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# EXPANSION OF MAPPINGS IN VARIOUS FUZZY METRIC SPACES: THEOREMS OF FIXED POINTS

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**ABSTRACT**

*Most academics work within the context of fuzzy metric space (F z- met spc), which employs fuzzy numbers. Based on fuzzy scalars, Xia and Guo [5] created the F z-met spc, which is similar to the traditional -met spc. This article uses fuzzy scalars to define fuzzy partial-met spc, a generalisation of F z-met spc. Fixed point theorems (FPTs) in partial F z-met spcs have been established using the concept of fuzzy scalars for extended mappings.*



**1. INTRODUCTION -**A met spc is simply a non - empty set involved with a two-variable function d that allows us to evaluate distance between points. We should indeed measure the distance not only between numerals and vectors, rather than between more intricate concepts including such sequences, sets, and numerous functions in various branches of mathematics. Several approaches have been proposed in this direction in order to determine a better conceptualization. Zadeh [12] devised the conceptualization of fuzzy sets in 1965 as a novel way to describe ambiguity in everyday life. Following Zadeh's pioneering work, there has already been a significant effort placed into generalizing fuzzy approximations of classical theories. A significant improvement is made possible in the direction of fuzzy topology, among several other disciplines.By adding the theory of fuzziness to classical metric and met spcs, Michalek and Kramosil [16] proposed the notion of $F_z -$ met spcs. In a metric linear space, Heilpern [7] developed a fuzzy contraction FPT and proposed the conceptualization of fuzzy mapping, a set-valued mapping generalization. The fuzzy extension of the point to set maps is this theorem. Simultaneously, fuzzy contraction for pair of mappings, Bose [11] generalized Heilpern's [7] fuzzy FPT. Furthermore, a FPT for non-expansive fuzzy mapping on a Banach space subset has been established. Fuzzy FPTs of fuzzy mappings or $F_z -$ met spcs have been studied widely by Lou[11], Azam [13, 14], Cho [17], Xia [5], Qiu[18], Rashman[19], and Zhang[10]. For complete met spcs, Wang [20] proved FPTs for expansion mappings pertaining to mappings which were contractive. Numerous researchers in this framework (Rhodes [21], Taniguchi [22], Khan [23], Daffer [24], Hunag [25], Górnicki [26], and Gurban [27]) have improved various Wang's conceptions.Matthews [2] introduced the conceptualization of generalized metric known as partial met spcand a non-Hausd or ffgeneralized version of the Banach theorem. Valero[3] has contributed significant conclusions on FPTs for partial met spc. Utilizing fuzzy scalars, Xia [5] established $F_z -$ met spc, which differ significantly from classical met spc. Comparable to $F_z -$ met spc, fuzzy partial met spcconstructed employing fuzzy scalars is a generalization of $F_z -$ met spcwith implications in computing theory. Haghi [4] examined various partial met spc generalizations of FPTs, the majority of which have been based on fundamental met spcresults.

Numerous researchers (Ciric[28], Altun [29], Aydi [30, 31], and Romaguera [32, 33]) have subsequently generalized FPTs for single-valued as well as multi-valued mappings to partial met spcs. They implemented Huang's [1] generalized contraction fixed point hypothesis to partial met spcs.FPTs for partial $F_z -$ met spcsemploying fuzzy scalars have been developed as well as evaluated for expanding mappings, which are comparable to partial met spcsexpanding mappings.

**Basic Preliminaries**

To establish the major theorems, the following propositions are included:

**Proposition 2.1. Fuzzy Set**

According to Zadeh [12], in a distinct universe of contemplation $\Upsilon = \{y_1, y_2, \ldots, y_j\}$, a fuzzy set $\mathcal{A}$ is described as:

$$\mathcal{A} = \{< y, \mu_{\mathcal{A}} >: y \in \Upsilon\}$$

where $\mu_{\mathcal{A}} : \Upsilon \rightarrow [0, 1]$ is the membership function of set $\mathcal{A}$.A fuzzy set is a continuation in the direction of classical set with various membership degrees.

