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Analysis of disturbances in a nonlocal thermoelastic medium with two-temperature

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Abstract

The goal of the present work is to analyze the disturbances in an isotropic, homogeneous, nonlocal half-space subjected to a mechanical load. The formulation is done in the frame of modified Green-Lindsay theory of generalized thermoelasticity with two-temperature. The normal mode method is adopted to obtain the exact expressions for the displacement components, stresses and temperature in the physical domain. Numerical results are evaluated and depicted graphically with the help of MATLAB software. Comparisons of the physical quantities are made to show the effects of nonlocal and time.

Keywords: Modified Green-Lindsay theory; Nonlocal thermoelasticity; Two-temperature; Mechanical load; Normal mode analysis.

1. Introduction

The generalized thermoelasticity theories involve hyperbolic-type governing equations where the notation hyperbolic reflects the fact that thermal waves are modelled, to alleviate this physical paradox of the infinite propagation speed of the classical model and agree with finite speed of thermal signals. The first generalized theory was invented by Lord and Shulman [1], who obtained a wave-type heat equation by postulating a new law of heat conduction and introducing one relaxation time. Secondly, Green and Lindsay [2] designed a new model known as temperature rate-reliant thermoelasticity model because the entropy, heat flux and stresses rely on both temperature change and temperature rate. In addition, this theory modifies all the equations of the classical theory of thermoelasticity, by introducing two different relaxation times in the constitutive relations. Recently, Yu et al. [3] established a new model of generalized thermoelasticity by introducing the strain rate in Green-Lindsay thermoelastic model with the aid of extended thermodynamics. In this paper, they observed that the strain rate may eliminate the discontinuity of the displacement at the elastic and thermal wave front and is safer in engineering practices.

Eringen [4, 5, 6] and Eringen and Edelen [7] developed the theory of nonlocal elasticity and derived the local forms of the balance laws and the entropy inequality from the basic global balance laws by including the localization residuals. Recently, the theory of nonlocal elasticity gained more importance because of concept behind these models encompasses the interacting forces between material points having the far-reaching character. Nonlocal elasticity is a type of generalized continuum theory, completely based on remote action forces between atoms. As for physical interpretation, the nonlocal theory incorporates long range interactions between points in a continuum model and so, an internal length scale parameter should be initiated in the arrangement of the nonlocal elasticity model. Restriction to single point of medium is the major constraint in local theory of elasticity, however nonlocal theory is not dependent at the single point rather explains the elasticity globally. Acharya and Mondal [8] investigated the propagation of Rayleigh waves on the surface of a viscoelastic solid under the linear theory of nonlocal elasticity. Yu et al. [9] proposed size-dependent generalized thermoelasticity using Eringen's nonlocal model.

Modifying the Classical-Duhem inequality, Gurtin and Williams [10, 11], Chen and Gurtin [12] and Chen et al. [13] proposed the two-temperature theory of thermoelasticity. This theory stated that the heat conduction on a deformable body relies upon two distinct temperatures, one is the conductive temperature and the other is thermodynamical temperature. The surface relevant temperature is mentioned to as thermodynamic temperature, while the volume relevant is mentioned to as the conductive temperature. The two-temperature relation can be expressed by the equation $\varphi - \vartheta = a^{*}\varphi_{ii}$, where a^{*} is the material parameter and for limiting value a^{*} approaches to zero then we obtain that both the temperatures are same and the classical theory is recovered. However, in the case of time dependent situation, the two temperatures are different, regardless of the heat supply. The material parameter, also known as temperature discrepancy, is the only key element which apart the two-temperature theory from the classical theory of thermoelasticity. The two-temperature theory and the classical theory have the main difference of thermal dependence. One of the advantages of two-temperature theory is that the thermodynamic behavior can be describe better in thermoelastic problems. Youssef [14] con- structed a new theory of generalized thermoelasticity by applying the theory of heat conduction in deformable bodies.

In the current manuscript, modified Green-Lindsay theory is applied to analyze the disturbances in a nonlocal, homogeneous half-space. By employing normal mode analysis, exact solutions for displacement components, stresses and temperature fields are received in the physical domain. Numerical estimates for the physical quantities are calculated and plotted graphically to explore the impacts of various parameters considered in the problem.

2. Derivation of fundamental equations

The constitutive relations for an isotropic, homogeneous, nonlocal medium with two-temperature in the context of the MGL model:

$$(1-\varepsilon^2\nabla^2)\sigma_{ij} = \sigma_{ij}^L = \left(1+t_0\tau_0\frac{\partial}{\partial t}\right)\left[\lambda u_{r,r}\delta_{ij} + \mu\left(u_{j,i}+u_{i,j}\right)\right] - \beta\left(1+\tau_0\frac{\partial}{\partial t}\right)\theta\delta_{ij}, \quad (1)$$

$$(1-\varepsilon^2\nabla^2)(\rho T_0 S) = (\rho T_0 S)^L = \beta T_0\left(1+t_1 v_0\frac{\partial}{\partial t}\right)e_{rr} + \rho C_E\left(1+v_0\frac{\partial}{\partial t}\right)\theta, \quad (2)$$

$$e_{ij} = \frac{1}{2}\left(u_{i,j}+u_{j,i}\right). \quad (3)$$

where σ_{ij} are the components of stress, e_{ij} are the components of strain, S is the specific entropy, $\beta = (3\lambda+2\mu)\alpha_t$, while α_t represents the coefficient of linear thermal expansion. λ, μ are lame constants and C_E is the specific heat at constant strain, $\theta = T - T_0, T$ is the absolute temperature, T_0 acts as reference temperature for $\left|\frac{\theta}{T_0}\right| \ll 1, \varepsilon = e_0$, ε is the nonlocal parameter, δ_{ij} is the kroneckor delta, e_{rr} is the cubical dilatation, τ_0, v_0, are the thermal relaxation parameters, t_0 and t_1 are some constant parameters, $\sigma_{ij}^L, (\rho T_0 S)^L$ quantities corresponding to local thermoelastic solid, differentiation w.r.t. time is represented by dot and derivatives of the coordinate system are indicated by a comma.

