MULTI DIMENSIONAL VECTOR SUBSPACES OF GENERALIZED HYPERGEOMETRIC FUNCTIONS BY USING MATLAB PROGRAMMING AND THEIR INTERPRETATION

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ABSTRACT

This paper is an attempt to create and plotted the number of subspaces from vector space of generalized hypergeometric functions $\mathbb{V} = {}_pF_q\left\{((a));((b));x\right\};\forall a_{i's}, b_{j's}, x \in \mathbf{R}$ .with the help of MATLAB, the dimensionality theorem has also been proved for these spaces

Keywords: MATLAB, Column Space, Null Space, Vector Space,Generalized Hypergeometric Function.

2. INTRODUCTION

Subspaces arise in connection with some matrix A, and provide useful information about the equation Ax = b. The vector space $\mathbb{V} = \left\{{}_pF_q\left\{((a));((b));x\right\};\forall a_{i's}, b_{j's}, x \in \mathbf{R}\right\}$ shall be a subspace of itself because it has the three properties required for a subspace. The column space of a matrix A shall be the set Col A of all linear combination of the columns of A. That is, if **a₁, a₂,…,aₙ**are the columns of a matrix A, then the column space of the matrix A can be defined as : Col (A) = Span {**a₁, a₂… an**}, Since the set C = {**a₁, a₂…aₙ**} already spans the column space we can imply delete dependent vectors one at a time till we get a linearly independent set.

The distinct scientific communities that are working using subspaces are in Hilbert Space, subspace clustering, signal processing, quantum (wave) mechanics, control theory, computer graphics, cryptography etc. We can see the works on generalized Hypergeometric Functions of Dwork B. (1990) and Daalhuis et al (2010). Elhamifaret al (2013) deals with sparse subspace clustering: algorithm, theory, and applications and Heckel et al (2014) on Subspace clustering of dimensionality-reduced data. We can see the work of Desmond J. Higham et al (2005) on MATLAB. Anotherimportant work of G. N. Georgiev et al(2003)deals with a new property of complex Kummer function and its application to waveguide propagation. We can also quote the work of R. P. Agrawal et al (1967) in generalized basic hypergeometric series with unconnected bases.

2.1 Let us consider the matrix 10 X 10 from vector space of Kummer function ${}_1F_1$ (-1; b; x), generated by MATLAB of the form

b ↓x→	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.1	0	-1	-2	-3	-4	-5	-6	-7	-8	-9
0.2	0.5	0	-0.5	-1	-1.5	-2	-2.5	-3	-3.5	-4
0.3	0.66667	0.33333	0	-0.3333	-0.6667	-1	-1.3333	-1.6667	-2	-2.3333
0.4	0.75	0.5	0.25	0	-0.25	-0.5	-0.75	-1	-1.25	-1.5
0.5	0.8	0.6	0.4	0.2	0	-0.2	-0.4	-0.6	-0.8	-1
0.6	0.83333	0.66667	0.5	0.33333	0.16667	0	-0.1667	-0.3333	-0.5	-0.6667
0.7	0.85714	0.71429	0.57143	0.42857	0.28571	0.14286	0	-0.1429	-0.2857	-0.4286
0.8	0.875	0.75	0.625	0.5	0.375	0.25	0.125	0	-0.125	-0.25
0.9	0.88889	0.77778	0.66667	0.55556	0.44444	0.33333	0.22222	0.11111	0	-0.1111
1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0

