International Journal of Engineering, Science and Mathematics
Vol. 10 Issue 04, April2021,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijmra.us, Email: editorijmie@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

Heat Transfer to MHD Flow in an inclined Channel with Heat Generation/Absorption

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Abstract

The present paper deals with the motion of an incompressible viscous fluid in an inclined channel. A Uniform magnetic field is applied normal to the channel, taking heat absorption, heat generation and viscous dissipation into account. The non-dimensional partial differential equations are transfer to ordinary differential equations and the perturbation method employed to solve basic differential equations. The velocity and temperature characteristics have been studied through graphs.

Key Words: MHD, Heat transfer, Viscous dissipation

MSC : 76D05,76W05

Introduction:

The magneto hydrodynamics flow with heat transfer in porous medium more significance in recent time for the reason that the boundary layer flow effects from magnetic fields and with the help of electrically conducting fluids performance of various systems can be controlled. Geothermal energy extractions, nuclear reactors, plasma studies and MHD generators are the examples of study. The phenomena of heat transfer are the subject of wide research due to its application in industry of chemical engineering such as production of polymer and processing of food. The study of flows through porous medium is important amongst mathematicians and engineers due to its applications in biochemical, electrochemical and petroleum. The petroleum engineering and hydrology depend on the properties of porous media. Petroleum engineering is mainly related to petroleum and natural gas exploration, well drilling logs and production. The exact solution of Hartmann plane coutte flow has been has been extensively [1]. Engineering problems, Geophysical and Astrophysical etc. are the applications of MHD. Nigam [2], Soundalgekar and Bhat [3], Raptis [4], Vajravelue [5], and Attia et al. [6]. Studied MHD flowsthrough a channel. Motionof a MHD fluid with heat transfer through between two flat plates with heat source and heat sink has been investigated by Bodosa and Borkakti[7]. Makinde and Mhone [8] investigated MHD flow through a channel by considering porous medium. In the presence of heat sourceMHD flow with heat transfer was studied by Choudhary and Jha [9]. Attia [10] analyzed heat transfer and MHD steady flow between two plates which are parallel and taking variations in physical variable into account. MHD motion of a fluid through two parallel plates with heat transfer was studied by Mebine [11]. Adesanya [12] and Hassan [13] taking radiation heat generation/absorption into account.Rathore et al. [14] presented a study by using homotopy perturbation sumudu transform method for viscous MHDflow on a sheet which is stretching. Gaur et al. [15] analyzed MHD flow of polar fluid bounded by horizontal parallel plates. In the present work we investigate unsteady MHD flow through in a inclined channel and non uniform walls.A study of C6H9NAO7 fluid by using entropy generation through an accelerated heated plate was given by Ahmed et al. [16]. A study for transient MHD Brinkman nanoliquid by using Caputo–Fabrizio fractional derivatives was given by Ali et al. [17].Hussain et al. [18] presentedconservation laws and explicit solution of a spatially Burgers–Huxley two dimensional equation.Sheikh et al. [19] introduce new model of fractional Casson fluid with heat and mass transfer based on generalized Fick's and Fourier's laws together.A study for Jeffery-Hamel flow in non-parallel walls by using hybrid computational approach was given by Singh et al. [20].For the partial differential equations arising in physical sciences an efficientcomputational scheme for systems of nonlinear time fractional has been given by Dubey et al. [21]. For constant heat source through porous medium with Mittag-Leffler, exponential and power Laws a new fractional exothermic reactions model was given by Kumar et al. [22]. Hanif et al. [23] provide an entropy analysis for motion of a magneto-hybrid nanofluid withheat transfer exaggerationin a vertical cone.Recently Khan et al. [24] provide comparative study by considering two AA7072 and AA7075 and hybridized nanomaterials alloys with activation energy through binary chemical reaction comprising nonlinear MHD radiative motion through a moving thin needle.

