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FUNDAMENTAL CONCEPTS OF FOURIER SERIES IN SUMMABILITY

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Abstract

Fourier series were invented by Fourier who was studying a physical problem. It is no wonder then that they have application of course not all creatures of "applied" mathematics have application in as wide an area as Fourier series. This solution of a problem of heat conduction by Fourier series give rise to an elegant mathematical theory with several applications diverse areas. Attempts to understand the behavior of these series, also laid down the foundation of rigorous analysis. Question like uniform convergence Cesarosummability and subjects like transmit cardinals and lebesgue measure are thought of as "pure" mathematics. Their history too is related to Fourier series.

Keyword:fourier series, summarbility

1. INTRODUCTION

The old hazy notation of convergence of infinite series was placed on sound foundation with the appearance of Cauchy's monumental work. "Cours d' Analysealgebrique" in 1821 and Abel'sresearches on the Binomial series in 1826 [1]. It was, however, noticed that there were certain no convergent series, which, particularly in Dynamical Astronomy, furnished nearly correct results. A theory of divergent series was formulated explicitly for the first time in 1890, when Cesaro published a paper on the multiplication of a series. Since then the theory of the series, whose sequence of partial sums oscillates, has been the center of attraction and fascination for most of the pioneering mathematical analysts [2]. After persistent efforts, in which a number of celebrated leading mathematicians took part, it was only in the closing decade of the last century and in the early years of the present century that satisfactory methods were devised so as to associate with them by processes closely connected with

Cauchy's concept of convergence, certain values which may be called their "sums" in a reasonable way. Such process of summationof series which were formerly tabooed being divergent have given rise to the modern rigorous theory of summability [3].

The idea of convergence having been thus generalized, it was quite natural to study the possibilities of extending the notion of absolute convergence. As a matter of fact, just as the notion of convergence had let to the development of its extension under the general title ofsummability, so also, by analogy, the concept of absolute convergence let to the formulation of the various processes of absolute summability [4].

As the ideas of ordinary and absolute convergence were instrumental to the development of ordinary and absolute summability respectively, so also, the notion of uniform convergence would have certainly insisted the analysts to think ofuniform summability.

Hardy and Littlewood, for the first time in 1913, introduced the notion of "strong summability" of Fourier series [5].

Fekete defined that a series $\sum a_n$is strongly summable

If$\sum_{v=1}^{n} |s_v - s| = o(n)$

as n→?, where s_v is the partial sums of the series $\sum a_n$. The series $\sum a_n$ in said to be summable [C, 1]. This type is now known as strong Cesaro summability of order 1, or [C, 1] summability.

It is important to note that strong summability is weaker than absolute summability and stronger than ordinary summability [6].

Lorentz, for the first time in 1948, defined almost convergence of a bounded sequence (S_n} ofan infinite series $\sum a_n$.

It is easy to see that a convergent sequence is almost convergent and the limits are the Mazhar& Siddiqi. The idea of almost convergence let to the formulation of almost summability methods.

2. INFINITE SERIES:

In this paper, our investigation are concerned with Fourier series and allied series and also double Fourier series. Let us mention a brief discussion related to these series.

Some of the most familiar method of an infinite series and with which we shall be concerned in the sequence are Fourier series, conjugate series of Fourier series, derived series of a Fourier series and double Fourier series [7].

In 1807, Fourier made a significant advance by saying that an arbitrary function given graphically by means of a curve, which may be broken by ordinary discontinuities, can be represented by means of singletrigonometric series. But the problem of convergence was taken up by Dirichlet in 1829 [8].

Dirichlet established that under certain restriction on the functions, the Fourier series actually converges to the value of the function. Let us define these series in the desirable range.

3. FOURIER SERIES:

Let f(t) be a 2π- periodic and Lebesque-integrable function of t in the interval (—π,π) and then periodically extended beyond this interval to the left and to the right so as to satisfy the functional equation $f(x \pm 2\pi) = f(x)$.

Then the Fourier series corresponding to the functionf(t) is defined by the correspondence [9].

$$f(t) \sim \frac{1}{2}a_o + \sum_{n=1}^{\infty} (a_n \cos nt + b_n \sin nt) = \sum_{n=o}^{\infty} A_n(t) \quad (1)$$

where the coefficients a_0, a_n and b_n are known as Fourier coefficients given by

$$a_o = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)dt \quad (2)$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\cos nt\, dt, \quad \text{where } n \geq 1. \tag{3}$$

And

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\sin nt\, dt, \quad \text{where } n \geq 1. \tag{4}$$

4. CONJUGATE SERIES OF FOURIER SERIES:

The notation in (1) is due to Hurwitz and the formulae (3) and (4) are known as Euler-Fourier Formulae [10].

The series

$$\sum_{n=1}^{\infty} (b_n \cos nt - a_n \sin nt) = \sum_{n=1}^{\infty} B_n(t) \tag{5}$$

is called the conjugate series of a Fourier series (1)

The series

$$\sum_{n=1}^{\infty} n\,(b_n \cos nt - a_n \sin nt) = \sum_{n=1}^{\infty} n\,B_n(t) \tag{6}$$

Equation (6) is obtained by differentiating the series (1) term by term is called the first differentiated series or the derived series of a Fourier series (1). It is well known that the derived series of a Fourier series may not itself be a Fourier series [11].

The series

$$\sum_{n=1}^{\infty} n\,(a_n \cos nt + b_n \sin nt) \tag{7}$$

is called the derived conjugate series of the Fourier series which is the conjugate series of the series (6).

5. DOUBLE FOURIER SERIES:

Let f(u, v) be a function of(u, v), periodic with respect to u and v in each case with period 2π and summable in the rectangle Q(-π,-π;π,π). The double Fourier series of function f(u, v) is given by [12].

$$f(u,v) \sim \sum_{m=o}^{\infty} \sum_{n=0}^{\infty} \lambda m,n\big[a_{m,n} \cos mu \cos nv + b_{m,n} \sin mu \cos nv + c_{m,n} \cos mu \sin nv + d_{m,n} \sin mu \sin nv\big] \tag{8}$$

Where

$$\lambda_{m,n} = \begin{cases} \frac{1}{4}, & \text{for } m = o,\ n = o \text{ or } m = n = o \\ \frac{1}{2}, & \text{for } m > o,\ n = o \text{ and } m = o,\ n > o. \\ 1, & \text{for } m > o,\ n > o \end{cases}$$

We write

$$\phi(u,v) = \phi(x,y;u,v) = \frac{1}{4}\big[f(x+u,y+v) + f(x+u,y-v) + f(x-u,y+v) + f(x-u,y-v) - 4f(x,y)\big]$$

We begin by giving a resume ofthe results obtained hitherto, which the backbone of the results is obtained in the sequel.

6. CONCLUSION

Among the purest branches of mathematics is number theory and surely it is subject quite independent of Fourier series. Yet, Hermann weyl used Fejer's convergence theorem for Fourier series to Weylequidistribution theorem.One of the areas where Fourier series and transforms have major applications, is crystallography. In 1985 the Noble prize in chemistry was given to Hauptman and Karle who developed a new method for calculation some crystallographic constants from their Fourier coefficients which can be inferred from measurements. Two crucial ingredients of their analysis were Wely'sequi-distribution theorem and theorems ofToeplitz on Fourier series of nonnegative functions.Fourier series is also being used at various steps indifferent branches of Engineering.

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