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Normal mode analysis to a nonlocal thermoelastic medium with gravity using Lord-Shulman theory

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Abstract

The main objective is to investigate interactions in a non localthermoelastic medium with gravityunder Lord-Shulman theory. The mechanical load is applied at the outer free surface of the half-space to obtain the complete solution of the physical field distributions.By employing the normal mode technique, the analytical expressions for the displacement components, stresses and temperature are obtained in the physical domain. The comparisons are made among the results obtained by taking into account the effect of gravity and the different values of nonlocal parameter. Theoretical and numerical findings show that the physical variables under consideration are significantly influenced by the nonlocal parameter and gravity field. All physical fields maintain the phenomena of the finite speed of propagation which is consistent with the generalized theory of thermoelasticity.

Keywords: Lord-Shulman theory; Non local; Gravitational force; Mechanical load; Normal mode analysis.

1. Introduction

The coupled theory of thermoelasticity developed by Biot (1956), removes the defect of uncoupled theory that mechanical causes have no effect on the temperature. However, this theory shares a defect of the uncoupled theory of predicting infinite speed of propagation for heat signals. Two well-known generalized thermoelastic models that drew attention of researchers are given by Lord and Shulman (1967) and Green and Lindsay (1972). Lord and Shulman (1967) formulateda theory of generalized thermoelasticity by including a flux rate term and one relaxation time in the Fourier's law of heat conduction and obtained a hyperbolic heat transport equation. This is known as L-S theory. Second generalized theory of thermoelasticity was introduced by Green and Lindsay (1972) which is known as G-L theory. This theory contains two constants that act as two relaxation times, which follows the classical Fourier's law of heat conduction and admits finite speed. Chandrasekharaiah (1998) brought out an extensive review article on the generalized theories of thermoelasticity.

Eringen and Edelen (1972) and Eringen (1972, 1974) extended the concept of nonlocality to elasticity and developed the theory of nonlocal elasticity. The nonlocal models differ from the classical (local) models by taking into account the principal balance laws that apply to the whole body. The main concept behind the nonlocal theory of elasticity is that the stress field at any reference point in the continuous body depends not only on the strain at that point but is also affected by the strains at all other neighborhood points. Balta and Suhubi (1977) developed another theory of nonlocal thermoelasticity. In this theory, the entropy inequality was used in classical form. By using the concept of nonlocal elasticity, Acharya and Mondal (2002) discussed the propagation of Rayleigh surface waves in a viscoelastic solid. Roy et al. (2015) examined the combined effect of magnetic field and rotation on nonlocal Rayleigh surface waves.

Generally, the impact of gravitational force is neglected in the classical model of elasticity. Gravitational influence on the problem of vibration of waves in materials, in particular on an elastic globe, was first considered by Bromwich (1898). Effects of angular velocity and gravitational field on a thermoelastic half-space were observed by Ailawalia and Narah (2009).

The current manuscript is an attempt to study the thermodynamical interactions in a homogeneous isotropic non local thermoelastic half-space with gravity under Lord–Shulman theory and Eringen's nonlocal elastic theory. The numerical results of physical quantities are discussed and illustrated graphically using MATLAB software. Non local and gravity show an increasing impact on all the considered physical quantities. Results carried out in this paper can be used in material science, designers of new materials, earthquake engineering, seismology, nuclear reactors and solid mechanics etc.

2. Governing equations

For an isotropic, homogeneous thermoelastic medium, the elastic, thermal transport equations with the effect of nonlocal based on L-S theory can be written as:

Constitutive equations

$$(1-\epsilon^2\nabla^2)\sigma_{ij} = \sigma_{ij}^L = 2\mu\epsilon_{ij} + (\lambda e - \gamma\theta)\delta_{ij}, \quad (1)$$

$$\epsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), e = u_{i,i}, \quad (2)$$

Equation of motion

$$\mu\nabla^2 u_i + (\lambda+\mu)\nabla e - \gamma\nabla\theta + F_i = \rho(1-\epsilon^2\nabla^2)\ddot{u}_i, \quad (3)$$

Heat conduction equation

$$K\nabla^2\theta - \gamma T_0\frac{\partial e}{\partial t} = \rho C_E\left(1+\tau_0\frac{\partial}{\partial t}\right)\frac{\partial\theta}{\partial t}, \quad (4)$$

where the physical quantities $\vec{u}$ and θ are the basic variables that denote to the components of the displacement, conductive temperature, σ_{ij} and σ_{ij}^L are the stress components, ϵ_{ij} are the strain components, λ and μ are Lame's constants, $\gamma = (3\lambda+2\mu)\alpha_t, \alpha_t$ is the thermal expansion coefficient, δ_{ij} is the Kronecker delta, K is the coefficient of thermal conductivity, ρ is the mass density, $\theta = T - T_0, T$ is the absolute temperature, T_0 is the reference temperature of the medium in its natural state assumed to be $\left|\frac{\theta}{T_0}\right| \leq 1$, $e = e_{kk}$ is the cubical dilatation, In the above equations. F_i is the body force, C_E is the specific heat at constant strain, τ_0 is the thermal relaxation time,

3. Formulation of problem

We consider a two-dimension problem, assumed that wave propagate in $x-y$ plane, hence all the considered functions will be depended on the time t and the coordinates x and y.