**Proposition 2.2. Fuzzy point**

In the terminology by Pu [15], for real-valued space,collection of fuzzy points known as fuzzy set, as described in the following:

2.2.1 $\mathcal{A} = \{(y, \boldsymbol{\Omega}) |\ \mathcal{A}(y) \geq \boldsymbol{\Omega}\}$, when set of $(y, \boldsymbol{\Omega}) \in \mathcal{A}$; $\boldsymbol{\Omega} \in [0, 1]$

2.2.2 $\mathcal{A} = \{(y, \boldsymbol{\Omega}) |\ \mathcal{A}(y) = \boldsymbol{\Omega}\}$, when set of $(y, \boldsymbol{\Omega}) = \mathcal{A}$; $\boldsymbol{\Omega} \in [0, 1]$

The set containing all the fuzzy points on $\Upsilon$ is defined by $P_F(\Upsilon)$

**Proposition 2.3. Fuzzy Scalars**

Specifically, if $\Upsilon = \Re$, $S_F(\Re)$ specifies the set of all fuzzy scalars and also the fuzzy points are often referred as fuzzy scalars.

Singh [6] proposes several fuzzy scalar definitions, which are as continues to follow:

For fuzzy scalars, $(\alpha, \boldsymbol{\Omega})$and$(\beta, \omega)$, then

2.3.1 $(\alpha, \boldsymbol{\Omega}) \geq (\beta, \omega)$, if $\alpha \succ \beta$ or $(\alpha, \boldsymbol{\Omega}) = (\beta, \omega)$

2.3.2 $(\alpha, \boldsymbol{\Omega})$ is said to be not less than $(\beta, \omega)$ if $\alpha \geq \beta$, then $(\alpha, \boldsymbol{\Omega}) \succ (\beta, \omega)$ or $(\beta, \omega) \prec (\alpha, \boldsymbol{\Omega})$

2.3.3 $(\alpha, \boldsymbol{\Omega})$ is non-negative fuzzy scalar for positive $\alpha$.

$S_F^+(\Re)$ signifies the set of all non-negative fuzzy scalars throughout the present article.

**Proposition 2.4. Partial$F_z -$ met spc**

FPTs in partial $F_z -$ met spcs are presented in this work. This section contains the proposed partial $F_z -$ met spc specification.

For a non-empty set$\Upsilon$, let us consider the mapping$p_F: P_F(\Upsilon) \times P_F(\Upsilon) \rightarrow S_F^+(\Re)$, then$(P_F(\Upsilon), p_F)$isknown as partial $F_z$ met spc, if for every

$$[(y, \boldsymbol{\Omega}), (y', \boldsymbol{\Omega}), (\ell, \rho)] \subset (P_F(\Upsilon), p_F).$$

The conditions specified below have been achieved.

2.4.1 $0 \leq p_F((y, \boldsymbol{\Omega}), (y, \boldsymbol{\Omega})) \leq p_F\big((y, \boldsymbol{\Omega}), (y', \omega)\big)$

2.4.2 $p_F((y, \boldsymbol{\Omega}), (y, \boldsymbol{\Omega})) = p_F((y, \boldsymbol{\Omega}), (y', \omega)) = p_F((y', \omega), (y', \omega))$, then $y = y'$and $\boldsymbol{\Omega} = \omega = 1$

2.4.3 $p_F((y, \boldsymbol{\Omega}), (y', \omega)) = p_F((y', \omega), (y, \boldsymbol{\Omega}))$

2.4.4 $p_F\big((y, \boldsymbol{\Omega}), (z, \rho)\big) \leq p_F\big((y, \boldsymbol{\Omega}), (y', \omega)\big) + p_F\big((y', \omega), (z, \rho)\big)$
$p_F((y', \omega), (y', \omega))$