Mathematical Formulation:

We coincide, electrically conducting incompressible viscousfluid which motionis unsteady through an inclined channel formed by two parallel plates standing in x-y direction and making an angle θ with the horizontal. The axis of x is consideredalong to the centre of the channel and the axis of y perpendicular to it. In the direction of y-axis a uniform magnetic field B_0 considered. The induced magnetic field compared with the applied magnetic field arising from induced current is negligible. The governing equations of the motion by assuming Boussinesq's approximation are given below:

$$\frac{\partial u'}{\partial t'} = -\frac{1}{\rho}\frac{\partial \rho'}{\partial x'} + \upsilon \frac{\partial^2 u'}{\partial y^2} - \frac{Ge \Pr B_0^{\ 2}}{\rho} u' + g \sin\theta - \frac{\upsilon \mathrm{u}}{K} \tag{1}$$

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2 - \frac{Cs\left(T - T_0\right)}{\rho C_p} \tag{2}$$

And the corresponding boundary conditions are

$$u' = -h_0,\ T' = T_0,\ at\ y' = -h$$

$$u' = 0,\ T' = T_1,\ at\ y' = h \tag{3}$$

Where T is temperature of the fluid, t is the time, u is the axial velocity, , g is ends force, C_p is the specific heat at constant temperature, k the thermal conductivity K is permeability of the porous medium, ρ is the density, G_e conductivity of the fluid, B_0 is the intensity of the magnetic field,P is the pressure, υ is the kinematic viscosity, Cs is the volumetric heat generation and T_0 is the wall temperature.

Non-dimensional parameters are:

$$x=\frac{x'u_0}{\upsilon},\ y=\frac{y'u_0}{\upsilon},t=\frac{t'u_0^{\ 2}}{\rho},\rho=\frac{\rho w}{u_0^{\ 2}},T=\frac{T'-T_0}{T_w-T_0},\mathrm{Re}=\frac{u_0h}{\upsilon},F_r=\frac{h_0^{\ 2}}{gh},S^2=\frac{\upsilon^2}{Ku} \tag{4}$$

Now using equation (4) in Equations (1), (2) and (3), we get dimensionless equations

$$\frac{\partial u}{\partial t}=-\frac{\partial p}{\partial x}+\frac{\partial^2 u}{\partial y^2}(H_a^{\ 2}+S^2)u+\frac{\sin\theta}{\mathrm{F_r\,Re}} \tag{5}$$

$$\frac{\partial T}{\partial t}=\frac{1}{\mathrm{Pr}}\frac{\partial^2 T}{\partial y^2}+Ec\left(\frac{\partial u}{\partial y}\right)^2-\alpha^2 T \tag{6}$$

Where $H_a^{\ 2}=\frac{\sigma B_0^{\ 2}\upsilon}{u_0^{\ 2}}$ (Magnetic field parameter)

$Q=\frac{Cs'\upsilon}{\rho C_p u0}$ (Heat source/sink parameter)

$Ec=\frac{u_0^{\ 2}}{\mu\rho C_p (T_w-T_0)^2}$ (Eckert number)

The corresponding boundary conditions becomes

$$u=-1,\ T=0,\ at\ y=-1$$
$$u=0,\ T=1,\ at\ y=1 \tag{7}$$

For solving equations (5) and (6) let

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$$\frac{\partial P}{\partial x} = ht,\; u = f(y)e^{-nt},\; T = g(y)e^{-nt}$$

Now from equations (5), (6) and (7), we get

$$u = \frac{\sinh a\,(y-1)}{\sinh 2a} + \frac{h_0 e^{-nt} + b}{a^2}\left[1 - \frac{\cosh ay}{\cosh a}\right] \quad (8)$$

Where $R^2 = {H_a}^2 + S^2,\; a^2 = R^2 - n,\; b = \frac{\sin\theta}{\mathrm{F_r\,Re}},\; h = h_0 e^{-nt}$

$$T = e^{-nt}\left[\begin{matrix}(C_1\cos Ay + C_2\sin \mathrm{M}y) + K_1\cosh 2a(y-1)e^{2nt} + \\ K_2 e^{2nt} + K_3\sinh 2ay - K_4 - K_5\sinh 2a(y-1) - K_6\end{matrix}\right] \quad (9)\; A^2 = (n-Q)\Pr,$$

