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DIFFERENT NORMS OF SUBSPACES CREATED BY NORMED VECTOR SPACES OF GENERALIZED MULTIPLE HYPERGEOMETRIC FUNCTION USING MATLAB

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ABSTRACT

This article attempts to generate the normed vector space of generalized hypergeometric function as its elements V={ }

from different generalized hypergeometric as well as multiple hypergeometric functions and found the different norms of these subspaces by using MATLAB.

Keywords: Generalized Hypergeometric Function, Multiple Hypergeometric Function, L1-Norm , L2-Norm, L∞-Norm, Frobenious-Norm, *Normed Vector Space, MATLAB.*

INTRODUCTION

The generalization of the three basic properties of vector space to more abstract vector spaces leads to the notion of norm. The generalization of the norms to an infinite number of components leads to the L^p spaces, with norms

$$\|x\|_p = \left(\sum_{i\in N}|x_i|^p\right)^{\frac{1}{p}} \quad \|f\|_{p,X} = \left(\int|f(x)|^p dx\right)^{\frac{1}{p}} \text{ resp.}$$

(for complex-valued sequences *x* resp. functions *f* defined on $X \subset R$).
In infinite dimensional normed vector spaces convergence can disappear if a different norm is used. Not all norms are equivalent in infinite dimensions. Infinite dimensional vector spaces are thus more interesting than finite dimensional ones. Each (inequivalent) norm leads to a different notion of convergence of sequences of vectors.
The work on multi dimensional vector space based evaluation method for the effectiveness of using ICTs in college teaching curriculum, Education Technology and Training, can be seen in the research paper of Chengcheng Zhang et al, (2009). Geocoading method using multidimensional vector space can seen in the work of Michael Asher at el, (2009). We can see the work of Zhenliang Zhang at el (2017) on Subspace Selection for Projection Maximization With Matroid Constraints they Consider the problem of selecting a subset of the ground set such that the projection of a vector of interest onto the subspace spanned by the vectors in the chosen subset reaches the maximum norm. Some microwave applications of the Kummer confluent hypergeometric function can also be seen in the work of G. N. Georgiev et al (2003). Carpentieri B. (2000) has worked on sparse pattern selection strategies for robust Frobenius-norm minimization preconditioners in electromagnetism. We can also quote the work on a normed space of genetic operators with applications to scalability issues by Jonathan E. Rowe (2006) where the genetic operators are elements. Other important applications of normed vector space can be seen in the works of Haizhang Zhang et al (2009) on reproducing Kernel Banach spaces for machine learning,

Martin Dyer et al (2009) on matrix norms and rapid mixing for spin systems, Mickaël D. Chekroun et al (2010) on Homeomorphisms group of normed vector space: Conjugacy problems and the Koopman operator.

Induced norm

If vector norms on F^m and F^n are given (*F* is field of real or complex numbers), then one defines the corresponding *induced norm* or operator norm on the space of *m*-by-*n* matrices as the following maxima:

$$\|A\| = \max\{\|Ax\| : x \in F^n \text{ with } \|x\| \le 1\}$$
$$= \max\left\{\frac{\|Ax\|}{\|x\|} : x \in F^n \text{ with } x \ne 0\right\}.$$

These are different from the entrywise *p*-norms and the Schatten *p*-norms for matrices treated below, which are also usually denoted by $\|A\|_p$.

The operator norm corresponding to the p-norm for vectors is:

$$\|A\|_p = \max_{x \ne 0} \frac{\|Ax\|_p}{\|x\|_p}.$$

In the case of $p = 1$ and $p = \infty$, the norms can be computed as:

$$\|A\|_1 = \max_{1 \le j \le n} \sum_{i=1}^{m} |a_{ij}|,$$

which is simply the maximum absolute column sum of the matrix

$$\|A\|_\infty = \max_{1 \le i \le m} \sum_{j=1}^{n} |a_{ij}|,$$

which is simply the maximum absolute row sum of the matrix

In the special case of $p = 2$ (the Euclidean norm) and $m = n$ (square matrices), the induced matrix norm is the *spectral norm*. The spectral norm of a matrix *A* is the largest singular value of *A* or the square root of the largest eigen value of the positive-semi definite matrix A^*A:

$$\|A\|_2 = \sqrt{\lambda_{\max}(A^*A)} = \sigma_{\max}(A)$$

where A^* denotes the conjugate transpose of *A*.