Thus, the components of the displacement and temperature will have the form

$$\vec{u}=(u,v,0),\quad u=u(x,y,t),\quad v=v(x,y,t),\quad \theta=\theta(x,y,t) \tag{5}$$

The components of gravitational force $\vec{F}=\left(F_x,F_y,F_z\right)$ in $x-y$ plane is defined as

$$\left(F_x=\rho g\frac{\partial v}{\partial x},F_y=-\rho g\frac{\partial u}{\partial x},F_z=0\right). \tag{6}$$

In view of expressions (5), the stresses arising from Eqs. (1) and (2) can be expressed as follows

$$(1-\epsilon^2\nabla^2)\sigma_{xx}=(2\mu+\lambda)\frac{\partial u}{\partial x}-\gamma\theta, \tag{7}$$

$$(1-\epsilon^2\nabla^2)\sigma_{yy}=(2\mu+\lambda)\frac{\partial v}{\partial y}-\gamma\theta, \tag{8}$$

$$(1-\epsilon^2\nabla^2)\sigma_{yx}=\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right). \tag{9}$$

Under to influence of gravity, Eq.(4) can be written as

$$\rho(1-\epsilon^2\nabla^2)\left(\frac{\partial^2 u}{\partial t^2}+g\frac{\partial v}{\partial x}\right)=\mu\nabla^2 u+(\lambda+\mu)\frac{\partial e}{\partial x}-\gamma\frac{\partial\theta}{\partial x}, \tag{10}$$

$$\rho(1-\epsilon^2\nabla^2)\left(\frac{\partial^2 u}{\partial t^2}-g\frac{\partial u}{\partial x}\right)=\mu\nabla^2 v+(\lambda+\mu)\frac{\partial e}{\partial y}-\gamma\frac{\partial\theta}{\partial y}. \tag{11}$$

Heat conduction equation in $x-y$ plane

$$K\left(\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right)=\gamma T_0\left(\frac{\partial^2 u}{\partial t\,\partial x}+\frac{\partial^2 v}{\partial t\,\partial y}\right)+\rho C_E\left(1+\tau_0\frac{\partial}{\partial t}\right)\frac{\partial\theta}{\partial t}. \tag{12}$$

The following dimensionless quantities are introduced to transform basic equations into a nondimensional form

$$(x',y',u',v')=\frac{1}{C_T t^*}(x,y,u,v),(t',\tau_0')=\frac{1}{t^*}(t,\tau_0),C_T^2=\frac{2\mu+\lambda}{\rho},$$
$$\theta'=\frac{\gamma}{2\mu+\lambda}\theta,\left(\sigma_{ij}',\sigma_{ij}^{L'}\right)=\frac{1}{\mu}\left(\sigma_{ij},\sigma_{ij}^{L}\right),t^*=\frac{K}{\rho C_E C_T^2}. \tag{13}$$

The displacement vector $\vec{u}$ can be written in the form

$$\vec{u}=\text{grad}\,\Pi+\text{curl}\,\Psi \tag{14}$$

Introducing the displacement potentials $\Pi(x,y,t)$ and $\Psi(x,y,t)$ which related to displacement components by the relations

$$u=\frac{\partial\Pi}{\partial x}+\frac{\partial\Psi}{\partial y},v=\frac{\partial\Pi}{\partial y}-\frac{\partial\Psi}{\partial x}, \tag{15}$$