Equation of motion

$$\sigma_{ji,j} = (1-\varepsilon^2\nabla^2)\rho\ddot{\vec{u}}_i. \quad (4)$$

By using equations (1) and (4), we get

$$\left(1+t_0\tau_0\frac{\partial}{\partial t}\right)\left[\lambda u_{r,rj} + \mu\left(u_{i,jj}+u_{j,ij}\right)\right] - \beta\left(1+\tau_0\frac{\partial}{\partial t}\right)\theta_{,j} = \rho(1-\varepsilon^2\nabla^2)\ddot{\vec{u}}_i. \quad (5)$$

The nonlocal generalization of Fourier's law for thermoelastic solid can be described as:

$$(1-\varepsilon^2\nabla^2)(q_i) = -K\theta_{,i}, \tag{6}$$

where q_i is the heat flux vector, K is the thermal conductivity.

Energy equation

$$q_{i,i} + \rho T_0 \dot{S}, \tag{7}$$

By virtue of (2), (6) and (7), the heat conduction equation reduces in the form

$$K\phi_{,ii} = \rho C_E\left(1 + v_0\frac{\partial}{\partial t}\right)\dot{\theta} + \beta T_0\left(1 + t_1 v_0\frac{\partial}{\partial t}\right)\dot{e}_{rr}, \tag{8}$$

$$\phi - \theta = a\phi_{,ii}, \tag{9}$$

where $a = \frac{a^*\omega^{*2}}{C_0^2}$.

3. Problem formulation

Consider an isotropic, nonlocal, homogeneous, micropolar thermodiffusive half space $z = 0$, with variable thermal conductivity and two temperature under the purview of the modified Green-Lindsay theory. We consider the rectangular cartesian coordinate system (x, y, z) with z-axis pointing vertically downwards into the medium. The displacement components take place the form as

$$u = u(x,z,t), \quad v = 0, \quad w = w(x,z,t). \tag{10}$$

In view of expression (10), the stress tensor components from equations (1) take the form:

$$(1-\varepsilon^2\nabla^2)\sigma_{xx} = \left(1 + t_0\tau_0\frac{\partial}{\partial t}\right)\left[(\lambda+2\mu)\frac{\partial u}{\partial x} + \lambda\frac{\partial w}{\partial z}\right] - \beta\left(1 + \tau_0\frac{\partial}{\partial t}\right), \tag{11}$$

$$(1-\varepsilon^2\nabla^2)\sigma_{zz} = \left(1 + t_0\tau_0\frac{\partial}{\partial t}\right)\left[(\lambda+2\mu)\frac{\partial w}{\partial z} + \lambda\frac{\partial u}{\partial x}\right] - \beta\left(1 + \tau_0\frac{\partial}{\partial t}\right), \tag{12}$$

$$(1-\varepsilon^2\nabla^2)\sigma_{zx} = \left(1 + t_0\tau_0\frac{\partial}{\partial t}\right)\left[\mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right]\right.. \tag{13}$$

Putting the components of stresses from equations (11) - (13) into equation (4), we obtain

$$\left(1 + t_0\tau_0\frac{\partial}{\partial t}\right)\left[(\lambda+\mu)\frac{\partial e}{\partial x} + \mu\nabla^2 u\right] - \beta\left(1 + \tau_0\frac{\partial}{\partial t}\right)\theta_{,x} = \rho(1-\varepsilon^2\nabla^2)\ddot{u}, \tag{14}$$

$$\left(1+t_0\tau_0\frac{\partial}{\partial t}\right)\left[(\lambda+\mu)\frac{\partial e}{\partial z}+\mu\nabla^2 w\right]+\beta\left(1+\tau_0\frac{\partial}{\partial t}\right)\theta_{,z}=\rho(1-\varepsilon^2\nabla^2)\ddot{w}. \quad (15)$$

The governing equations can be place into more convenient forms by presenting the following non-dimensional quantities:

$$(x',z',\varepsilon')=\frac{\omega^*}{C_0}(x,z,\varepsilon),(u',w')=\frac{\rho\omega^* C_0}{\beta_1 T_0}(u,w),(t',\tau_0',v_0')=\omega^*(t,\tau_0,v_0,),$$
$$(\theta',\phi')=\frac{\theta,\phi}{T_0},\sigma_{ij}'=\frac{1}{\beta_1 T_0}\sigma_{ij}. \quad (16)$$

Introducing Helmholtz decomposition, the potential functions on the dimensionless form can be defined as:

$$u=\frac{\partial\xi_1}{\partial x}+\frac{\partial\xi_2}{\partial z}, \qquad w=\frac{\partial\xi_1}{\partial z}-\frac{\partial\xi_2}{\partial x}. \quad (17)$$

