{\infty}M_k\left[\frac{|u_k(Z^p(\Delta^r x))_k|}{\rho}\right]^{p_k}\geq\epsilon\right\}\in I\right\}.$$

$$Z_\infty^I(\mathcal{M},\Delta^r,p,u) =$$

$$\left\{x=(x_n):\left\{k\in\mathbb{N}:\exists\, K>0,\sum_{k=1}^{\infty}M_k\left[\frac{|u_k(Z^p(\Delta^r x))_k|}{\rho}\right]^{p_k}\geq K\right\}\in I\right\}.$$

If we take $(M_k) = M$, $(p_k) = 1$, $(u_k) = 1$ for all $k \in N$ and $r = 0$, then we get the sequence spaces defined [6].

Theorem 2.1 The spaces $z_0^I(M, \Delta^r, p, u)$ and $m_{z0}^I(M, \Delta^r, p, u)$ are solid and monotone.

Proof. We shall prove the result for $z_0^I(M, \Delta^r, p, u)$. For $m_{z0}^I(M, \Delta^r, p, u)$, the result can be proved similarly [7]. Let $x = (x_n) \in Z_0^I\,(M, \Delta^r, p, u)$, then there exists $\rho > 0$ such that

$$\left\{k \in \mathbb{N} : \sum_{k=1}^{\infty} M_k \left[\frac{|u_k(Z^p(\Delta^r x))_k|}{\rho}\right]^{p_k} \geq \epsilon\right\} \in I. \tag{1}$$

Let (α_n) be a sequence scalar with $|\alpha n| \leq 1$ for all $n \in N$. Then the result follows from (1) and the following inequality

$$M_k \left[\frac{|\alpha_n u_k(Z^p(\Delta^r x))_k|}{\rho}\right]^{p_k} \leq |\alpha_n| M_k \left[\frac{|u_k(Z^p(\Delta^r x))_k|}{\rho}\right]^{p_k} \leq M_k \left[\frac{|u_k(Z^p(\Delta^r x))_k|}{\rho}\right]^{p_k}$$

for all $n \in N$. The space $z_0^I(M, \Delta^r, p, u)$ is monotone follows from the Lemma, [8].

Theorem 2.2 The spaces $Z^I(M, \Delta^r, p, u)$ and $m_Z^I(M, \Delta^r, p, u)$ are neither monotone nor solid in general [9].

Proof. The proof of this result follows from the following example. Let $I = I_f$, $M_k(x) = x$ for all $x \in [0,\infty)$, $p = (p_k) = 1$, $u = (u_k) = 1$, for all k and $r = 0$. Consider the K-step space T_K of T defined as follows [10] Let $(x_n) \in T$ and $(y_n) \in T_K$ be such that

$$y_n = \begin{cases} x_n, & \text{if } n \text{ is odd.} \\ 0, & \text{otherwise.} \end{cases}$$

Consider the sequence (x_n) defined by $x_n = 1\ 2$ for all $n \in N$. Then $(x_n) \in Z^I\,(M, \Delta^r, p, u)$ but its K- step space preimage does not belongs to $Z^I(M, \Delta^r, p, u)$. Thus $Z^I\,(M, \Delta^r, p, u)$ are not monotone.

Theorem 2.3 The spaces $Z^I(M, \Delta^r, p, u)$ and $z_0^I(M, \Delta^r, p, u)$ are not convergence free in general.

Proof. The proof of this result follows from the following example. Let $I = I^f$, $M_k(x) = x^2$ for all $x \in [0,\infty)$, $u = (u_k) = 1$, $p = (p_k) = 1$ and $r = 0$ for all $k \in N$ [11].

Consider the sequence (x_n) and (y_n) defined by $x_n = 1/n^2$ and $y_n = n^2$ for all $n \in N$. Then (x_n) belongs to $Z^I(M, \Delta^r, p, u)$ and $z_0^I(M, \Delta^r, p, u)$, but (y_n) does not belongs to both $Z^I(M, \Delta^r, p, u)$ and $z_0^I(M, \Delta^r, p, u)$. Hence, the spaces are not convergence free.