$$A = \begin{bmatrix} 0 & -1 & -2 & -3 & -4 & -5 & -6 & -7 & -8 & -9 \\ 0.5 & 0 & -0.5 & -1 & -1.5 & -2 & -2.5 & -3 & -3.5 & -4 \\ 0.66667 & 0.33333 & 0 & -0.3333 & -0.6667 & -1 & -1.3333 & -1.6667 & -2 & -2.3333 \\ 0.75 & 0.5 & 0.25 & 0 & -0.25 & -0.5 & -0.75 & -1 & -1.25 & -1.5 \\ 0.8 & 0.6 & 0.4 & 0.2 & 0 & -0.2 & -0.4 & -0.6 & -0.8 & -1 \\ 0.83333 & 0.66667 & 0.5 & 0.33333 & 0.16667 & 0 & -0.1667 & -0.3333 & -0.5 & -0.6667 \\ 0.85714 & 0.71429 & 0.57143 & 0.42857 & 0.28571 & 0.14286 & 0 & -0.1429 & -0.2857 & -0.4286 \\ 0.875 & 0.75 & 0.625 & 0.5 & 0.375 & 0.25 & 0.125 & 0 & -0.125 & -0.25 \\ 0.88889 & 0.77778 & 0.66667 & 0.55556 & 0.44444 & 0.33333 & 0.22222 & 0.11111 & 0 & -0.1111 \\ 0.9 & 0.8 & 0.7 & 0.6 & 0.5 & 0.4 & 0.3 & 0.2 & 0.1 & 0 \end{bmatrix} \quad \ldots\ldots\ldots(2.1)$$

$$U = \begin{bmatrix} 1 & 0 & -1 & -2 & -3 & -4 & -5 & -6 & -7 & -8 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \ldots\ldots\ldots(2.2)$$

The reduced row echelon form of A

If we label the column vectors of the matrix U with $u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}$, then it is easy to see that $u_3 = -u_1 + 2 u_2$. It is important to observe that the columns of A have the same dependency relation; that is, $a_3 = -a_1 + 2 a_2$. We can eliminate a_3 and the remaining vectors will still span the column space of matrix A. Col (A) = Span $\{a_1, a_2, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}\}$. We also observe that $u_4, u_5, u_6, u_7, u_8, u_9, u_{10}$ columns of A will have the same dependency relation. We can eliminate all $a_4, a_5, a_6, a_7, a_8, a_9, a_{10}$ step by step and the remaining vectors will still span the column space of matrix A.

$$B_1 = \left\{ \begin{bmatrix} 0 \\ 0.5 \\ 0.66667 \\ 0.75 \\ 0.8 \\ 0.83333 \\ 0.85714 \\ 0.875 \\ 0.88889 \\ 0.9 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 0.33333 \\ 0.5 \\ 0.6 \\ 0.66667 \\ 0.71429 \\ 0.75 \\ 0.77778 \\ 0.8 \end{bmatrix} \right\} \qquad Q = \left\{ \begin{bmatrix} -0.89082 \\ -0.37689 \\ -0.20559 \\ -0.11993 \\ -0.068537 \\ -0.034275 \\ -0.0098026 \\ 0.008552 \\ 0.022828 \\ 0.034248 \end{bmatrix}, \begin{bmatrix} -0.17057 \\ 0.14122 \\ 0.24515 \\ 0.29711 \\ 0.32829 \\ 0.34908 \\ 0.36392 \\ 0.37506 \\ 0.38372 \\ 0.39065 \end{bmatrix} \right\}$$

Basis of the column space (A) Orthonormal basis of the column space (A)

Col (A) = Span {a_1, a_2}. Because u_1 and u_2 are linearly independent and so is with a_1, $a_2$is the basis for the column space of A. The dimension of the column space of the matrix A shall be the Rank of the matrix A and hence the column space of A is a subspace of R^{10}.

THE NULL SPACE

The null space of a matrix A is the set Nul A of all solutions to the homogeneous equationA**x = 0.** Symbolically Nul(A) = {**x** : A**x**=**0**}.The null space can be composed by taking all solutions of the matrix equation A**x** = 0 in following form:

$$
\begin{bmatrix}
0 & -1 & -2 & -3 & -4 & -5 & -6 & -7 & -8 & -9 \\
0.5 & 0 & -0.5 & -1 & -1.5 & -2 & -2.5 & -3 & -3.5 & -4 \\
0.66667 & 0.33333 & 0 & -0.3333 & -0.6667 & -1 & -1.3333 & -1.6667 & -2 & -2.3333 \\
0.75 & 0.5 & 0.25 & 0 & -0.25 & -0.5 & -0.75 & -1 & -1.25 & -1.5 \\
0.8 & 0.6 & 0.4 & 0.2 & 0 & -0.2 & -0.4 & -0.6 & -0.8 & -1 \\
0.83333 & 0.66667 & 0.5 & 0.33333 & 0.16667 & 0 & -0.1667 & -0.3333 & -0.5 & -0.6667 \\
0.85714 & 0.71429 & 0.57143 & 0.42857 & 0.28571 & 0.14286 & 0 & -0.1429 & -0.2857 & -0.4286 \\
0.875 & 0.75 & 0.625 & 0.5 & 0.375 & 0.25 & 0.125 & 0 & -0.125 & -0.25 \\
0.88889 & 0.77778 & 0.66667 & 0.55556 & 0.44444 & 0.33333 & 0.22222 & 0.11111 & 0 & -0.1111 \\
0.9 & 0.8 & 0.7 & 0.6 & 0.5 & 0.4 & 0.3 & 0.2 & 0.1 & 0
\end{bmatrix}
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \end{bmatrix}
=
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \dots(2.3)
$$

$$
M =
\begin{bmatrix}
0 & -1 & -2 & -3 & -4 & -5 & -6 & -7 & -8 & -9 & 0 \\
0.5 & 0 & -0.5 & -1 & -1.5 & -2 & -2.5 & -3 & -3.5 & -4 & 0 \\
0.66667 & 0.33333 & 0 & -0.3333 & -0.6667 & -1 & -1.3333 & -1.6667 & -2 & -2.3333 & 0 \\
0.75 & 0.5 & 0.25 & 0 & -0.25 & -0.5 & -0.75 & -1 & -1.25 & -1.5 & 0 \\
0.8 & 0.6 & 0.4 & 0.2 & 0 & -0.2 & -0.4 & -0.6 & -0.8 & -1 & 0 \\
0.83333 & 0.66667 & 0.5 & 0.33333 & 0.16667 & 0 & -0.1667 & -0.3333 & -0.5 & -0.6667 & 0 \\
0.85714 & 0.71429 & 0.57143 & 0.42857 & 0.28571 & 0.14286 & 0 & -0.1429 & -0.2857 & -0.4286 & 0 \\
0.875 & 0.75 & 0.625 & 0.5 & 0.375 & 0.25 & 0.125 & 0 & -0.125 & -0.25 & 0 \\
0.88889 & 0.77778 & 0.66667 & 0.55556 & 0.44444 & 0.33333 & 0.22222 & 0.11111 & 0 & -0.1111 & 0 \\
0.9 & 0.8 & 0.7 & 0.6 & 0.5 & 0.4 & 0.3 & 0.2 & 0.1 & 0 & 0
\end{bmatrix}
M \square
\begin{bmatrix}
1 & 0 & -1 & -2 & -3 & -4 & -5 & -6 & -7 & -8 & 0 \\
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

An augmented matrix form of 2.3Reduced row echelon form of thematrix M

The reduced row echelon form of the matrix M indicates that the solutions of equation 2.3 can be written as:
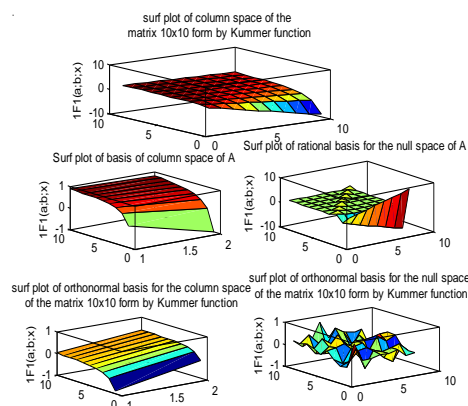
$$
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \end{bmatrix}
=
\begin{bmatrix}
x_3 + 2x_4 + 3x_5 + 4x_6 + 5x_7 + 6x_8 + 7x_9 + 8x_{10} \\
-2x_3 - 3x_4 - 4x_5 - 5x_6 - 6x_7 - 7x_8 - 8x_9 - 9x_{10} \\
x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10}
\end{bmatrix}
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \end{bmatrix}
= x_3 \begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
+ x_4 \begin{bmatrix} 2 \\ -3 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
+ x_5 \begin{bmatrix} 3 \\ -4 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
+ x_6 \begin{bmatrix} 4 \\ -5 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
+ x_7 \begin{bmatrix} 5 \\ -6 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}
+ x_8 \begin{bmatrix} 6 \\ -7 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}
+ x_9 \begin{bmatrix} 7 \\ -8 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}
+ x_{10} \begin{bmatrix} 8 \\ -9 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \dots(2.4)
$$