Using the non-dimensional quantities defined in (16) and dropping the primes and using potential functions from (17) into equations (8), (9), (14) and (15), we get

$$\left[A_6^*\left(1+t_0\tau_0\frac{\partial}{\partial t}\right)\nabla^2-(1-\varepsilon^2\nabla^2)\frac{\partial^2}{\partial t^2}\right]\xi_1-A_2\left(1+\tau_0\frac{\partial}{\partial t}\right)\theta=0, \quad (18)$$

$$\left[A_3\left(1+t_0\tau_0\frac{\partial}{\partial t}\right)\nabla^2-(1-\varepsilon^2\nabla^2)\frac{\partial^2}{\partial t^2}\right]\xi_2=0, \quad (19)$$

$$\nabla^2\phi=A_{13}\left(1+v_0\frac{\partial}{\partial t}\right)\frac{\partial\theta}{\partial t}+A_{14}\left(1+t_2 v_0\frac{\partial}{\partial t}\right)\nabla^2\dot{\xi}_1, \quad (20)$$

$$\phi-\theta=a\frac{\partial^2\phi}{\partial z^2}. \quad (21)$$

4. Solution methodology

The solution of the considered physical variables can be decomposed in terms of normal modes in the following form:

$$\left[u,w,\phi,\theta,\xi_1,\xi_2,\sigma_{ij},\sigma_{ij}^L\right](x,z,t)=\left[u^*,w^*,\phi^*,\theta^*,\xi_1^*,\xi_2^*,\sigma_{ij}^*,\sigma_{ij}^{L*}\right](z)e^{(\omega t+\imath m x)}. \quad (22)$$

where $u^*,w^*,\phi^*,\theta^*,\xi_1^*,\xi_2^*,\sigma_{ij}^*$ are the amplitudes of the physical quantities, ω is the angular frequency,ı is the imaginary unit and m is the wave number in x-direction.

By virtue of expression (22), Eqs. (18) - (21) can takes the form as:

$$(B_1D^2-B_2)\xi_1^*+(h_1D^2-h_2)\phi^*=0, \quad (23)$$

$$(B_{20}D^2 - B_{21})\xi_1^* - (h_6 D^2 - h_7)\phi^* = 0, \tag{24}$$

$$(B_5 D^2 - B_6)\xi_2^* = 0. \tag{25}$$

The above constants are mentioned in Appendix A.

The condition for the existence of a non-trivial solution of the system of equations (23) and (24) gives us the differential equation as:

$$[D^4 - h_4^{'} D^2 + h_5^{'}](\xi_1^*(z), \phi^*(z)) = 0, \tag{26}$$

where $h_4^{'} = \frac{h_2^{'}}{h_1^{'}}$, $h_5^{'} = \frac{h_3^{'}}{h_1^{'}}$, $h_1^{'} = (B_1 h_6 + B_{20} h_1)$,

$$h_2^{'} = (B_1 h_7 + B_2 h_6 + B_{20} h_2 + B_{21} h_1), \quad h_3^{'} = (B_2 h_7 + B_{21} h_2).$$

The general solution of Eqs. (25) and (26) which is bounded as $z \to \infty$, will be of the form:

$$[\xi_1^*, \phi^*](z) = \sum_{i=1}^{2} [1, H_{1i}] M_i(m, \omega) e^{-\lambda_i z}, \tag{27}$$

$$(\xi_2^*)(z) = M_3(m, \omega) e^{-\lambda_3^* z}. \tag{28}$$

where $\lambda_i (i = 1,2)$ are the roots of the characteristic equations of (25) and (26) with positive real parts, $M_i (i = 1,2,3)$ are arbitrary constants depending upon m and ω and H_{1i} is the coupling constant obtained from Eqs. (23)-(24).

where $H_{1i} = \frac{B_{20}\lambda_i^2 - B_{21}}{h_6 \lambda_i^2 - h_7}$. (29)

In order to obtain the displacement components u^*, w^* using equations (22), (27) and (28) into equation (17), we obtain

$$u^* = \imath m \sum_{i=1}^{2} M_i(m, \omega) e^{-\lambda_i z} - \lambda_3 M_3(m, \omega) e^{-\lambda_3 z}, \tag{30}$$

$$w^* = -\lambda_i \sum_{i=1}^{2} M_i(m, \omega) e^{-\lambda_i z} - \imath M_3(m, \omega) e^{-\lambda_3 z}. \tag{31}$$

We obtain the expressions for the stresses and temperature distributions, using the above relations in similar manner:

$$\sigma_{xx}^{L*} = \sum_{i=1}^{2} H_{4i} M_i(m, \omega) e^{-\lambda_i z} + N_{44} M_3(m, \omega) e^{-\lambda_3 z}, \tag{32}$$

$$\sigma_{zx}^{L*} = \sum_{i=1}^{2} H_{3i} M_i(m,\omega) e^{-\lambda_i z} + N_{24} M_3(m,\omega) e^{-\lambda_3 z}, \tag{33}$$

$$\sigma_{zz}^{L*} = \sum_{i=1}^{2} H_{2i} M_i(m,\omega) e^{-\lambda_i z} + N_{14}^{*} M_3(m,\omega) e^{-\lambda_3 z}\ , \tag{33}$$

$$[\phi^*, \theta^*] = \sum_{i=1}^{2} (H_{1i}, H_{1i}^{*}) M_i(m,\omega) e^{-\lambda_i z}. \tag{34}$$

The detail of above constants is given in Appendix A.

5. Application: Mechanical load acting on the surface

Consider the following non-dimensional boundary conditions to determine the coefficients $M_n(i = 1,2,3)$ and ignore the positive exponential to avoid the unbounded solutions at infinity. Applications of normal mode analysis technique modifies the following admissible boundary conditions (we assume that the boundary conditions are equivalent to local boundary conditions):

Following are mechanical and thermal boundary conditions:

$$\sigma_{zz}^{L*}(x,0,t) = -f^*(x,t), \quad \sigma_{zx}^{L*}(x,0,t) = 0. \tag{35}$$

where $f^* = \frac{f(x,t)}{\beta_1 T_0}$ and $f(x,t)$ is a given function of x and t.

$$\frac{\partial \phi}{\partial z} = 0. \tag{36}$$

With the help of these boundary conditions, a non-homogeneous system can be obtained as:

$$\begin{bmatrix} H_{21} & H_{22} & N_{14}^{*} \\ H_{31} & H_{32} & N_{24} \\ \lambda_1 H_{11} & \lambda_2 H_{12} & 0 \end{bmatrix} \begin{bmatrix} M_1 \\ M_2 \\ M_3 \end{bmatrix} = \begin{bmatrix} -f^* \\ 0 \\ 0 \end{bmatrix}, \tag{37}$$