"Entrywise" Norms

These vector norms treat a $m \times n$ matrix as a vector of size *mn*, and use one of the familiar vector norms.

For example, using the *p*-norm for vectors, we get:

$$\|A\|_p = \left(\sum_{i=1}^{m}\sum_{j=1}^{n} |a_{ij}|^p\right)^{1/p}.$$

This is a different norm from the induced *p*-norm and the Schatten *p*-norm, but the notation is the same.

The special case $p = 2$ is the Frobenius norm, and $p = \infty$ yields the maximum norm.

Frobenius Norm

For $p = 2$, this is called the Frobenius norm or the Hilbert–Schmidt norm, though the latter term is often reserved for operators on Hilbert space. This norm can be defined in various ways:

$$\|A\|_F = \sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n} |a_{ij}|^2} = \sqrt{\operatorname{trace}(A^*A)} = \sqrt{\sum_{i=1}^{\min\{m,n\}} \sigma_i^2}$$

Where A^* denotes the conjugate transpose of A, σ_i are the singular values of A, and the trace function is used. The Frobenius norm is very similar to the Euclidean norm on F^n and comes from an inner product on the space of all matrices.

Max Norm

The max norm is the element wise norm with $p = \infty$:

$$\|A\|_{\max} = \max\{|a_{ij}|\}.$$

This norm is not sub multiplicative.

The most familiar cases are $p = 1, 2, \infty$. The case $p = 2$ yields the Frobenius norm, introduced before. The case $p = \infty$ yields the spectral norm, which is the matrix norm induced by the vector 2-norm.

INFINITE DIMENSIONAL CASE

The generalization of the above norms to an infinite number of components leads to the L^p spaces, with norms

$$\left\|{}_pF_q\left\{((a));((b));x\right\}\right\|_p = \left(\sum_{i\in N}|x_i|^p\right)^{\frac{1}{p}} \text{ resp. } \quad \|f\|_{p,X} = \left(\int_X |f(x)|^p\,\mathrm{d}x\right)^{1/p}$$

(for complex-valued sequences x resp. functions f defined on $X \subset R$),.
Any inner product induces in a natural way the norm

$$\left\|{}_pF_q\left\{((a));((b));x\right\}\right\| = \sqrt{\left\langle {}_pF_q\left\{((a));((b));x\right\}, {}_pF_q\left\{((a));((b));x\right\}\right\rangle}$$

Other examples of infinite dimensional normed vector spaces can be found in the Banach space.

NORM OF A DATABASE 1 OF KUMMER HYPERGEOMATRIC FUNCTION $_1F_1$ (1; 0.1; x)

x	1F1(1;0.1;x)									
0.1:0.1:1	2.0954	3.4006	4.9473	6.7711	8.9129	11.419	14.341	17.738	21.678	26.236
1.1:0.1:2	31.498	37.561	44.533	52.539	61.717	72.224	84.237	97.955	113.6	131.43
2.1:0.1:3	151.73	174.81	201.04	230.82	264.6	302.9	346.28	395.4	450.97	513.81
3.1:0.1:4	584.82	665.03	755.58	857.76	973	1102.9	1249.3	1414.2	1599.9	1808.9
4.1:0.1:5	2044	2308.5	2605.9	2940.1	3315.7	3737.6	4211.4	4743.3	5340.4	6010.3
5.1:0.1:6	6761.8	7604.7	8549.8	9609.3	10797	12127	13618	15288	17157	19251
6.1:0.1:7	21594	24217	27152	30436	34110	38219	42814	47951	53695	60116
7.1:0.1:8	67292	75311	84272	94282	105460	117950	131900	147480	164870	184280
8.1:0.1:9	205950	230140	257130	287250	320860	358360	400190	446850	498900	556940
9.1:0.1:10	621670	693840	774310	864020	964030	1075500	1199800	1338200	1492500	1664500

Database 1

Norm 1 = 2493585.032
Norm 2 = 3753947.186
Norm (infinite) = 10688399.74
Norm (frobenious) = 3753949.516

