

BOUNDARY CONDITION OF COMPOSITE LAMINATED PLATE

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ABSTRACT

In this paper we discussed about the boundary conditions at outer edges and at crack-axis. The displacement in-plane and out of plane are obtained in terms of two constants, one for in-plane & one for transverse direction, in previous article we obtained the components of displacement and then in-plane stresses and out of plane bending moments and twisting couple in the presence of Griffith-Crack(s).

KEYWORDS :crack axis, composite plate, Griffith-Crack, Strain, stress, laminated plate, plane band.

1 .INTRODUCTION

We divide the problem into two aspect :

[A] Buckling due to transverse load

[B] The opening of crack at due to extensional stress at of equal intensity.

In this chapter we shall consider the [A] part only. We are considering a thin rectangular plate of length $2a$ and breadth. There is a crack opened at we considered the rectangular plate orthotropic as in elastic property in middle plane and orthotropic in bending, also. It is called a special orthotropy. The plate is a composite plate with four laminated plies and having the property in which coupling stiffness coefficients are all zero.

There is symmetry about the line Therefore, the shear stress and twisting moment must be zero.

2. BOUNDARY CONDITIONS

(BUCKLING)

The following boundary conditions, see figure(1) , will given the in plane problem.

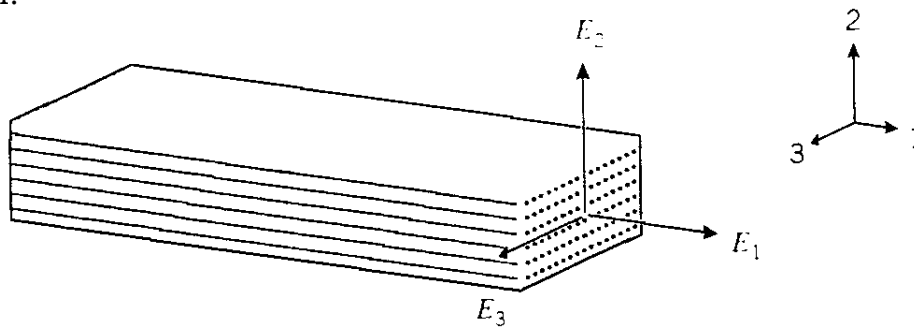


Fig (1)

$$(2.1) \quad u^0(\pm a, y) = 0,$$

$$(2.2) \quad w(\pm a, y) = 0,$$

$$N_{xy}(\pm a, y) = 0,$$

$$(2.3)$$

$$M_{xy}(\pm a, y) = 0,$$

$$(2.4)$$

Now, using some equations in previous paper then we see that these equation are identically satisfied.

Now the boundary conditions at $y=\delta$, are given as.

$$(2.5) \quad N_{xy}(x, \delta) = 0, \quad N_y(x, \delta) = Q(x)$$

$$M_y(x, \delta) = 0, \quad M_{y,y} + 2 M_{xy,x} = 0 \quad (2.6)$$

Where Q_x is applied strength at $y = \pm \delta$,

we get,

$$t_1^1 [A_n \sinh(\alpha_n \gamma_1 \delta) + B_n \cosh(\alpha_n \gamma_1 \delta)] +$$

$$t_2^1 [C_n \sinh(\alpha_n \gamma_2 \delta) + D_n \cosh(\alpha_n \gamma_2 \delta)] = 0$$

(2.7)

$$\sum \alpha_n \cos \alpha_n x \left\langle t_1^0 [A_n \cosh(\alpha_n \gamma_1 \delta) + B_n \sinh(\alpha_n \gamma_1 \delta)] + \right.$$

$$t_2^0 [C_n \cosh(\alpha_n \gamma_2 \delta) + D_n \sinh(\alpha_n \gamma_2 \delta)] \left. \right\rangle = Q(x_n)$$

(2.8)

$$(2.8) (a) \quad Q(x_n) = \frac{1}{\alpha_n} \int_0^a Q(x) \cos \alpha_n x dx$$

$$\alpha_n^2 \left[t_1^3 \langle E_n \cosh(\alpha_n \gamma_3 \delta) + F_n \sinh(\alpha_n \gamma_3 \delta) \rangle \right.$$

$$+ t_2^3 \langle G_n \cosh(\alpha_n \gamma_4 \delta) + H_n \sinh(\alpha_n \gamma_4 \delta) \rangle \left. \right]$$

$$+ \sum_{m=1}^{\infty} \frac{\beta m^2 q_{sc}(\alpha_n, \beta_m) \cos(m\pi)}{W} = 0$$