From Eq.(37), the expressions of $M_i(i = 1,2,3)$ can be obtained as:

$$M_1 = \frac{\Delta_1}{\Delta}, M_2 = \frac{\Delta_2}{\Delta}, M_3 = \frac{\Delta_3}{\Delta}. \tag{38}$$

Where$\Delta = \lambda_1(H_{22}H_{11}N_{24} - N_{14}^{*}H_{11}H_{32}) + \lambda_2(N_{14}^{*}H_{31}H_{12} - H_{21}H_{12}N_{24}), \Delta_1 = f^*\lambda_2 H_{12}N_{24}, \Delta_2 = -f^*\lambda_1 H_{11}N_{24}, \Delta_3 = -f^*(\lambda_2 H_{31}H_{12} - \lambda_1 H_{11}H_{32})$.

Substituting the expression (38) into Eqs. (30)-(35), we get the expressions for displacement components, stresses and temperature distribution as:

$$u^* = im\frac{1}{\Delta}\sum_{i=1}^{2} \Delta_i e^{-\lambda_i z} - \lambda_3 \frac{\Delta_3}{\Delta} e^{-\lambda_3 z}, \quad (39)$$

$$w^* = -\lambda_i \frac{1}{\Delta}\sum_{i=1}^{2} \Delta_i e^{-\lambda_i z} - \imath m \frac{\Delta_3}{\Delta} e^{-\lambda_3 z}, \quad (40)$$

$$\sigma_{xx}^{L*}(z) = \frac{1}{\Delta}\sum_{i=1}^{2} H_{4i}\Delta_i e^{-\lambda_i z} + \frac{\Delta_3}{\Delta} N_{44} e^{-\lambda_3 z}, \quad (41)$$

$$\sigma_{zz}^{L*}(z) = \frac{1}{\Delta}\sum_{i=1}^{2} H_{2i}\Delta_i e^{-\lambda_i z} + \frac{\Delta_3}{\Delta} N_{14}^* e^{-\lambda_3 z}, \quad (42)$$

$$\sigma_{zx}^{L*}(z) = \frac{1}{\Delta}\sum_{i=1}^{2} H_{3i}\Delta_i e^{-\lambda_i z} + \frac{\Delta_3}{\Delta} N_{24} e^{-\lambda_3 z}, \quad (43)$$

$$[\phi^*, \theta^*] = \frac{1}{\Delta}\sum_{i=1}^{2} (H_{1i}, H_{1i}^*)\Delta_i e^{-\lambda_i z}. \quad (44)$$

6. Computational results and discussion

In order to illustrate the field variables from a numerical perspective, the following constants are considered:

$\lambda = 9.4 \times 10^{10}\text{kgm}^{-1}\text{s}^{-2}, \quad \mu = 4.0 \times 10^{10}\text{kgm}^{-1}\text{s}^{-2}, \rho = 1.74 \times 10^{3}\text{kgm}^{-3},$
$\alpha_t = 2.36 \times 10^{-5}\text{K}^{-1}, K = 2.510\text{Wm}^{-1}\text{K}^{-1}, \quad T_0 = 293\text{K}, \tau_0 = 0.05\text{s}, \nu_0 = 0.05\text{s},$
$a = 1.2 \times 10^{4}\text{m}^2\text{s}^{-2}\text{K}^{-1}, \quad C_E = 9.623 \times 10^{2}\text{Jkg}^{-1}\text{K}^{-1}, \quad m = 1.2.$

Since ω is the complex constant (i.e. $\omega = \omega_0 + l\omega_1$), so that $e^{\omega t} = e^{\omega_0 t}[\cos(\omega_1 t) + \imath\sin(\omega_1 t)]$. For small values of time, we can take ω as real i.e. $\omega = \omega_0$.

6.1EffectofNonlocalparameter

Figures 1 represents the space variations of normal displacement *w* with distance *x* for two values of nonlocal parameter ε $(= 0.195 \times 10^{-9}, 0.195 \times 10^{-1})$.). Moreover, we found that the curves corresponding to different values follow similar pattern with differences in magnitude. Figure 2 shows the variations of normal stress with distance *x*. In this figure, the curve starts with some negative value and then tends to zero values. It is noticed that the nonlocal parameter has decreasing impact on the profile of normal stress.

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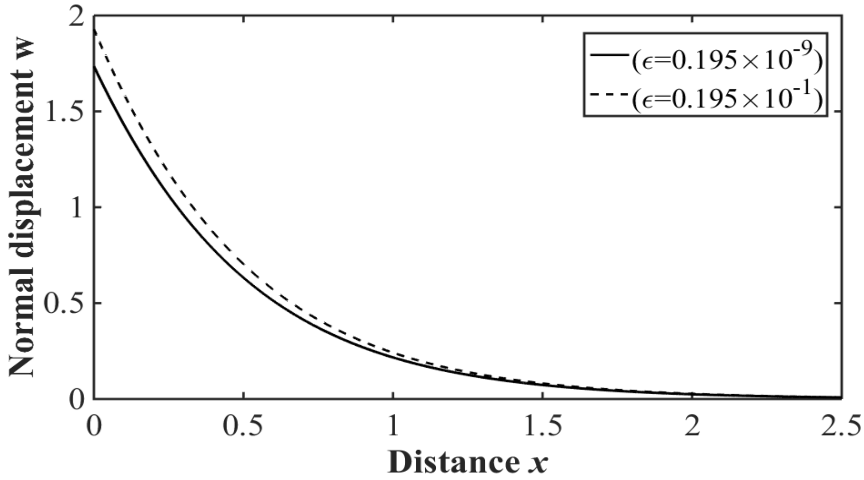

Figure 1: Variations of normal displacement with distance

Figure 3 depicts the spatial variations of tangential stress with distance *x* for two different values of nonlocal parameter. Here, tangential stress begins with a zero value for all the curves, which leads to satisfy the boundary condition. Increase in the value of nonlocal parameter results in decrease in the numerical values of tangential stress, which shows that the nonlocal parameter has decreasing effect on the profile of tangential stress.