$$K_1 = \frac{a^2}{(2\sinh^2 2a)(4a^2 + A^2)},$$

$$K_2 = \frac{a^2}{4A^2\sinh^2 2a},$$

$$K_3 = \frac{(h_0 e^{-nt} + b)}{2a(4a^2 + A^2)\cosh^2 a},$$

$$K_4 = -\frac{(h_0 e^{-nt} + b)^2}{4A^2},$$

$$K_5 = \frac{(h_0 e^{-nt} + b)^2 e^{nt}}{2\sinh 2a\,\sinh a},$$

$$K_6 = \frac{(h_0 e^{-nt} + b)\sinh a}{2\sinh 2a\,\sinh a}$$

Results and Discussions:

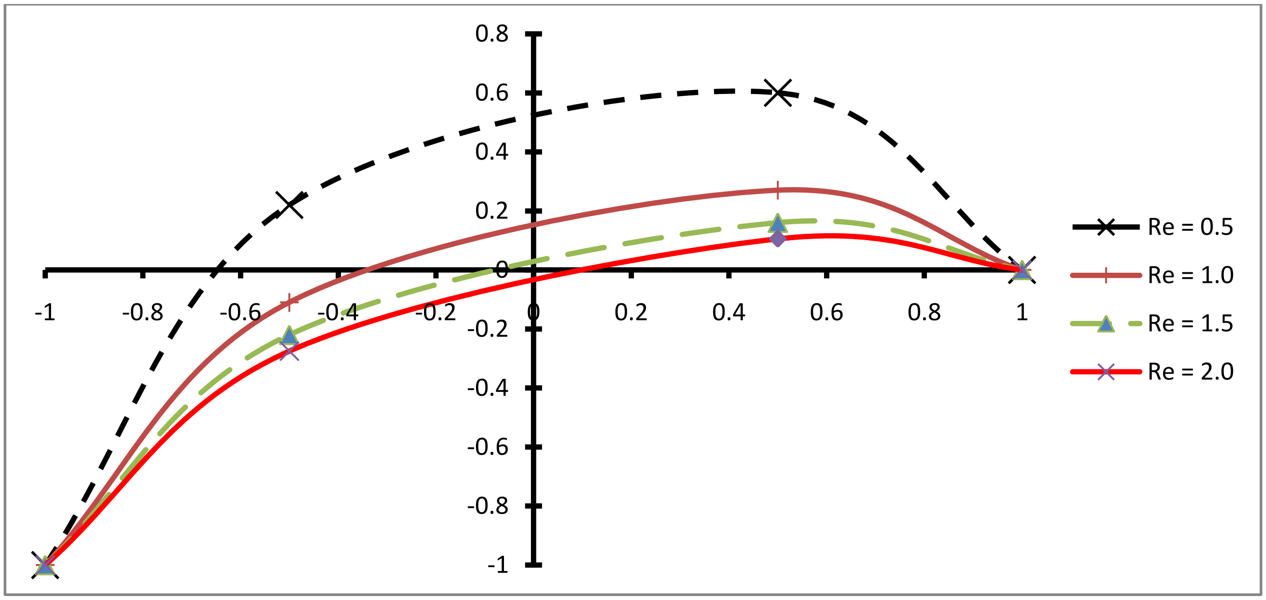

Fig.-1. Effect of Re on velocity

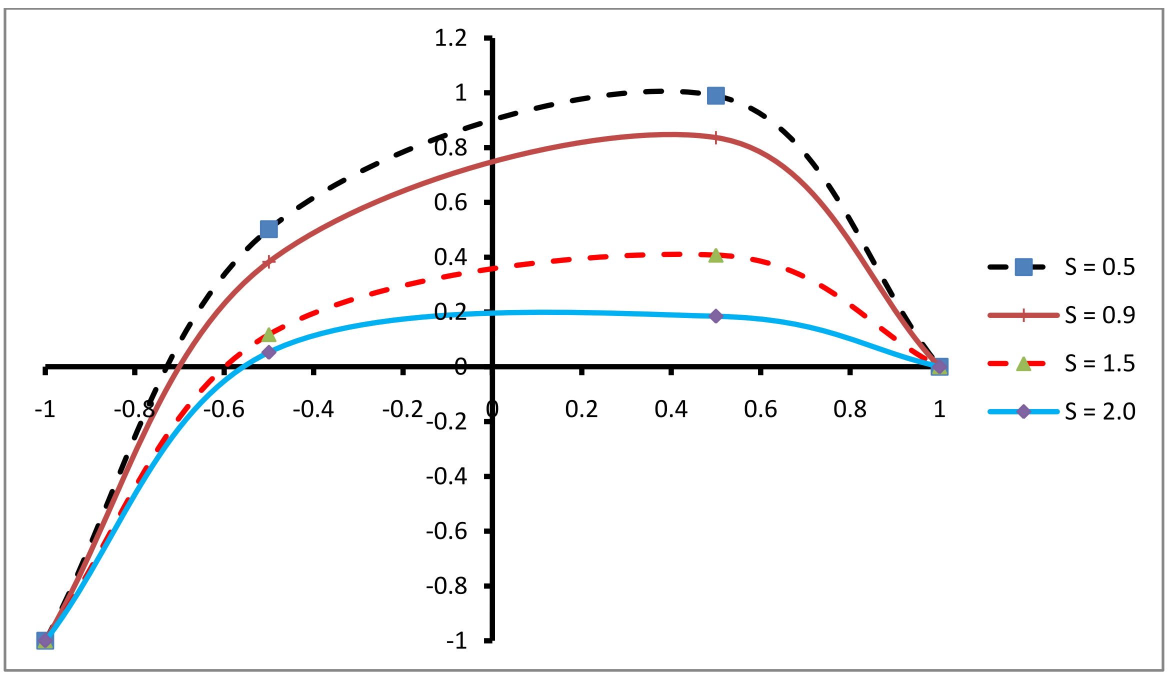

Fig.-2.Effect of S on velocity

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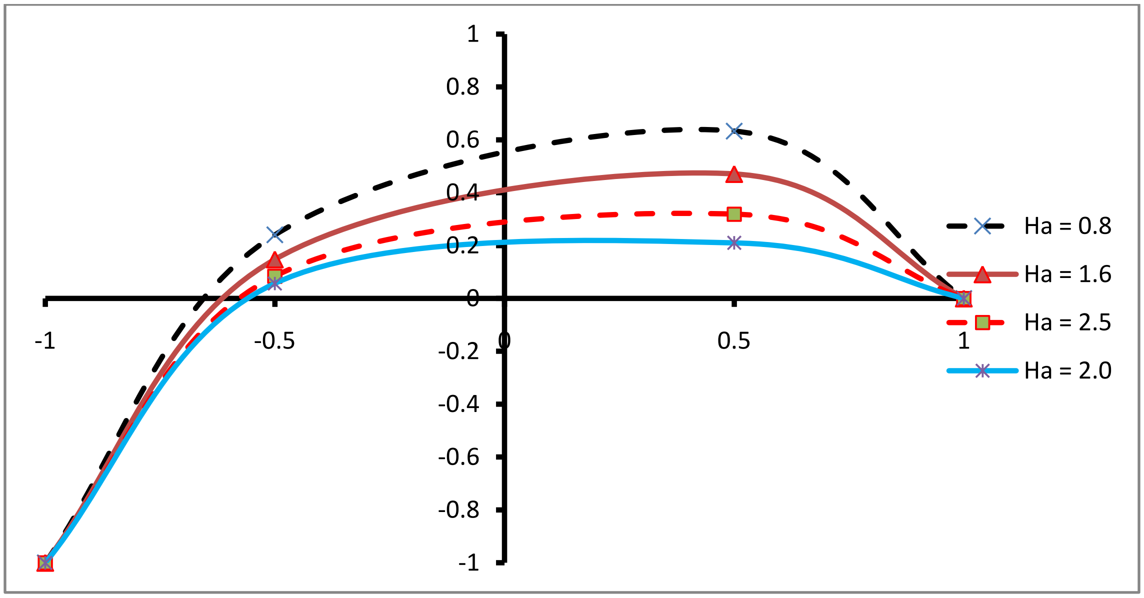