(2.9)

$$\alpha_n^3 \left[\gamma_3 t_1^3 \langle E_n \sinh(\alpha_n \gamma_3 \delta) + F_n \cosh(\alpha_n \gamma_3 \delta) \rangle + \right. \\ \left. \gamma_4 t_2^3 \langle G_n \sinh(\alpha_n \gamma_4 \delta) + H_n \cosh(\alpha_n \gamma_3 \delta) \rangle \right] = 0$$

$$\left[t_1^4 \langle E_n \sinh(\alpha_n \gamma_3 \delta) + F_n \cosh(\alpha_n \gamma_3 \delta) \rangle \right. \\ \left. + t_2^4 \langle G_n \sinh(\alpha_n \gamma_4 \delta) + H_n \cosh(\alpha_n \gamma_4 \delta) \rangle \right] = 0$$

Simplifying

$$E_n \langle \gamma_3 t_1^3 + 2t_1^4 \rangle \sinh(\alpha_n \gamma_3 \delta) + F_n \langle \gamma_3 t_1^3 + 2t_1^4 \rangle \\ \cosh(\alpha_n \gamma_3 \delta) + G_n \langle \gamma_4 t_2^3 + 2t_2^4 \rangle \sinh(\alpha_n \delta \gamma_4) \\ + H_n \langle \gamma_4 t_2^3 + 2t_2^4 \rangle \cosh(\alpha_n \gamma_4 \delta) = 0 \quad (2.10)$$

Where, $t_i^0, t_i^1, t_i^2, t_i^3, t_i^4$ are defined by in different equation of previous article respectively .

3 . BOUNDARY CONDITIONS AT X-AXIS

The boundary conditions at $y = 0$

$$N_{xy}(x, 0) = 0, \quad 0 \leq x \leq a \quad (3.1)$$

$$M_{xy}(x, 0) = 0, \quad 0 \leq x \leq a \quad (3.2)$$

we get,

$$A_{66} \sum_{n=1}^{\infty} \alpha_n \sin \alpha_n x \left[t_1^1 B_n + t_2^1 D_n \right] = 0$$

$$-\sum_{n=1}^{\infty} \alpha_n^2 \cos(\alpha_n x) \left[t_1^3 F_n + t_2^3 H_n \right] = 0$$

Then

$$t_1^1 B_n + t_2^1 D_n = 0 \quad (3.3)$$

$$t_1^3 F_n + t_2^3 H_n = 0 \quad (3.4)$$

Thus, out of eight constant $A_n, B_n, C_n, D_n, E_n, F_n, G_n, H_n$. we can express three out of four in terms of remaining fourth one. Thus we are going to find A_n, B_n, C_n in terms of D_n and E_n, F_n, G_n in terms of H_n .

From above , we get

$$B_n = -t_5 D_n, F_n = -p_5 H_n, (3.5)$$

$$t_5 = t_2^1 / t_1^1, p_5 = t_2^3 / t_1^3 (3.6)$$

Conclusion

Present analysis can be extended to most general type of composite laminated plates (coupling or without coupling). So we evaluated analytically, the boundary condition of the composite laminated plate

Reference

1. Wu Z, Chen R, Chen W (2005) Refined laminated composite plate element based on global-local higher order shear deformation theory. *Composite Structure* 70:135–152
2. Akavci SS (2010) Two new hyperbolic shear displacement models for orthotropic laminated composite plates. *Mechanical Composite Mater* 46(2):215–226
3. Roylance, D., *Mechanics of Materials*, Wiley&Sons, New York, 1996.
4. Powell, P.C, *Engineering with Polymers*, ChapmanandHall, London, 1983.
5. S Gohari, S Sharifi, Z Vrcelj, MY. Yahya ,First-ply failure prediction of an Unsymmetrical laminated ellipsoidal woven GFRP composite shell with incorporated surface-bounded sensors and internally pressurized Compos. Part B, 77 (2015), pp. 502-518,
6. PD Gosling, Polit O Faimun ,A high-fidelity first-order reliability analysis for shear deformable laminated composite *Plates Compos. Struct.*, 115 (2014), pp. 12-28,
7. Whitney, J.M. and Leissa, A.W. (1969), “Analysis of Heterogeneous Anisotropic Plates”, *Journal of Applied Mechanics*, Vol. 36, 2, pp. 261-266.
8. Bartholomew, P. (1976), “Ply Stacking Sequence for Laminated Plates Having In plane and Bending Orthotropy”, Royal Aircraft Establishment, TR-76003.