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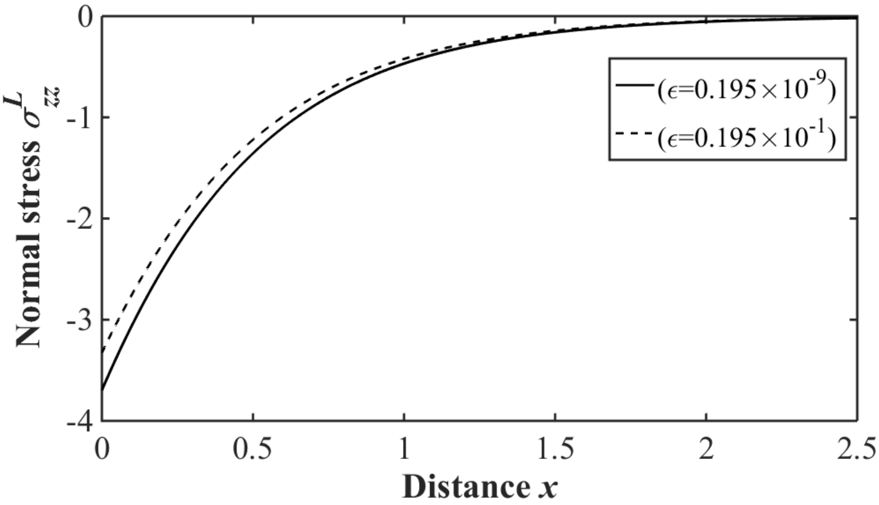

Figure 2: Variations of normal stress with distance

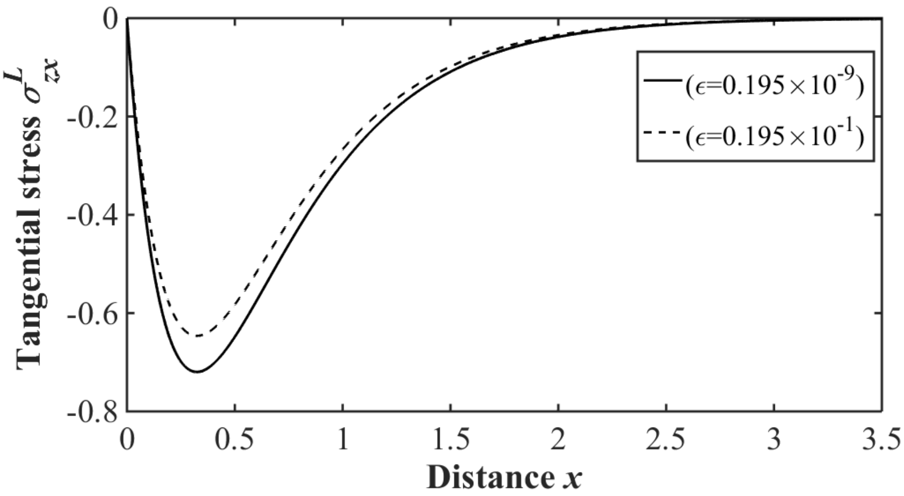

Figure 3: Variations of normal stress with distance

Figures 4-5 elucidate the space variation of conductive and thermodynamical temperature with distance *x* for two values of nonlocal parameter. The nonlocal parameter has an increasing impact on the profile of conductive and thermodynamical temperature. We have found wave type heat propagation in the medium by the distribution of the temperatures.

