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Certain Results on Quasi- Hadmard products

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ABSTRACT

In this paper,we found certain results on Quasi-Hadmard products. For analytic starlike convex p-valent general function.

KEYWORDS:

- p-valent, infinite series, summation formula

1. **Introduction:** Many authors [1],[2],[5],[8],[9],[10] etc., studied the various classes of the analytics starlikes convex (Univalent as well p-valent) functions and there important properties. In this paper we study the Quasi–Hadamard Products for the class $T^*_{p,n}\ (A, B, \alpha)$ which introduce by Singh and Sohi[5].

Let $S_{p,n}$ denote the class of functions[5] of the form

$f(z) = z^p + \sum_{k=n+p} a_k z^k\ ;\ \ a_k \geq 0$ (1.1)

Where $n, p \in N$ and $f(z)$ is analytic and p-valent in the unit disc

$E = \{z; |z| < 1\}$,for fixed A and B, $-1 \leq B < A < 1$; a function of the class $S_{p,n}$ is said to be in class $S^*_{p,n}\ (A, B, \alpha)$ iff

$\frac{zf'(z)}{f(z)} < \frac{p+[(p-\alpha)A+\alpha B]z}{1+Bz}, z \in E$ (1.2)

Or equivalently $f(z) \in S^*_{p,n}\ (A, B, \alpha)$ iff

$|(\frac{zf'(z)}{f(z)} - p)/((p-\alpha)A + \alpha B - B\frac{zf'(z)}{f(z)}| < 1, z \in E$ (1.3)

Again $f(z)$ is said to belong to the class $K^{*}_{p,n}\ (A,B,\alpha)$ iff

$\frac{zf'(z)}{p} \in S^{*}_{p,n}\ (A,B,\alpha)$ (1.4)

Also, if$T_{p,n}$ denotes the subclass of $S_{p,n}$ consisting of functions of the form

$f(z) = z^p - \sum_{k=n+p} a_k z^k\ ;\ \ a_k \geq 0$ (1.5)

(Where $n, p \in N$, and $f(z)$ is analytic and p-valent in E) then we define

$T^{*}_{p,n}(A,B,\alpha) = S^{*}_{p,n}\ (A,B,\alpha) \cap T_{p,n}$ (1.6)

$C_{p,n}(A,B,\alpha) = K_{p,n}\ (A,B,\alpha) \cap T_{p,n}$ (1.7)

Here, we study the class $T_{p,n}$ of p-valent functions and their corresponding subclass $T^{*}_{p,n}(A,B,\alpha)$and $C_{p,n}(A,B,\alpha)$ of starike and convex functions. Our classes are generalizations of several subclasses available in the mathematical literature, we mention below some important special cases:

(i) For $\alpha = 0$,we gets the class $T^{*}_{p,n}(A,B)$ and $C_{p,n}(A,B)$ studied by Sohi[7].

(ii) For $p = 1$, we get the class of functions $f(z)$ which are analytic and univalent in unit disc E. In further subsituations $\alpha = 0; A = 1 - 2\delta$ and B=-1, we get the subclass that was studied by Srivastavaet. al.[9].

(iii) For $n = 1$ and $p = 1$, we get the class of functions $f(z)$ which are analytic and univalent in E, and the corresponding classes $T^{*}_{1,1}(A,B)$ and $C_{1,1}(A,B)$ were studied by Amrik Singh and N.S Sohi[6].

(iv) On setting $A = \beta - 2\alpha\beta\gamma$ and $B = -\beta\gamma$ in [2]. further for $\gamma = 1$, we get the class of functions that were studied by seikinet al.[4]

For the class $T^{*}_{p,n}(A,B,\alpha)$and $C_{p,n}(A,B,\alpha)$, we have the following coefficient [5] contained in.

Lemma 1: If functions $f(z) \in T_{p,n}$ satisfy the condition

$$\sum_{k=n+p}^{\infty}[(1-B)k-(1-A)p-(A-B)]\alpha_k \leq (A-B(p-\alpha)(1.8)$$

$\boldsymbol{f(z)} \in T_{p,n}^{*}(A,B,\alpha)$. The equality in (1.8) is attained by the function

$$f_1(z) = z^p - \frac{(A-B)(p-\alpha)z^k}{[(1-B)k-(1-A)p-(A-B)\alpha]}(k \geq n+p)(1.9)$$

Lemma2 If functions If functions $f(z) \in T_{p,n}$ satisfy the condition

$$\sum_{k=n+p}^{\infty}\binom{k}{p}[(1-B)k-(1-A)p-(A-B)\alpha]\alpha_k \leq (A-B)(p-\alpha).$$

(1.10)

Then $f(z) \in C_{p,n}(A,B,\alpha)$. The equality in (1.10) is attained by the function

$$f_2(z) = z^p - \frac{p(A-B)(p-\alpha)z^k}{k[(1-B)k-(1-A)p-(A-B)\alpha]}(k \geq n+p) \qquad (1.11)$$