$$
N = \left\{
\begin{bmatrix} 0.76243 \\ -0.37934 \\ -0.41725 \\ -0.20018 \\ 0.1077 \\ -0.15137 \\ 0.022416 \\ 0.12729 \\ 0.039856 \\ 0.088449 \end{bmatrix},
\begin{bmatrix} -0.030469 \\ 0.38032 \\ -0.2792 \\ -0.47312 \\ 0.17781 \\ 0.10017 \\ 0.368 \\ -0.38471 \\ 0.40043 \\ -0.25924 \end{bmatrix},
\begin{bmatrix} -0.018018 \\ -0.57117 \\ 0.12438 \\ 0.14284 \\ 0.29476 \\ 0.57 \\ 0.069131 \\ -0.45471 \\ -0.047976 \\ -0.10924 \end{bmatrix},
\begin{bmatrix} 0.082991 \\ 0.16015 \\ 0.19562 \\ -0.32941 \\ 0.090951 \\ -0.06891 \\ -0.14879 \\ -0.45091 \\ -0.25193 \\ 0.72024 \end{bmatrix},
\begin{bmatrix} 0.031406 \\ 0.072987 \\ 0.015427 \\ -0.083301 \\ 0.16544 \\ 0.16097 \\ -0.8307 \\ 0.028265 \\ 0.48973 \\ -0.050234 \end{bmatrix},
\begin{bmatrix} 0.14267 \\ 0.14091 \\ -0.36407 \\ 0.50416 \\ -0.56598 \\ 0.14267 \\ -0.023897 \\ -0.38498 \\ 0.2131 \\ 0.19541 \end{bmatrix},
\begin{bmatrix} -0.17749 \\ 0.27176 \\ -0.621 \\ 0.20559 \\ 0.45344 \\ 0.21096 \\ -0.13263 \\ 0.12189 \\ -0.42012 \\ 0.087603 \end{bmatrix},
\begin{bmatrix} -0.11074 \\ -0.10978 \\ -0.0045117 \\ 0.42014 \\ 0.45308 \\ -0.66603 \\ 0.0407 \\ -0.29342 \\ 0.2491 \\ 0.021458 \end{bmatrix}
\right\}
$$

Hence Rank-nullity theorem satisfied for the above matrix form by Kummer function

Rank (A) = 2Nullity (A) = 8Dim (A) = 10

Dim (A) = Rank (A) + Nullity (A) ………………………………(2.5)

MATLAB PROGRAM

```
>> X = [1:1:10];
>>A=[hypergeom(-1,1,X);hypergeom(-1,2,X); hypergeom(-1,3,X);
hypergeom(-1,4,X);hypergeom(-1,5,X);hypergeom(-1,6,X);hypergeom
(-1,7,X); hypergeom(-1,8,X); hypergeom(-1,9,X); hypergeom(-1,10,X)];
>> U = rref (A);
>> B1 = A (:, [1 2]);
>> Q = orth (A);
>> B2 = null (A, 'r');
>> N= null (A);
>> Dim A= size (A, 2);
>> Rank A = size (B1, 2);
>> Nullity = size (B2, 2);
>> Subplot (3, 2, 1:2); surf (A);
>> Subplot (3, 2, 3); surf (B1);
>> Subplot (3, 2, 4); surf (B2);
>> Subplot (3, 2, 5); surf (Q);
>> Subplot (3, 2, 6); surf (N);
```
FIG 1.1