**Proposition 2.5.**Huang[1], consider$(\Upsilon, \partial)$as partial $F_z -$ met spc and the mapping,$\tau: \Upsilon \rightarrow \Upsilon$ is said to be a expanding mapping, if $\forall$ $(y, \boldsymbol{\Omega}), (y', \omega)$ $\exists$ $\kappa > 1$such that

$$p(\tau y, \tau y') \geq \kappa\, p(y, y')$$

**Proposition 2.6.**Huang [1], two selfmappings$\eta(y)$and$\mu(y)$ of a partial $F_z -$ met spc$(\Upsilon, \partial)$ is commuting, if$\forall$ $y \in \Upsilon$, $\eta(\mu(y)) = \mu(\eta(y))$.

Haghi [4], for selfmappings $\eta, \mu: \Upsilon \rightarrow \Upsilon$of a set$\Upsilon$,if$y \in \Upsilon$, $\exists$ $\eta(y) = \mu(y)$, then point $y$ is known as a point of coincidence of $\eta(y)$and$\mu(y)$.

Selfmappings are weakly compatible, if for some$y \in \Upsilon$, $\exists$ $\eta(y) = \mu(y)$, then

$$\eta(\mu(y)) = \mu(\eta(y)),$$

i.e.mappings commute at their coincidence point.

**Lemma 2.7.**Huang [1], consider$(\Upsilon, p)$as a partial met spc, then

2.7.1 If$\{y_j\}$ is a Cauchy sequence in met spc$(\Upsilon, p)$, it is also a Cauchy sequence in $(\Upsilon, p^s)$ and vice a versa.

2.7.2 A partial met spc$(\Upsilon, p)$is complete iffthemet spc$(\Upsilon, p^s)$is complete. Furthermore,

$$\lim_{j\to\infty} p^s(\alpha, y_j) = 0 \text{ iff } p(\alpha,\alpha) = \lim_{j\to\infty} p(\alpha, y_j) = \lim_{j,m\to\infty} p(y_j, y_m)$$

**3. Main Results**

Fixed points theorems for generalizing mappings have been devised in partial $F_z -$ met spcin this section, as described in the following:

**Theorem 3.1.**Consider that for surjective mapping$\tau: \Upsilon \to \Upsilon$ and complete partial $F_z -$ met spc$(P_F(\Upsilon), p_F)$, $\exists\ \alpha_1, \alpha_2, \alpha_3 \geq 0$with $\alpha_1 + \alpha_2 + \alpha_3 > 1$ such that

$$p_F\left(\tau(y_j, \boldsymbol{\Omega}_j), \tau(y'_j, \omega_j)\right) \geq \alpha_1\ p_F\left((y_j, \boldsymbol{\Omega}_j), (y'_j, \omega_j)\right) + \alpha_2\ p_F\left((y_j, \boldsymbol{\Omega}_j), \tau(y_j, \boldsymbol{\Omega}_j)\right)$$
$$+\alpha_3\ p_F\big((y'_j, \omega_j), \tau(y'_j, \omega_j)\big) \quad (3.1.1)$$

$\forall\ (y, \boldsymbol{\Omega}), (y', \omega) \in \Upsilon; (y, \boldsymbol{\Omega}) \neq (y', \omega)$.

Then, there is an existence of fixed point for the mapping$\tau$in the space $\Upsilon$ .

**Proof.**Since, given mapping $\tau$ is surjective. So, let$(y_0, \boldsymbol{\Omega}_0) \in \Upsilon$, considering $(y_1, \boldsymbol{\Omega}_1) \in \Upsilon$ such that$\tau(y_1, \boldsymbol{\Omega}_1) = (y_0, \boldsymbol{\Omega}_0)$.Defining a sequence $\{(y_j, \boldsymbol{\Omega}_j)\} \in \Upsilon$,using induction, such that

$$(y_{j-1}, \boldsymbol{\Omega}_{j-1}) = \tau(y_j, \boldsymbol{\Omega}_j).$$

Retaining generality, consider

$$(y_{j-1}, \boldsymbol{\Omega}_{j-1}) \neq (y_j, \boldsymbol{\Omega}_j);\ \ \forall\ j = 1,2,3, \ldots\ldots..$$