MATLAB PROGRAM FOR THE NORM OF A DATABASE 1 GENERATED FOR KUMMER FUNCTION 1F1(1; 0.1; x)

```
>> x = [.1:.1:1; 1.1:.1:2; 2.1:.1:3; 3.1:.1:4; 4.1:.1:5; 5.1:.1:6; 6.1:.1:7; 7.1:.1:8; 8.1:.1:9; 9.1:.1:10];
>> y = [hypergeom(1,0.1,x)];
>> norm (y,1);
>> norm (y,2);
>> norm (y,inf);
>> norm (y,'fro');
```

NORM OF A DATABASE 2 OF HYPERGEOMATRIC FUNCTION OF HIGHER ORDER $_4F_3$ (1, 2, 3, 4; 3, 4, 5; X)

Database 2

x	$_4F_3$ (1,2,3,4; 3,4,5; x)				
2 to 10 (step 2)	1 - 2.3562i	-0.52598 - 1.3254i	-0.59478 - 0.72722i	-0.52153 - 0.45099i	-0.44557 - 0.30536i
12to 20 (step 2)	-0.38319 - 0.21998i	-0.33366 - 0.16585i	-0.29419 - 0.12943i	-0.26232 - 0.10379i	-0.23622 - 0.085059i
22to 30 (step 2)	-0.21453 - 0.070971i	-0.19628 - 0.060109i	-0.18073 - 0.051561i	-0.16735 - 0.044712i	-0.15573 - 0.039142i
32 to 40 (step 2)	-0.14555 - 0.034551i	-0.13657 - 0.030722i	-0.1286 - 0.027495i	-0.12147 - 0.024751i	-0.11506 - 0.022399i
42 to 50 (step 2)	-0.10927 - 0.020366i	-0.10402 - 0.018598i	-0.099233 - 0.01705i	-0.094855 - 0.015688i	-0.090837 - 0.014482i

Norm 1 = 3.488179222
Norm 2 = 3.295516024
Norm (infinite) = 6.154656032
Norm (frobenious) = 3.337466278

MATLAB PROGRAM FOR NORM OF A DATABASE 2.2 OF HYPERGEOMATRIC FUNCTION OF HIGHER ORDER 4F3 (1, 2, 3, 4; 3, 4, 5; x)

```
>> x = [2:2:10; 12:2:20; 22:2:30; 32:2:40; 42:2:50];
>> y = [hypergeom([1 2 3 4],[3 4 5],x)];
>> norm (y,1)
>> norm (y,2)
>> norm (y,inf)
>> norm (y,'fro')
```

NORM OF A DATBASE 3 APPELL'S DOUBLE HYPERGEOMETRIC FUNCTION F_2 (a, b, b'; c; x,y) <u>for a = -1</u>

<u>Database 3</u>

b,b'c,c'↓x,y→	2,1	4,3	6,5	8,7	10,9	12,11	14,13	16,15	18,17	20,19
2,3,3,4	-0.08333	2.0833	8.25	18.417	32.583	50.75	72.917	99.083	129.25	163.42
3,4,4,5	-0.1	2.8	10.5	23	40.3	62.4	89.3	121	157.5	198.8
4,5,5,6	-0.1	3.3	12.033	26.1	45.5	70.233	100.3	135.7	176.43	222.5
5,6,6,7	-0.09524	3.6667	13.143	28.333	49.238	75.857	108.19	146.24	190	239.48
6,7,7,8	-0.08929	3.9464	13.982	30.018	52.054	80.089	114.13	154.16	200.2	252.23
7,8,8,9	-0.08333	4.1667	14.639	31.333	54.25	83.389	118.75	160.33	208.14	262.17
8,9,9,10	-0.07778	4.3444	15.167	32.389	56.011	86.033	122.46	165.28	214.5	270.12
9,10,10,11	-0.07273	4.4909	15.6	33.255	57.455	88.2	125.49	169.33	219.71	276.64
10,11,11,12,	-0.06818	4.6136	15.962	33.977	58.659	90.008	128.02	172.7	224.05	282.07
11,12,12,13	-0.0641	4.7179	16.269	34.59	59.679	91.538	130.17	175.56	227.73	286.67

Norm 1 = 2482.52832445332
Norm 2 = 1224. 23209936978
Norm (infinite) = 1041.43956043956
Norm (frobenious) = 1224.23647090243