3. LACUNARY STRONG ZWEIER DOUBLE CONVERGENT SEQUENCE SPACES

An attempt has been made to introduce double Zweierlacunary strongly convergent sequence spaces over n−normed spaces and study different properties of these spaces like linearity, paranorm, solidity and monotone etc. Some inclusion relations between these spaces are also established. Finally, we study the concept of the double Zweierlacunary statistical convergence over n−normed spaces in this section, [12].

Zweier sequence spaces for single sequences were defined and studied. It is defined the double Zweier sequence spaces $[W^2, Z]$, $[N_{\theta r,s}, Z]0$, $[N_{\theta r,s}, Z]$ and $[N_{\theta r,s}, Z]_\infty$ as the set of all double sequences such that Z−transforms of them are in $[W^2]$, $[N_{\theta r,s}]_0$, $[N_{\theta r,s}]$ and $[N_{\theta r,s}]_\infty$ which were introduced, [13].

We denote the double sequences v = (vkl) and w = (wkl) which will be used throughout this section, the Z− transform of a sequence x = (xkl) and y = (ykl) is defined as

$$v_{kl} = \frac{1}{2}(x_{kl} + x_{kl-1}) \quad \text{and} \quad w_{kl} = \frac{1}{2}(y_{kl} + y_{kl-1}); \quad (k, l \in N).$$

Let $(X, \|\cdot, ..., \cdot\|)$ be an n-normed space and W(n−X) denotes the space of Xvalued sequences. Let $M = (M_{kl})$ be a sequence of Orlicz functions, $p = (p_{kl})$ be a bounded double sequence of positive real numbers and $u = (u_{kl})$ be a double sequence of strictly positive real numbers. Now we define double Zweier sequence spaces as follows [14]:

$$[N_{\theta_{r,s}}, Z, \mathcal{M}, \Delta^n, p, u, \|\cdot, ..., \cdot\|]_0 = \left\{x = (x_{kl}) : P - \lim_{r,s} \frac{1}{h_{r,s}} \sum_{(k,l)\in I_{r,s}} M_{kl}\left[u_{kl}\left(\left\|\frac{\Delta^r v_{kl}}{\rho}, z_1, ..., z_{n-1}\right\|\right)\right]^{p_{kl}} = 0\right.$$

for some ρ > 0 } ,

$$[N_{\theta_{r,s}}, Z, \mathcal{M}, \Delta^r, p, u, \|\cdot, ..., \cdot\|] = \left\{x = (x_{kl}) : P - \lim_{r,s} \frac{1}{h_{r,s}} \sum_{(k,l)\in I_{r,s}} M_{kl}\left[u_{kl}\left(\left\|\frac{\Delta^r v_{kl} - L}{\rho}, z_1, ..., z_{n-1}\right\|\right)\right]^{p_{kl}} = 0\right.$$

for some L and ρ > 0} ,

$$[N_{\theta_{r,s}}, Z, \mathcal{M}, \Delta^r, p, u, \|\cdot, ..., \cdot\|]_\infty = \left\{x = (x_{kl}) : \sup_{r,s} \frac{1}{h_{r,s}} \sum_{(k,l)\in I_{r,s}} M_{kl}\left[u_{kl}\left(\left\|\frac{\Delta^r v_{kl}}{\rho}, z_1, ..., z_{n-1}\right\|\right)\right]^{p_{kl}} < \infty\right.$$

for some ρ > 0} ,

$$[W^2, Z, \mathcal{M}, \Delta^r, p, u, \|\cdot, ..., \cdot\|] = \left\{x = (x_{kl}) : P - \lim_{m,n} \frac{1}{mn} \sum_{k,l=1,1}^{m,n} M_{kl}\left[u_{kl}\left(\left\|\frac{\Delta^r v_{kl} - L}{\rho}, z_1, ..., z_{n-1}\right\|\right)\right]^{p_{kl}} = 0\right.$$

for some L and ρ > 0 } .