Alternatively, there is an existence ofsome $j_0$such that

$(y_{j_0-1}, \boldsymbol{\Omega}_{j_0-1}) = (y_{j_0}, \boldsymbol{\Omega}_{j_0})$,

which results the existence of a fixed point $(y_{j_0}, \boldsymbol{\Omega}_{j_0})$for mapping $\tau$.Following are the results:

$$p_F((y_{j-1}, \boldsymbol{\Omega}_{j-1}), (y_j, \boldsymbol{\Omega}_j)) = p_F\left(\tau(y_j, \boldsymbol{\Omega}_j), \tau(y_{j+1}, \boldsymbol{\Omega}_{j+1})\right)$$

$$\geq \alpha_1 p_F\left((y_j, \boldsymbol{\Omega}_j), (y_{j+1}, \boldsymbol{\Omega}_{j+1})\right) + \alpha_2 p_F\left((y_j, \boldsymbol{\Omega}_j), \tau(y_j, \boldsymbol{\Omega}_j)\right)$$

$$+\alpha_3((y_{j+1}, \boldsymbol{\Omega}_{j+1}), \tau(y_{j+1}, \boldsymbol{\Omega}_{j+1}))$$

$$= \alpha_1 p_F\left((y_j, \boldsymbol{\Omega}_j), (y_{j+1}, \boldsymbol{\Omega}_{j+1})\right) + \alpha_2 p_F\left((y_j, \boldsymbol{\Omega}_j), (y_{j-1}, \boldsymbol{\Omega}_{j-1})\right)$$

$$+\alpha_3((\mathcal{y}_{j+1},\boldsymbol{\Omega}_{j+1}),(\mathcal{y}_j,\boldsymbol{\Omega}_j)$$

$$(1-\alpha_2)p_F\left((\mathcal{y}_{j-1},\boldsymbol{\Omega}_{j-1}),(\mathcal{y}_j,\boldsymbol{\Omega}_j)\right) \geq (\alpha_1+\alpha_3)p_F\left((\mathcal{y}_{j+1},\boldsymbol{\Omega}_{j+1}),(\mathcal{y}_j,\boldsymbol{\Omega}_j)\right).$$

Considering,$(\alpha_1+\alpha_3)=0$, then$\alpha_2<1$, whichis a contradiction. This implies,

$$\alpha_1+\alpha_3 \neq 0,$$

Hence,

$$(1-\alpha_2)<0.$$

Thus,

$$p_F((\mathcal{y}_{j+1},\boldsymbol{\Omega}_{j+1}),(\mathcal{y}_j,\boldsymbol{\Omega}_j)) \leq h\ p_F\left((\mathcal{y}_{j-1},\boldsymbol{\Omega}_{j-1}),(\mathcal{y}_j,\boldsymbol{\Omega}_j)\right)(3.1.2)$$

where,

$$h=\frac{1-\alpha_2}{\alpha_1+\alpha_3}< 1.$$

A sequence $\langle \mathcal{y}_j,\boldsymbol{\Omega}_j\rangle$ is a Cauchy sequence in$(P_F(\Upsilon),p_F)$iscomplete.

Since, the space is complete hence every Cauchy sequence must be convergent and hence, for the Cauchy sequence$\langle \mathcal{y}_j,\boldsymbol{\Omega}_j\rangle$converges to $(P_F(\Upsilon),p_F)$, also, there is an existence of a point $(\ell,\rho')\in P_F(\Upsilon)$, such that $\lim_{j\to\infty}\left((\mathcal{y}_j,\boldsymbol{\Omega}_j),(\ell,\rho')\right)=0.$