MATLAB PROGRAM FOR NORM OF A DATBASE 3 OF APPELL' S DOUBLE HYPERGEOMETRIC FUNCTION F_2 (a, b, b'; c; x, y)

```
>> x = [2:2:20];
>>y = [1:2:19];
>>Y=[(hypergeom([-1 2],3,x).*hypergeom([-1 3],4,y)); (hypergeom([-1 3],4,x).*hypergeom([-1 4],5,y));(hypergeom([-1 4],5,x).*hypergeom([-1 5],6,y));(hypergeom([-1 5] ,6,x).*hypergeom([-1 6],7,y));(hypergeom([-1 6],7,x).*hypergeom([-1 7],8,y));(hypergeom([-1 8],9,x).*hypergeom([-1 9],10,y)); (hypergeom([-1 9],10,x).*hypergeom([-1 10],11,y));(hypergeom([-1 10],11,x).*hypergeom([-1 11],12,y));(hypergeom([-1 11], 12,x) .*hypergeom([-1 12],13,y));(hypergeom([-1 12],13,x).*hypergeom([-1 13],14,y))];
>>norm(Y, 1)
>>norm(Y, 2)
>>norm(Y, inf)
>>norm(Y, 'fro')
```

CONCLUSION

Normed Vector space of generalized hypergeomatric function used in Hilbert spaces. They are indispensable tools in the theories of partial differential equations, quantum mechanics, Fourier analysis (which includes applications to signal processing and heat transfer)—and ergodic theory. The Kummer confluent hypergeometric function belongs to an important class of special functions of the mathematical physics with a large number of applications in different branches of the quantum (wave) mechanics. Norms are different names for essentially the same thing, though generally they'll be used in different contexts. All represent a norm corresponding to our ordinary notion of distance. The *Euclidean Norm or L_1* – Norm is our usual notion of distance applied to an n-dimensional space. It is the square root of the sum of squares of the distances in each dimension.The Manhattan Norm or L_2- Norm is a special case of the L_p -Norm where L_2 is equivalent to the Euclidean norm and would be used only in the context where another L_p norm is relevant. The above result clearly show that the *Frobenius Norm* is also equivalent to the Euclidean norm generalised to matrices instead of vectors. A matrix norm is how much a matrix can stretch a vector to a maximum. If norm of a matrix is say 10; it means it can stretch a vector x by 10 maximum. For most things we want the L_2 norm. One important property is that it is invariant under rotation. Measure the distance between two points, then rotate everything and measure the distance again. I will get the same answer using the L_2 norm but not the other. The L_1 norm is a very popular penalty function in applied math because it promotes sparsity and is robust against outliers. L_1 norm regularization is used a lot in areas like machine learning, image processing, and compressed sensing. In probability, I have random variables who are part of the L_1 space but not the L_2 space, so here it is natural for we use this L_1 norm. L_2 is the natural norm associated with the Euclidian distance, so if we are working in a Euclidian space, I might want to use this one. The max norm might be wise if we just want to cap our error in "each point". Lp norm might be of use when we are looking at certain complex space (nonlinear spaces) where I might want to look at the penalty on derivatives of different orders. I am finding it harder to justify to engineers where they will use Normed vector spaces- I know it is important for time domain (state space) control theory and you describe stresses in materials using tensors. For other fields of engineering, computer memory extensively uses the conception of partition of matrices. If the matrix size gets larger than the space of computer memory it divides the matrices into sub matrices and does calculation. Again linear operator plays a key role in computer graphics. For many CAD software generates drawing using linear operators. I would like to tell about one of the important application of my research it deals as a Sparse Matrices. sparse matrices are useful for savings in storage: There are lesser non zero elements than zeros and saving in computing time: have a smart storage algorithm and save the computing time. If you have a very large matrix with few non-zero values, it doesn't always make sense to store the zero values. It can make operations very expensive when the dimensions are large. You can take advantage of having less information by representing your matrix in a way that only uses the non-zero entries. The matrix is converted into a table of index-value pairs. The index (m,n) is two integers representing the row and column, and the value is the entry for that element. You can see how this will save memory, at the very least, if your matrix is huge but only has a few (non-zero) entries. The sparse matrix would be small in memory, while the actual matrix would be huge for storing all the zeros. This article open new vista for the researchers.

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