2. **Quasi- Hadamard Products: Let** $f_j(z)(j = 1, \dots \dots . m)$ satisify(1.5), that is

$$f_j(z) = z^p - \sum_{k=n+p}^{\infty} a_{k_j} z^k \ (j = 1, \dots \dots . m) \qquad (2.1)$$

We denote by $f_1 * f_2 * \dots \dots f_m *$, the Quasi- Hadamard Product of the functions

$f_1, f_2, \dots \dots, f_m$ and defined as

$$f_1(z) * f_2(z) * \dots \dots f_m(z) = z^p - \sum_{k=n+p}^{\infty} a_{k_1} a_{k_2 a_{k_3}} \dots \dots a_{k_m} z^k$$

With the help of Lemma 1 and 2, we obtain the Quasi- Hadamard Products for

theclasses $T_{p,n}^{*}(A,B,\alpha)$ and $C_{p,n}(A,B,\alpha)$ given by.

Theorem1: If $f_j(z) \in T_{p,n}^{*}(A,B,\alpha_j)(j = 1,2,3 \dots m)$ then

$$(f_1 * f_2 * \dots \dots f_m)(z) \in T_{p,n}^{*}(A,B,\beta)$$

where

$$\beta = p - \frac{(1-B)\prod_{j=1}^{m}(p-\alpha_j)}{\prod_{j=1}^{m}[(1-B)n+(A-B)(p-\alpha_j)]-(A-B)\prod_{j=1}^{m}(p-\alpha_j)} \qquad (2.2)$$

The result is sharp for functions

$$f_j(z) = z^p - \frac{(A-B)(p-\alpha_j)z^{n+p}}{[(1-B)n-(A-B)(p-\alpha_j)]}(j = 1, \dots m) \qquad (2.3)$$

Theorem2 $f_j(z) \in C_{p,n}(A,B,\alpha_j)(j = 1,2,3 \dots m)$then

$$(f_1 * f_2 * \dots \dots f_m)(z) \in C_{p,n}(A,B,\delta)$$

where

$$\delta = p - \frac{(1-B)p^{m-1}\prod_{j=1}^{m}(p-\alpha_j)}{(n+p)^{m-1}\prod_{j=1}^{m}[(1-B)n+(A-B)(p-\alpha_j)]-(A-B)p^{m-1}\prod_{j=1}^{m}(p-\alpha_j)} \qquad (2.4)$$

The result is sharp for the function the function

$$f_j(z) = z^p - \frac{(A-B)(p-\alpha_j)z^{n+p}}{(n+p)[(1-B)n-(A-B)(p-\alpha_j)]}(j = 1, \dots m) \qquad (2.5)$$

Theorem3: If $f_j(z) \in T^*_{p,n}(A,B,\alpha_j)(j = 1,2,3 \dots m)$ and

$g_i(z) \in C_{p,n}(A,B,\alpha_j)(i = 1,2,3 \dots q)$then

Then

$$(f_1 * f_2 * \dots \dots * f_m * g_1 * g_2 * \dots \dots * g_q)(z) \in C_{p,n}(A,B,\gamma)$$

Where

$$\gamma = p - \frac{1}{L}[(1-B)np^{q-1}\prod_{j=1}^{m}(p-\alpha_j)\prod_{i=1}^{q}(p-\beta_i) \qquad (2.6)$$

and

$$L = \{(n+p)^{q-1}\prod_{j=1}^{m}[(1-B)n+(A-B)(p-\alpha_j)]\prod_{i=1}^{q}[(1-B)n +$$

$$(A-B)(p-\beta_i) - p^{q-1}(A-B)\prod_{j=1}^{m}(p-\alpha_j)\prod_{j=1}^{m}(p-\beta_i)\}$$

(2.7)

The result is sharp for functions

$$f_j(z) = z^p - \frac{(A-B)(p-\alpha_j)z^{n+p}}{[(1-B)n-(A-B)(p-\alpha_j)]}(j = 1, \dots m) \qquad (2.8)$$

$$g_i(z) = z^p - \frac{(A-B)(p-\beta_i)z^{n+p}}{(n+p)[(1-B)n+(A-B)(p-\beta_i)]}(i = 1, \dots q) \qquad (2.9)$$

Proof of theorem1

We invoke the principle of mathematical induction to prove the theorem for $m = 1$, we find that $\beta = \alpha_1$

Now, for $m = 2$, we have only to find the largest β such that

$$\sum_{k=n+p}^{\infty} \frac{[(1-B)k-(1-A)p-(A-B)\beta}{(A-B)(p-\beta)} \alpha_{k_1}\alpha_{k_2} \leq 1 \quad (2.10)$$

Since $f_j(z) \in T_{p,n}^*(A, B, \alpha_j)(j = 1,2)$, from lemma 2 we have

$$\sum_{k=n+p}^{\infty} \left(\frac{[(1-B)k-(1-A)p-(A-B)\alpha_j}{(A-B)(p-\alpha_j)} \right) \alpha_j \leq 1\,; (j = 1,2) \quad (2.11)$$

Then, using Cauchy-Schwarz inequality and following the Principle Mathematical Induction rule we can prove it.