Consequently, to find $(u,\omega^{'})\in p_F(\Upsilon)$, such that $(\ell,\rho')=\tau(u,\omega')$. Using lemma (2.7),

$$p_F\{(\mathcal{y}_j,\boldsymbol{\Omega}_j),(\ell,\rho')\} = \lim_{j\to\infty} p_F\left((\mathcal{y}_j,\boldsymbol{\Omega}_j),(\ell,\rho')\right) = \lim_{j,\ m\to\infty} p_F\left((\mathcal{y}_j,\boldsymbol{\Omega}_j),(\mathcal{y}_m,\boldsymbol{\Omega}_m)\right)=0$$

(3.1.3)

Since,

$$\max\left\{\begin{matrix} p_F\left((\mathcal{y}_j,\boldsymbol{\Omega}_j),(\mathcal{y}_j,\boldsymbol{\Omega}_j)\right), \\ p_F\left((\mathcal{y}_{j+1},\boldsymbol{\Omega}_{j+1}),(\mathcal{y}_{j+1},\boldsymbol{\Omega}_{j+1})\right)\end{matrix}\right\} \leq p_F\left((\mathcal{y}_j,\boldsymbol{\Omega}_j),(\mathcal{y}_{j+1},\boldsymbol{\Omega}_{j+1})\right)(3.1.4)$$

Using (3.1.2),

$$\max\left\{\begin{matrix} p_F\left((\mathcal{Y}_j,\boldsymbol{\Omega}_j),(\mathcal{Y}_j,\boldsymbol{\Omega}_j)\right), \\ p_F\left((\mathcal{Y}_{j+1},\boldsymbol{\Omega}_{j+1}),(\mathcal{Y}_{j+1},\boldsymbol{\Omega}_{j+1})\right)\end{matrix}\right\} \le h^j p_F((\mathcal{Y}_1,\boldsymbol{\Omega}_1),(\mathcal{Y}_0,\boldsymbol{\Omega}_0)) \quad (3.1.5)$$

We shall show that$(u,\omega') = (\ell,\rho')$. Using (3.1.1),

$$p_F\{(\mathcal{Y}_j,\boldsymbol{\Omega}_j),(z,\rho)\} = p_F\left(\tau(\mathcal{Y}_{j_{+1}},\boldsymbol{\Omega}_{j+1}),\tau_u\right)$$

$$\ge \alpha_1 p_F\left((\mathcal{Y}_{j+1},\boldsymbol{\Omega}_{j+1}),u\right) + \alpha_2 p_F\left((\mathcal{Y}_{j+1},\boldsymbol{\Omega}_{j+1}),(\mathcal{Y}_j,\boldsymbol{\Omega}_j)\right)$$

$$+\alpha_3 p_F((u,\omega'),\tau(u,\omega'))$$

As, $j \to \infty$,

$$0 = p_F((\ell,\rho'),(\ell,\rho')) \ge (\alpha_1+\alpha_3)p_F(u,(\ell,\rho'))$$

which results,

$p_F((u,\omega'),(\ell,\rho')) = 0$

Thus,$(u,\omega') = (\ell,\rho') = T(u,\omega')$, which proved that $(\ell,\rho')$is a fixed point of$\tau$.

**Theorem 3.2.** Considerthat fora continuous surjective mapping $\tau:\Upsilon \to \Upsilon$ and complete partial $F_z -$ met spc$(P_F(\Upsilon),p_F)$. For$\boldsymbol{\Omega} > 1$, for each and every

$$(\mathcal{Y},\boldsymbol{\Omega}),(\mathcal{Y}',\omega) \in P_F(\Upsilon),$$

we have,

$$P_F(\tau(\mathcal{Y},\boldsymbol{\Omega}),\tau(\mathcal{Y}',\omega)) \ge \lambda(u,\omega')(3.2.1)$$

where,

$$(u,\omega') \in \{P_F((\mathcal{Y},\boldsymbol{\Omega}),(\mathcal{Y}',\omega)),P_F((\mathcal{Y},\boldsymbol{\Omega}),\tau(\mathcal{Y},\boldsymbol{\Omega})),P_F((\mathcal{Y}',\omega),\tau(\mathcal{Y}',\omega))\}.$$

Then, there is an existence of fixed point for the mapping$\tau$ in$\Upsilon$.