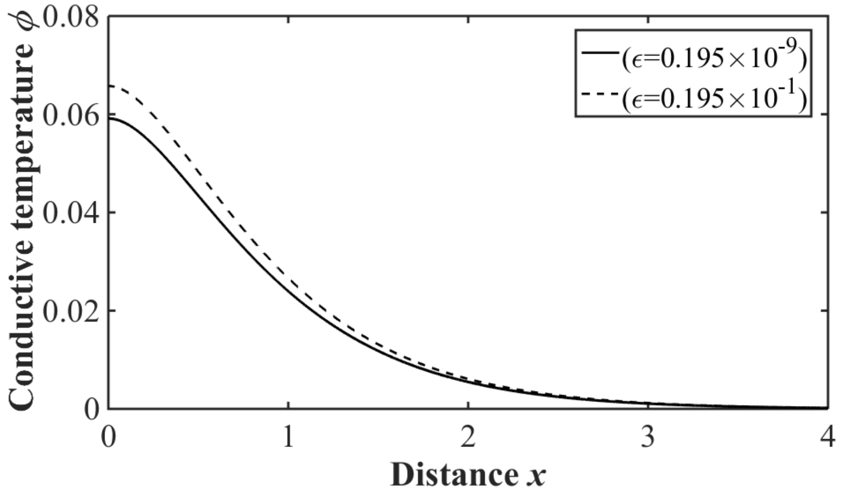

Figure 4: Variations of normal stress with distance

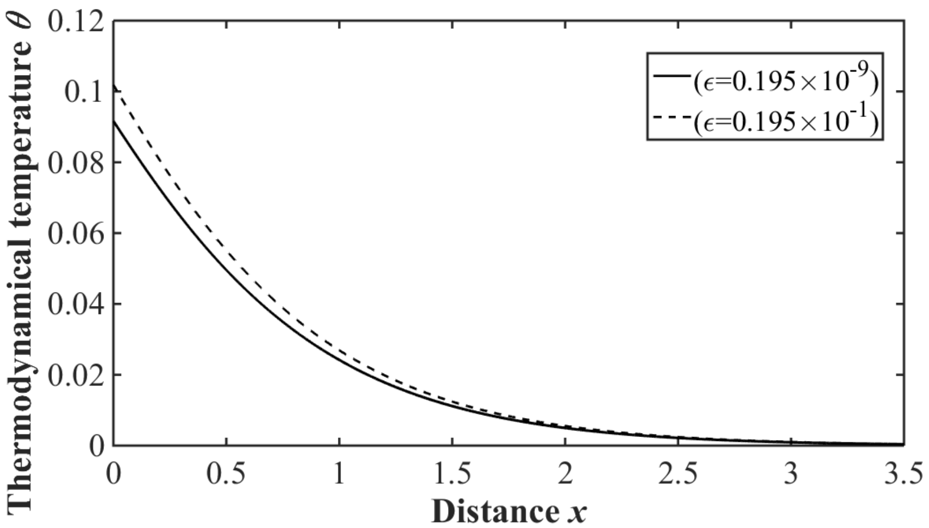

Figure 5: Variations of normal stress with distance

6.2 Effect of Time parameter

Figure6displaystheeffectoftimeparameteronnormaldisplacementforthetwodifferentvalues of time t(=0.01,0.1).The time parameter has an increasing effect on the profile of normal displacement.Figure 7 demonstrates the variation of normal stress with distance x for both values of time parameter.It can be noticed from the figure that an increase in the values of time parameter results in increase in the magnitudes of the normal stress for both the valuesof time.The effect of nonlocal parameter on tangential stress is analyzed in Figure 8.

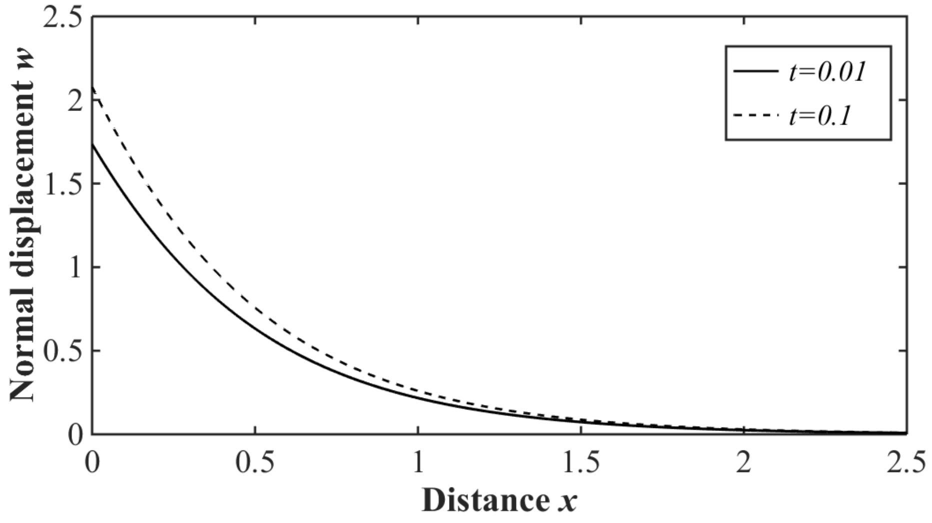

Figure 6: Variations of normal stress with distance

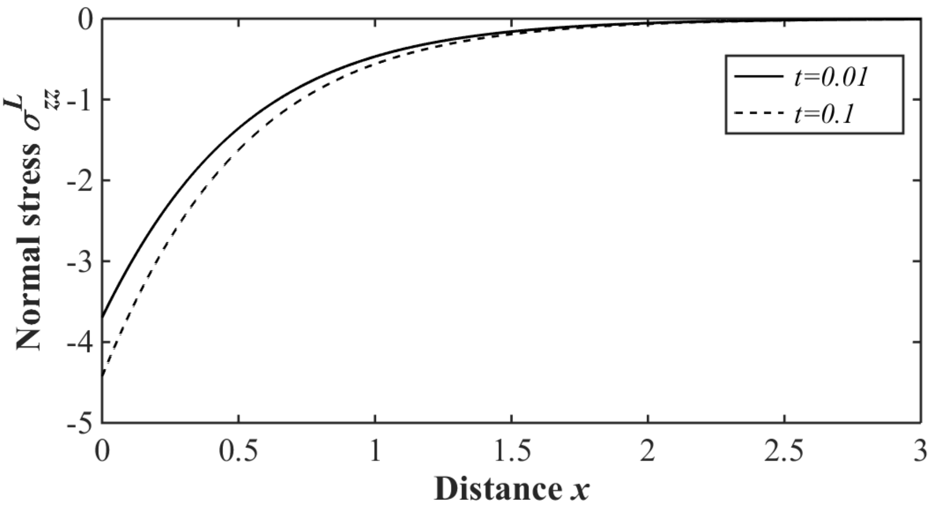

Figure 7: Variations of normal stress with distance

In this figure, all the curves start with value zero, which is totally in the favour of boundary condition. The parameter *t* has a prominent increasing effect on the profile of tangential stress for both the values of time. The pattern of variation of conductive and thermodynamical temperaturs for two values of time parameter illuminated in Figures 9 and 10. We noticed from the plots that as we increase the value of time, the numerical values of temperature fields increase. Moreover, the graphs start with some positive value and then graphs tend to zero, which is physically reasonable.