Theorem 3.1 If $0 < p_{kl} < q_{kl} < \infty$ for each k and l, then we have $[N\theta_{rs}$, Z,M, Δr , p, u, k·, ..., ·k]∞ ⊂ $[N\theta_{rs}$, Z,M, Δr , q, u, k·, ..., ·k]∞.

Proof. Let x = (x_{kl}) ∈ $[N\theta_{rs}$, Z,M, Δr , p, u, k·, ..., ·k]∞. Then there exists ρ > 0 such that

$$\sup_{r,s} \frac{1}{h_{r,s}} \sum_{(k,l)\in I_{r,s}} M_{kl}\left[u_{kl}\left(\left\|\frac{\Delta^r v_{kl}}{\rho}, z_1, ..., z_{n-1}\right\|\right)\right]^{p_{kl}} < \infty.$$

This implies that $M_{kl}\left[u_{kl}\left(\left\|\frac{\Delta^r v_{kl}}{\rho}, z_1, ..., z_{n-1}\right\|\right)\right]^{p_{kl}} < 1$ for sufficiently large values of k and l. Since M_{kl}'s are non-decreasing, we get

$$\begin{aligned}&\sup_{r,s}\frac{1}{h_{r,s}}\sum_{(k,l)\in I_{r,s}} M_{kl}\left[u_{kl}\left(\left\|\frac{\Delta^r v_{kl}}{\rho}, z_1, ..., z_{n-1}\right\|\right)\right]^{q_{kl}}\\ \leq\ & \sup_{r,s}\frac{1}{h_{r,s}}\sum_{(k,l)\in I_{r,s}} M_{kl}\left[u_{kl}\left(\left\|\frac{\Delta^r v_{kl}}{\rho}, z_1, ..., z_{n-1}\right\|\right)\right]^{p_{kl}}\\ <\ & \infty.\end{aligned}$$

Thus, x = (x_{kl}) ∈ [Nθ_{rs} , Z,M, Δr , q, u, k·, ..., ·k]∞. This completes the proof.

Theorem 3.2 The double Zweier sequence space [Nθ_{rs} , Z,M, Δr , p, u, k·, ..., ·k]∞ is solid.

Proof. Suppose x = (x_{kl}) ∈ [Nθ_{rs} , Z,M, Δr , p, u, k·, ..., ·k]∞

$$\sup_{r,s}\frac{1}{h_{r,s}}\sum_{(k,l)\in I_{r,s}} M_{kl}\left[u_{kl}\left(\left\|\frac{\Delta^r v_{kl}}{\rho}, z_1, ..., z_{n-1}\right\|\right)\right]^{p_{kl}} < \infty, \text{ for some } \rho > 0.$$

Let (α_{kl}) be a double sequence of scalars such that |α_{kl}| ≤ 1 for all k, l ∈ N [15]. Then we get

$$\begin{aligned}&\sup_{r,s}\frac{1}{h_{r,s}}\sum_{(k,l)\in I_{r,s}} M_{kl}\left[u_{kl}\left(\left\|\frac{\Delta^r \alpha_{kl} v_{kl}}{\rho}, z_1, ..., z_{n-1}\right\|\right)\right]^{p_{kl}}\\ &\quad\leq \sup_{r,s}\frac{1}{h_{r,s}}\sum_{(k,l)\in I_{r,s}} M_{kl}\left[u_{kl}\left(\left\|\frac{\Delta^r v_{kl}}{\rho}, z_1, ..., z_{n-1}\right\|\right)\right]^{p_{kl}}\\ &\quad< \infty.\end{aligned}$$

This completes the proof.

4. CONCLUSION

It is concluded that the purpose of introducing Zweier sequence spaces. This paper comprises of three sections. The first section of this paper deals with the study of spaces Z I (M, Δr , p, u), Z I 0 (M, Δr , p, u) and Z I ∞(M, Δr , p, u). We have explored some topological properties and algebraic properties between these spaces. In the second section we introduce double Zweierlacunary convergent sequence spaces and study different properties of these spaces like linearity, paranorm, solid, monotone etc. Some interesting inclusion relations between these sequence spaces have also been established by using counter examples. We prove that these spaces are not solid, not convergence free in general.

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