Fig.-3.Effect of Ha on velocity

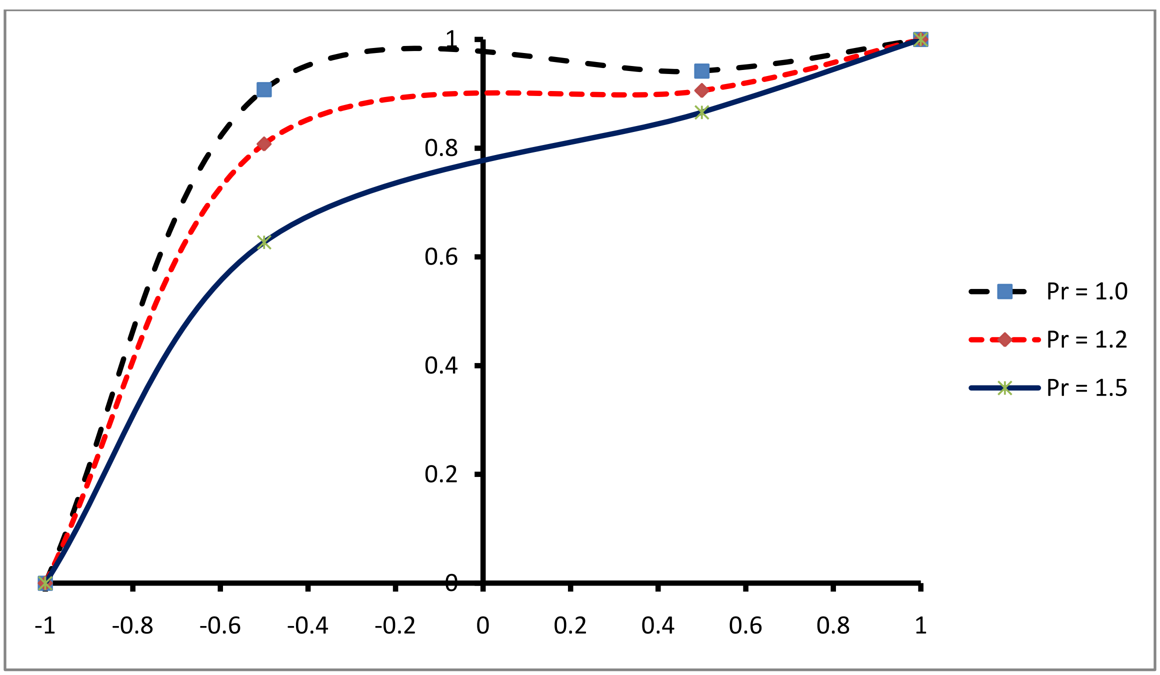

Fig.-4.Effect of Pr on temperature

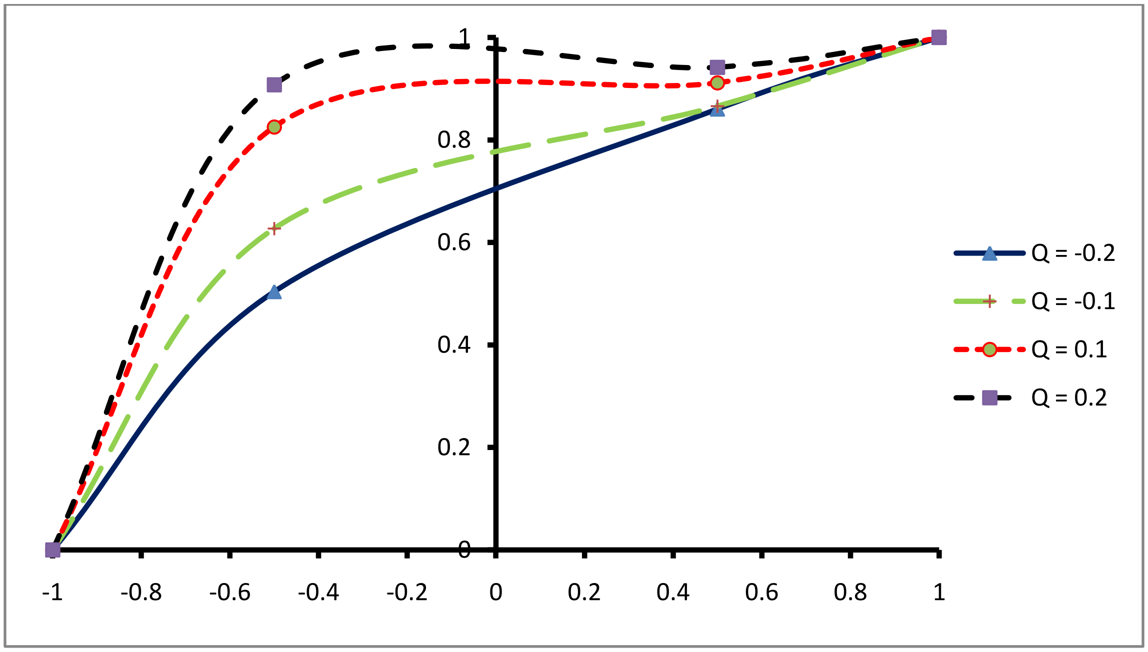

Fig.-5. Effect of Q on temperature

In the preceding section, numerical solution of the velocity profile and temperature profile are obtained and shown in graph 1-5. In order to obtain physical insight into the problem and establish the influence of different parameters on velocity profile and temperature profile, numerical calculation are performed and presented graphically. Figure 1 displays the dimensionless velocity distribution for different values of Reynolds number. It is found that the magnitude of velocity is enhanced with the raise of the Re. This is consistent with the fact that if Reynolds numberis small, in the entire flow field viscosity will be felt down and the viscous force will be predominant. Figure 2 shows the effect of permeability parameter (S) on the velocity distribution. It is noted that thickness of the boundary layer increases as S increases. Figure 3 depict the effect of transverse magnetic field on velocity. We observe that for the increasing value of magnetic field parameter Ha the velocity distribution decreases. That temperature plummets as the value of Prandtl number increases, shown in figure 4. Because the thickness of thermal boundary layer decreases as the value of Prandtl number increases. Figure 5is for knowing the effects of heat absorption and heat generation parameter (Q) on heat transfer. We observe that temperature increases for the increasing strength of heat source ($Q > 0$) while reverse effect is noted for heat sink (Q < 0). Thus width of the thermal boundary layer enlarges corresponding to heat generation parameter but thickness reduces for increasing heat absorption.