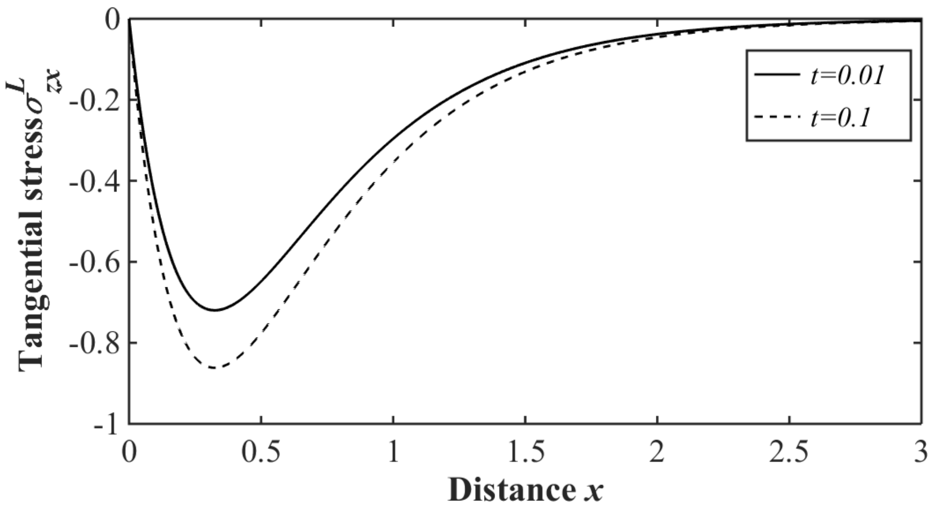

Figure 8: Variations of normal stress with distance

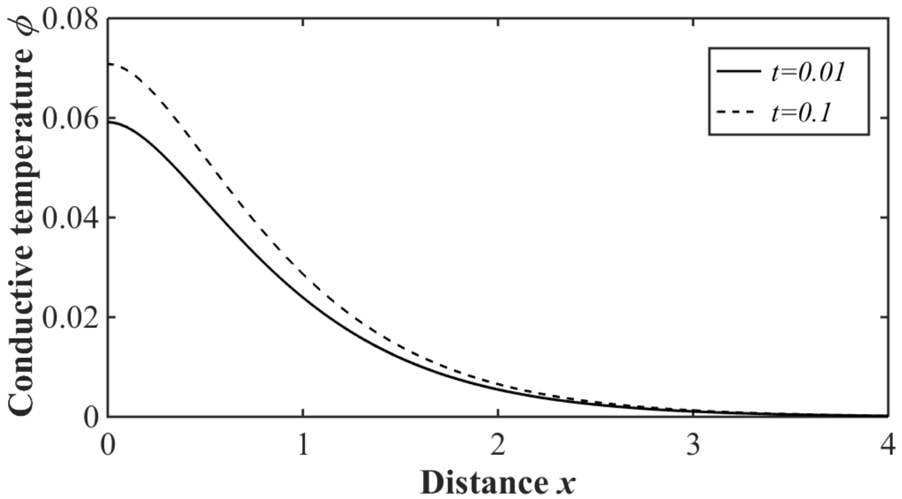

Figure 9: Variations of normal stress with distance

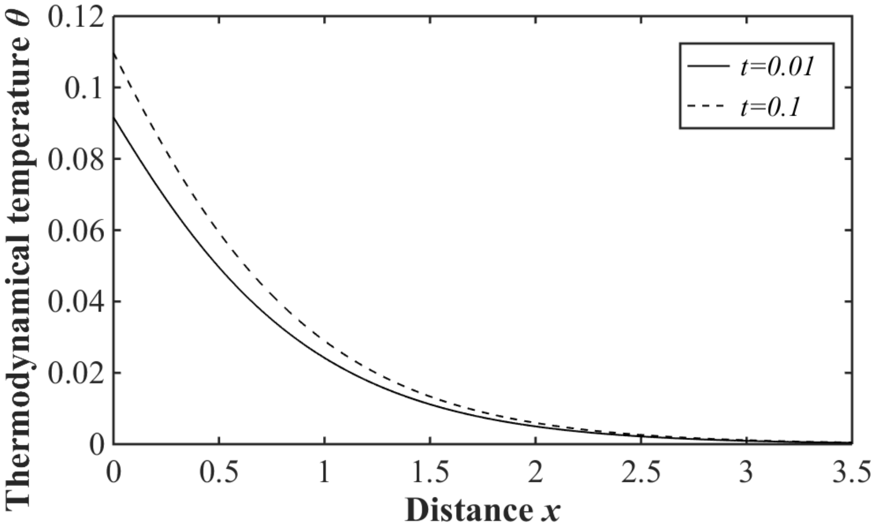

Figure 10: Variations of normal stress with distance

5. Concluding remarks

The present investigation is to furnish a mathematical model to acquire the behaviour of normal displacement, stresses and temperatures in a homogeneous, nonlocal half space with two temperature within the context of modified Green-Lindsay theory. The normal mode analysis is adopted in this work. This mathematical technique provides exact solutions without any assumed restrictions on the actual physical quantities appearing in the governing equation. The theoretical and numerical results reveal that all the parameters have significant effect on the considered physical variables. We can make the following conclusion based on the above analysis:

- From all the figures, it is apparent that all the physical variables are restricted to a limited domain of space, which is in accordance with the notion of generalized thermoelasticity theory and supports all the physical facts.
- All the field variables satisfy the boundary conditions.

- All the considered field variables are highly influenced by nonlocality. Increasing effect of nonlocality is observed in temperature and displacement fields while stress fields have received decreasing effect.

- From all figures, it is observed that time increases the magnitude of all physical fields.

- From the distribution of temperature fields, we have found a wave-type heat propagation in the medium. The heat wave front moves forward with a finite speed with the passage of time.
- Due to generalized theory, the speed of propagation remains in bounded region, which indicates that the hyperbolic theory is more appropriate than the classical theory of thermoelasticity.