**Proof.**As a result of the Theorem (3.1), it is given that the mapping $\tau$ is continuous surjective.

Using induction, defining a sequence $\{(\mathcal{Y}_j,\boldsymbol{\Omega}_j)\} \in \Upsilon$ such that

$$(\mathcal{Y}_{j-1},\boldsymbol{\Omega}_{j-1}) = \tau(\mathcal{Y}_j,\boldsymbol{\Omega}_j).$$

Retaining the generality, assuming that $(\mathcal{Y}_{j-1},\boldsymbol{\Omega}_{j-1}) \neq (\mathcal{Y}_j,\boldsymbol{\Omega}_j)$; $\forall\ j = 1,2,3,\ldots.$.

Alternatively, there is an existence of some $j_0$ such that $(y_{j_0-1}, \boldsymbol{\Omega}_{j_0-1}) = (y_{j_0}, \boldsymbol{\Omega}_{j_0})$, then $(y_{j_0}, \boldsymbol{\Omega}_{j_0})$ is a fixed point of $\tau$.From (3.2.1), following are the results:

$$p_F\left((y_{j-1}, \boldsymbol{\Omega}_{j-1}), (y_j, \boldsymbol{\Omega}_j)\right) = p_F\left((\tau(y_j, \boldsymbol{\Omega}_j), \tau(y_{j+1}, \boldsymbol{\Omega}_{j+1})\right)$$
$$\geq \lambda(u_j, \omega'_j)$$

where,

$$(u_j, \omega'_j) = \left\{p_F\left((y_j, \boldsymbol{\Omega}_j), (y_{j+1}, \boldsymbol{\Omega}_{j+1})\right), p_F\left((y_j, \boldsymbol{\Omega}_j), (y_{j-1}, \boldsymbol{\Omega}_{j-1})\right)\right\}$$

Now consider the following two possibilities:

**Case I.**If $(u_j, \omega'_j) = p_F\left((y_j, \boldsymbol{\Omega}_j), (y_{j+1}, \boldsymbol{\Omega}_{j+1})\right)$,then

$$p_F((y_{j-1}, \boldsymbol{\Omega}_{j-1}), (y_j, \boldsymbol{\Omega}_j)) \geq \lambda p_F((y_j, \boldsymbol{\Omega}_j), (y_{j+1}, \boldsymbol{\Omega}_{j+1}))$$

For$(x_{j-1}, \lambda_{j-1}) = (x_j, \lambda_j)$, $p_F((y_{j-1}, \boldsymbol{\Omega}_{j-1}), (y_j, \boldsymbol{\Omega}_j)) = 0$,

which is a contradiction.

**Case II.**If$p_F((y_{j-1}, \boldsymbol{\Omega}_{j-1}), (y_j, \boldsymbol{\Omega}_j)) = 0$,then

$$p_F((y_{j-1}, \boldsymbol{\Omega}_{j-1}), (y_j, \boldsymbol{\Omega}_j)) \geq \boldsymbol{\Omega} p_F((y_j, \boldsymbol{\Omega}_j), (y_{j-1}, \boldsymbol{\Omega}_{j-1})).$$

Hence, sequence, $\langle y_j, \boldsymbol{\Omega}_j \rangle$, is Cauchy in$(P_F(\Upsilon), p_F)$ in completespace.

Every Cauchy sequence $\langle y_j, \boldsymbol{\Omega}_j \rangle$ in complete space is convergent andconverges tothe point $(\ell, \rho) \in (P_F(\Upsilon), p_F)$. Because of the continuity of $\tau$,$(\ell, \rho)$is a fixed point of $\tau$.

## Author'sContributions

The authors contributed equally. The complete research paper is pursued and authorized by both contributors.

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