Further, if the function $f_j(z)$ are defined by (2.3), then we have

$$f_1(z) * f_2(z) * \ldots \ldots f_m(z) = z^p - \frac{(A-B)\sum_{j=1}^{m}(p-\alpha_j)}{\prod_{j=1}^{m}[(1-B)n+(A-B)(p-\alpha_j)} z^{n+p}$$

$$= z^p - A_{n+p}z^{n+p} \quad (2.12)$$

Which shows that

$$\sum_{k=n+p}^{\infty} \left(\frac{[(1-B)k+(1-A)p-(A-B)\beta}{(A-B)(p-\beta)} \right) A_k$$

$$= \left(\frac{[(1-B)n+(A-B)(p-\beta)}{(A-B)(p-\beta)} \left\{ (A-B) \prod_{j=1}^{m} \frac{(p-\alpha_j)}{[(1-B)n+(A-B)(p-\alpha_j)} \right\} \right. \quad (2.13)$$

Consequently, the result stated in theorem 1 is sharp for functions $f_j(z)$ defined by (2.3)

Proof of theorem2:

The result is obvious for $m = 1$.

For $m = 2$, we have to find the largest δ such that

$$\sum_{k=n+p}^{\infty} \left\{ \left(\frac{k}{p}\right) \frac{[(1-B)k-(1-A)p-(A-B)\delta]}{(A-B)(p-\delta)} \right\} \alpha_{k_1}\alpha_{k_2} \leq 1 \quad (2.14)$$

Using lemma 3 and proceeding and similar lines as in theorem1 we can we prove the theorem 2.

Proof of theorem3: we know that if $f(z) \in T^*_{p,n}(A,B,\alpha)$ and $g(z) \in$

$C_{p,n}(A,B,\beta)$,

Then $(f * g)(z) \in C_{p,n}(A,B,\gamma)$, where

$$\gamma = p - \frac{(1-B)(p-\alpha)(p-\beta)}{[(1-B)n+(A-B)(p-\alpha)][(1-B)n+(A-B)(p-\beta)]-(A-B)(p-\alpha)(p-\beta)}$$

Thus theorem1 and theorem2 together lead to the desired result.

3. Special Cases of theorem1

The theorem 1 through 3 is quite integral as they involve general classes of function. For the sake of illustrations, we mention below some interesting(new and known) special theorem for theorem 1 only

(I) Letting $\alpha_j = \alpha\ (j = 1, \ldots\ldots m)$ in theorem 1, we get

Corollary1.

If $f_j(z)$ $\in T^*_{p,n}(A,B,\alpha); (j = 1,2, \ldots\ldots m)$, then

$$(f_1(z) * f_2(z) * \ldots\ldots f_m)(z) \in T^*_{p,n}(A,B,\beta^*)$$

Where

$$\beta^* = \frac{(1-B)(p-\alpha)^m}{[(1-B)n+(A-B)(p-\alpha)]^m-(A-B)(p-\alpha)^m} \quad (3.1)$$

The result is sharp for functions

$$f_j(z) = z^p - \frac{(A-B)(p-\alpha_j)z^{n+p}}{[(1-B)n-(A-B)(p-\alpha)]}(j = 1, \ldots m) \quad (3.2)$$

Further, for $m = 2, p = 1, A = 1$and $\beta = 0$, we get the result obtained earlier by srivastava and Chatterjea[8]'

(II) Setting $p = 1; n = 1$ in theorem1, we have

Corollary 2

If $f_j(z) \in T^*_{1,1}(A, B, \alpha_j); (j = 1,2, \dots \dots m)$, then

$(f_1 * f_2 * \dots \dots f_m)(z) \in T^*_{1,1}(A, B, \beta^1)$, where

$$\beta' = \frac{(1-B)\prod_{j=1}^{m}(1-\alpha_j)}{\prod_{j=1}^{m}[1+A-2B-(A-B)\alpha_j]-\prod_{j=1}^{m}(1-\alpha_j)} \quad (3.3)$$

$$f_j(z) = z - (A - B)\left(\frac{1-\alpha_j}{(1+A-2B)-(A-B)\alpha_j}\right)z^2 (j = 1, \dots m) \quad (3.4)$$

Further, for A=1, B=0, we get the known result obtained by owa[1], Also for m=2 (with A = 1, B = 0) we arrive at another known result silverman[3].

(III) Again, if we take A=1, B = 0 in theorem 1-3, we get the result for the classes studied by Tariq[10, p. 159 Eqs (4.5.2) and (4.5.3)] and by owa[2]. Also, taking $\alpha = 0$ in theorems 1-3, we get the result for the classes studied by Singh and sohi[6].

Conclusion: In this paper we studied certain classes of MultivalentFunctions, for certain results on Quasi Hadmard products. We also prove the corollary on it, with known and unknown result.

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