A mathematical model presented here is supposed to be useful for the researchers/scientists working in the field of seismology, geomechanics, earthquake engineering, material science, designers of new materials, solid dynamics, as well as for those working on the development of a theory of hyperbolic thermoelasticity. Nonlocal continuum mechanics plays an important role in analysis related to nano-technology applications. The formation of the nonlocal thermoelastic materials gives useful information to the scientists in material chemistry for developing one dimensional nano-composites in order to apply in pharmaceutical industry and also in environment science.

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Appendix

A

$D=\frac{\partial}{\partial z}, B_1=A_6^*(1+t_0\tau_0\omega)+\varepsilon^2\omega^2, B_2=m^2+(1+\varepsilon^2m^2)(\omega^2-\Omega^2), B_3=A_2(1+\tau_0\omega),$
$B_4=(1+\tau_1\omega), B_5=A_3+\varepsilon^2\omega^2, B_6=A_3m^2+\omega^2+\varepsilon^2m^2\omega^2, B_7=A_5,\quad B_8=(1+A_7\varepsilon^2\omega^2),$
$B_9=m^2+A_7(1+\varepsilon^2m^2)\omega^2+A_8,\quad B_{10}=A_9(1+t_0\tau_0\omega),\quad B_{11}=m^2A_9(1+t_0\tau_0\omega),$
$B_{12}=1+\tau_1\omega+A_{10}\omega\varepsilon^2,\quad B_{13}=m^2(1+\tau_1\omega)+\omega A_{10}(1+\varepsilon^2\omega^2), B_{14}=A_{11}(1+t_1\tau_0\omega),$
$B_{15}=2m^2A_{11}(1+t_1\tau_0\omega),\quad B_{16}=m^4A_{11}(1+t_1\tau_0\omega),\quad B_{17}=A_{12}(1+\tau_0\omega),$
$B_{18}=m^2A_{12}(1+\tau_0\omega), B_{20}=A_{14}(1+t_2v_0\omega)\omega, B_{21}=m^2A_{14}(1+t_2v_0\omega)\omega,$
$B_{22}=A_{15}\omega(1+v_1\omega), B_{23}=a, B_{24}=1+am^2, h_1=B_3B_{23}, h_2=B_3B_{24},$
$h_3=B_{17}B_{23},\quad h_4=(B_{17}B_{24}+B_{18}B_{23}), h_5=B_{18}B_{24}, h_6=1+B_{19}B_{23},$
$h_7=m^2+B_{19}B_{24},\quad h_8=(B_{22}h_3+h_6B_{12}), h_9=(B_{22}h_4+h_6B_{13}+h_7B_{12}),$
$h_{10}=(B_{22}h_5+h_7B_{13}),\quad h_{11}=(B_{22}B_{14}+B_{12}B_{20}),$
$h_{12}=(B_{22}B_{15}+B_{12}B_{21}+B_{13}B_{20}), h_{13}=(B_{22}B_{16}+B_{13}B_{21}), h_{14}=(B_{20}h_3-h_6B_{14}),$
$h_{15}=(B_{20}h_4+B_{21}h_3-h_7B_{14}-h_6B_{15}), h_{16}=(B_{20}h_5+B_{21}h_4-B_{16}h_6-B_{15}h_7),$
$h_{17}=(B_{21}h_5-B_{16}h_7), h_1^*=(B_1h_8+h_1h_{11}+B_4h_{14}),$
$h_2^*=(B_1h_9+B_2h_8+h_1h_{12}+h_2h_{11}+h_{15}B_4), h_3^*=(B_1h_{10}+B_2h_9+h_1h_{13}+h_2h_{12}+B_4h_{16}),$
$h_4^*=(B_2h_{10}+h_2h_{13}+h_{17}B_4), H_{4i}=\sum_{i=1}^{3}\ (p_2\lambda_i^2-m^2p_1-p_3H_{1i}^*-p_4H_{2i}^*),$
$N_{4i}=\sum_{i=4}^{5}\ (\imath p_2m\lambda_i-\imath mp_1\lambda_i),\quad H_{2i}=\sum_{i=1}^{3}\ (p_1\lambda_i^2-p_2m^2-p_3H_{1i}^*-p_4H_{2i}^*),$
$N_{1i}^*=\sum_{i=4}^{5}\ (p_1\lambda_i lm-p_2\lambda_i\imath m), H_{3i}=\sum_{i=1}^{3}\ -(\lambda_i lmp_5+\lambda_i lmp_6),$
$H_{1i}^*=H_{11}(B_{24}-B_{23}\lambda_i^2), N_{2i}=\sum_{i=4}^{5}\ (p_5\lambda_i^2+p_6m^2-A_5N_{1i}),$
$p_1=(1+t_0\tau_0\omega), p_2=A_1(1+t_0\tau_0\omega), p_3=A_2(1+\tau_0\omega), p_4=(1+\tau_1\omega),$
$p_5=A_3(1+t_0\tau_0\omega), p_6=A_4(1+t_0\tau_0\omega), N_{34}=\frac{\gamma\omega^{*2}}{\rho c_0^4}\lambda_4N_{14}, N_{35}=\frac{\gamma\omega^{*2}}{\rho c_0^4}\lambda_5N_{15},$
$H_{4i}^*=\sum_{i=1}^{2}\ (p_2\lambda_i'^2-m^2p_1-p_3H_{1i}^*), H_{2i}^*=\sum_{i=1}^{2}\ (p_1\lambda_i'^2-p_2m^2-p_3H_{1i}^*),$
$C_{1i}=\frac{B_{20}\lambda_i'^2-B_{21}}{h_6\lambda_i'^2-h_7}, C_{1i}^*=C_{1i}(B_{24}-B_{23}), N_{24}^*=p_5\lambda_4^{*2}+p_6m^2.$