References:

1. Cramer, K.R. and Pai, S. I. (1973), *Magnetofluid dynamics for engineers and applied physicists*, MacGraw-Hill, New York.
2. Nigam, S.D. and Singh, S.N. (1960). Heat transfer by Laminar flow between parallel plates under the action of transverse magnetic field, *Q.J. Mech. Appl. Math.*, **13**, 85-87.
3. Soundalgekar, V.M. and Bhat, J.P. (1971). Oscillatory MHD channel flow and heat transfer, *Indian J. Pure Appl. Math.*,**15**, 819-828.
4. Raptis, A, Massias, C. and Tzivanidis, G. (1982). Hydromagnetic free convection flow through porous medium between two parallel plates, *Phys. Lett*., **90(A)**, 288-299.
5. Vajravelu, K. (1988). An exact periodic solution of hydromagnetic flow in a horizontal channel, *ASME Journal of Applied Mechanics*, **55**, 981-983.
6. Attia, H.A. and Kotb, N.A. (1996). MHD. Flow between two parallel plates with heat transfer, *Acta Mechanics*, **117**, 215-220.
7. Bodosa, G. and Borkakti, A.K. (2003). Unsteady free convection MHD flow and heat transfer of a visco-elastic fluid past between two heated horizontal plates with heat sources or sinks, *Indian Theo. Phys.*, **51(1)**, 39-46.
8. Makinde, O.D. and Mhone, P.Y. (2005). Heat transfer to MHD oscillatory flow in a channel filled with porous medium, *Rom. Journ. Phys*., **50**, 931-938.
9. Attia, H.A. (2006). On the effectiveness of variation in the physical variance on MHD steady flow between parallel plates with heat transfer, *Int. J. for numerical methods in Engineering*, **65**, 224-235.
10. Chaudhary, R.C., Jha, A.K. and Dhayal, S.S. (2006). Heat transfer to magneto-hydrodynamic flow in a horizontal channel with heat source, *J. Rajasthan Acad. Phy. Sci*,, **5(2)**, 233-238.