2.2 Let us consider the matrix 10x10 from vector space of higher hypergeometric function from Dixon's Theorem $_3F_2$ (a, b, c; 1+a-b, 1+a-c; x)

b,c ↓ x→	2	3	4	5	6	7	8	9	10	11
2,3	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10
3,4	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10
4,5	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10
5,6	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10
6,7	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10
7,8	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10
8,9	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10
9,10	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10
10,11	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10
11,12	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10

$$A = \begin{bmatrix} -1 & -2 & -3 & -4 & -5 & -6 & -7 & -8 & -9 & -10 \\ -1 & -2 & -3 & -4 & -5 & -6 & -7 & -8 & -9 & -10 \\ -1 & -2 & -3 & -4 & -5 & -6 & -7 & -8 & -9 & -10 \\ -1 & -2 & -3 & -4 & -5 & -6 & -7 & -8 & -9 & -10 \\ -1 & -2 & -3 & -4 & -5 & -6 & -7 & -8 & -9 & -10 \\ -1 & -2 & -3 & -4 & -5 & -6 & -7 & -8 & -9 & -10 \\ -1 & -2 & -3 & -4 & -5 & -6 & -7 & -8 & -9 & -10 \\ -1 & -2 & -3 & -4 & -5 & -6 & -7 & -8 & -9 & -10 \\ -1 & -2 & -3 & -4 & -5 & -6 & -7 & -8 & -9 & -10 \\ -1 & -2 & -3 & -4 & -5 & -6 & -7 & -8 & -9 & -10 \end{bmatrix} \quad U = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The reduced row echelon form of A

$$B_1 = \left\{ \begin{bmatrix} -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \end{bmatrix} \right\} \quad Q = \left\{ \begin{bmatrix} -0.31623 \\ -0.31623 \\ -0.31623 \\ -0.31623 \\ -0.31623 \\ -0.31623 \\ -0.31623 \\ -0.31623 \\ -0.31623 \\ -0.31623 \end{bmatrix} \right\}$$

Basis for the column space (A)   Orthonormal basis for the column space of A

$$B_2 = \left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -6 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -7 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -8 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -9 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -10 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

$$N = \begin{bmatrix} -0.071933 & -0.0057054 & 0.0049433 & -0.036377 & -0.049183 & -0.14977 & 0.18122 & 0.96181 & 0.089871 \\ -0.14387 & 0.37778 & 0.046023 & -0.063351 & 0.40064 & 0.25458 & -0.56792 & 0.10279 & 0.51151 \\ -0.061082 & 0.099066 & 0.93406 & 0.077073 & 0.029955 & -0.044723 & 0.25551 & -0.077135 & 0.10267 \\ -0.28773 & -0.15012 & -0.096363 & -0.73923 & 0.16579 & 0.29345 & 0.4064 & -0.092446 & 0.10032 \\ -0.050232 & 0.086123 & 0.026271 & -0.012405 & -0.82904 & 0.44541 & -0.15427 & 0.027615 & 0.11841 \\ -0.12216 & 0.19813 & -0.057844 & -0.32966 & -0.246 & -0.77943 & -0.18271 & -0.14803 & 0.13024 \\ 0.42476 & 0.55861 & -0.2616 & 0.19042 & 0.08565 & 0.050358 & 0.48902 & -0.068986 & 0.1476 \\ -0.57546 & -0.30024 & -0.19273 & 0.54184 & 0.086079 & -0.050653 & 0.14677 & -0.095043 & 0.20294 \\ 0.59033 & -0.58853 & 0.061067 & -0.054524 & 0.10797 & -0.0069708 & -0.19291 & 0.036595 & 0.19441 \\ -0.10046 & 0.17225 & 0.052542 & -0.02481 & 0.18562 & 0.1166 & -0.2431 & 0.10978 & -0.76021 \end{bmatrix}$$

Rational basis of null space (A)Orthonormal basis for the null space of A

Hence Rank-nullity theorem satisfied for the above matrix form by Dixon's Theorem Rank (A) = 1, Nullity (A) = 9,Dim (A) = 10,