11. Mebine, P. (2007). Radiation effects on MHD flow with heat transfer between two parallel plates. *Global J. of Pure and Applied Mathematics*, **3**, 191-202.
12. Adesanya, S.O. (2012). Study of magneto-hydrodynamic visco-elastic heat generating /absorbing slip flow through porous medium with radiation effect, *Int. J. of Heat and Technology*, **30**, 69-74.
13. Rathore, S., Singh, J. and Shishodia, Y. (2013). An efficient analytical approach for MHD viscous flow over a stretching sheet via homotopy perturbation sumudu transform method. *Ain Shams Engineering Journal*. 4. 549-555. 10.1016/j.asej.2012.12.002.
14. Hassan, A.R., Gbadeyan, A.J. (2015). A reacting hydro-magnetic internal heat generating fluid flow through channel, *Int. J. of Heat and Technology*, **33(3)**, 43-50.
15. Gaur, P.K., Jha, A.K. and Sharma, R.P. (2017). MHD flow of a polar fluid through a porous medium of variable permeability bounded by horizontal parallel plates. *Acta Technica*, **62(2-B)**.1-12.
16. Ahmed, TN. and Khan, I. (2019). Entropy Generation in C6H9NAO7 Fluid over an Accelerated Heated Plate, *Front. Phys*. 7: 250. doi: 10.3389/fphy.
17. Ali, F., Sheikh, NA., Khan, I. and Nisar, KS. (2019). Caputo–Fabrizio fractional derivatives modeling of transient MHD Brinkman nanoliquid: Applications in food technology. *Chaos, Solitons & Fractals*, **131**, 109489.
18. Hussain, A., Bano, S., Khan, I., Baleanu, D. and Nisar, KS. (2019). Lie Symmetry Analysis, Explicit Solutions and Conservation Laws of a Spatially Two-Dimensional Burgers–Huxley Equation. *Symmetry*,**12 (1),** 170.
19. Sheikh, NA., Ching, DLC., Khan, I., Kumar, D. and Nisar, KS. (2019). A new model of fractional Casson fluid based on generalized Fick's and Fourier's laws together with heat and mass transfer. *Alexandria Engineering Journal.*
20. Singh, J., Rashidi, M.M., Rathore, S. and Kumar, D. (2019). A hybrid computational approach for Jeffery-Hamel flow in non-parallel walls, *Neural Computing and Applications*, **31**, 2407-2413.

21. Kumar, D., Singh, J., Tanwar, K. and Baleanu, D.(2019). A new fractional exothermic reactions model having constant heat source in porous media with power, exponential and Mittag-Leffler Laws, *International Journal of Heat and Mass Transfer*, **138**, 1222-1227.
22. Dubey, V.P., Kumar, R., Kumar, D., Singh, J. and Khan, I. (2020). An efficient computational scheme for nonlinear time fractional systems of partial differential equations arising in physical sciences. *Adv Differ Equ*.**, 46**. https://doi.org/10.1186/s13662-020-2505-6.
23. Hanif, H., Khan, I. and S., Shafie (2020). Heat transfer exaggeration and entropy analysis in magneto-hybrid nanofluid flow over a vertical cone: a numerical study. *Journal of Thermal Analysis and Calorimetry*, 1-17.
24. U Khan, U., Zaib, A., Khan, I., Baleanu, D., and Sherif, ESM. (2020). Comparative investigation on MHD nonlinear radiative flow through a moving thin needle comprising two hybridized AA7075 and AA7072 alloys nanomaterials through binary chemical reaction with activation energy, *Journal of Materials Research and Technology*.