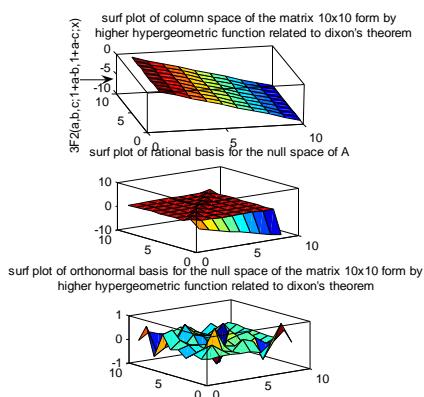Dim (A) = Rank (A) + Nullity (A)…………………………………….... (2.7)

FIG 2.3

MATLAB PROGRAM

```
>> X = [2:1:11];
>>A = [hypergeom([-1 2 3], [-2 -3], x);hypergeom([-1 3 4], [-3 -4], x);
hypergeom([-1 4 5], [-4 -5], x);hypergeom([-1 5 6], [-5 -6], x);
hypergeom([-1 6 7], [-6 -7], x);hypergeom([-1 7 8], [-7 -8], x);
hypergeom([-1 8 9], [-8 -9], x); hypergeom([-1 9 10], [-9 -10], x);
hypergeom([-1 10 11], [-10 -11], x); hypergeom([-1 11 12], [-11 -12], x)];
>> U = rref (A);
>> B1 = A (: 1 );
>> Q = orth(A);
>> B2 = null (A, 'r');
>> N = null (A);
>> Dim A = size (A, 2);
>> Rank A = size (B1, 2);
>> Nullity A =size (B2, 2);
>> Subplot (3, 2, 1:2); surf (A);
>> Subplot (3, 2, 3:4); surf (B2);
>> Subplot (3, 2, 5: 6); surf (N);
```

3. Applications of Vector Subspaces Generalized Hypergeomatric Function

Vector space and their subspaces of generalized hypergeomatric function used in Hilbert spaces. They are indispensable tools in the theories of partial differential equations, quantum mechanics, Fourier analysis (which includes applications to signal processing and heat transfer)—and ergodic theory. The Kummer confluent hypergeometric function belongs to an important class of special functions of the mathematical physics with a large number of applications in different branches of the quantum (wave) mechanics. I am finding it harder to justify to engineers where they will use vector spaces- I know it is important for time domain

(state space) control theory and you describe stresses in materials using tensors. For other fields of engineering, computer memory extensively uses the conception of partition of matrices.If the matrix size gets larger than the space of computer memory it divides the matrices into sub matrices and does calculation. Again linear operator plays a key role in computer graphics. For many CAD software generates drawing using linear operators. And don't forget about cryptography.Matrices can be cleverly used in cryptography. Exchanging secret information using matrix is very robust and easy in one sense. I would like to tell about one of the important application of my research it deals as a Sparse Matrices.sparse matrices are useful for savings in storage: There are lesser non zero elements than zeros and saving in computing time: have a smart storage algorithm and save the computing time.If you have a very large matrix with few non-zero values, it doesn't always make sense to store the zero values. It can make operations very expensive when the dimensions are large. You can take advantage of having less information by representing your matrix in a way that only uses the non-zero entries.The matrix is converted into a table of index-value pairs. The index (m,n) is two integers representing the row and column, and the value is the entry for that element. You can see how this will save memory, at the very least, if your matrix is huge but only has a few (non-zero) entries. The sparse matrix would be small in memory, while the actual matrix would be huge for storing all the zeros.

4. Interpretation

We have generated the vector subspaces of generalized hypergeomatric function of vector space of hypergeometric function by using MATLAB, which has been used in the generation and plotting of basis and orthonormal basis of column space and null space of a matrix of generalized hypergeometric function.

Multi dimensional vector space of generalized hypergeometric function is the generalization of general vector space which shall be better suited to mathematical concepts studied in a spiral. For this purpose we have taken a concept in conceptual settings which have been generalized to a more complex setting by increasing dimensions and again to an even more complex setting by selecting characteristics that appear useful in this exercise.

I would hope that the variety and novelty of the result already found might provide inspiration for further research.

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