Using Eq. (13) and (14) in Eqs. (7)-(12) along with some simplifications, we obtain the equations (after dropping the primes) are

$$(1-\epsilon^2\nabla^2)\sigma_{xx}=\beta^2\nabla^2\Pi-2\left(\frac{\partial^2\Pi}{\partial y^2}-\frac{\partial^2\Psi}{\partial y\,\partial x}\right)-\beta^2\theta, \tag{16}$$

$$(1-\epsilon^2\nabla^2)\sigma_{yy}=(\beta^2-2)\frac{\partial^2\Pi}{\partial x\,\partial y}+\frac{\partial^2\Psi}{\partial y^2}+\beta^2\left(\frac{\partial^2\Pi}{\partial y\,\partial x}-\frac{\partial^2\Psi}{\partial x^2}\right)-\beta^2\theta, \tag{17}$$

$$(1-\epsilon^2\nabla^2)\sigma_{yx}=\mu\left(\frac{\partial^2\Pi}{\partial x\,\partial y}+\frac{\partial^2\Psi}{\partial y^2}+\frac{\partial^2\Pi}{\partial y\,\partial x}-\frac{\partial^2\Psi}{\partial x^2}\right), \tag{18}$$

$$\left(\nabla^2-(1-\epsilon^2\nabla^2)\frac{\partial^2}{\partial t^2}\right)\Pi-(1-\epsilon^2\nabla^2)\left(g\frac{\partial\Psi}{\partial x}\right)-\theta=0, \tag{19}$$

$$\left(\nabla^2-(1-\epsilon^2\nabla^2)\beta^2\frac{\partial^2}{\partial t^2}\right)\Psi+(1-\epsilon^2\nabla^2)\beta^2\left(g\frac{\partial\Pi}{\partial x}\right)=0, \tag{20}$$

$$\nabla^2\theta-\left(1+\tau_0\frac{\partial}{\partial t}\right)\frac{\partial\theta}{\partial t}-\epsilon_1\nabla^2\dot{\Pi}=0. \tag{21}$$

4. Normal mode analysis

The normal mode analysis method is used, which has the advantage to get the exact solution without any assumed constraints on the physical quantities. In this method, the solution of the physical quantities is transformed in terms of normal modes, which appear in the governing equations of the problem considered. So, the solution of the physical variables under consideration can be decomposed in term of normal modes in the following form:

$$\left(\Pi, \Psi, \theta, \sigma_{ij}, \sigma_{ij}^{L}\right)(x, y, t) = \left(\Pi^*, \Psi^*, \theta, \sigma_{ij}^*, \sigma_{ij}^{L^*}\right)(y)\exp(\omega t + \iota m x), \quad (22)$$

where ω is the angular frequency, ι is the imaginary unit and m is the wave number in the x-direction.

The following set of equations can be obtained by incorporating expression (22) into Eqs. (18)-(21)

$$(A_1 D^2 + A_2)\Pi^* + (A_3 D^2 + A_4)\Psi^* - \theta^* = 0, \quad (23)$$

$$(A_5 D^2 + A_6)\Psi^* + (A_7 D^2 + A_8)\Pi^* = 0, \quad (24)$$

$$(D^2 - A_9)\theta^* - A_{10}(D^2 - m^2)\Pi^* = 0, \quad (25)$$

where $\epsilon_1 = \frac{\gamma^2 T_0 t^*}{K\rho}, \beta^2 = \frac{2\mu+\lambda}{\mu}, A_1 = 1 + \omega^2\epsilon^2, A_2 = -m^2 - \omega^2 - \omega^2\epsilon^2 - \omega^2\epsilon^2 m^2,$

$A_3 = \iota g m\epsilon^2, A_4 = -\iota g m - \iota g m^3\epsilon^2, A_5 = 1 + \beta^2\omega^2\epsilon^2, A_6 = -m^2 - \beta^2\omega^2 - \beta^2\omega^2\epsilon^2 m^2 - \beta^2\omega^2, A_7 = -\beta^2\iota m\epsilon^2, A_8 = \beta^2\iota m + \beta^2\iota m^3\epsilon^2, A_9 = m^2 + (1 + \tau_0\omega)\omega, A_{10} = \omega\epsilon_1.$

Eliminating $\Pi^*(y), \Psi^*(y)$ and $\theta^*(y)$ between eqs. (23)-(25), we obtain the following sixth order ordinary differential equation

$$\left(D^6 + P_1' D^4 + P_2' D^2 + P_3'\right)(\Pi^*, \Psi^*, \theta^*) = 0, \quad (26)$$

Eqn. (26) can be factored as:

$$(D^2 - k_1^2)(D^2 - k_2^2)(D^2 - k_3^2)(\Pi^*, \Psi^*, \theta^*) = 0, \quad (27)$$

where $k_n (n = 1,2,3)$ are the roots of the characteristic equation(27).

The general solutions of Equation (27), bounded as $y \to \infty$, are given by:

$$[\Pi^*, \Psi^*, \theta^*](y) = \sum_{n=0}^{3} [1, H_{1n}, H_{2n}] M_n e^{-k_n y}, \tag{28}$$

where $M_n (n = 1,2,3)$ are arbitrary coefficients depending upon m and ω.

Introducing the nondimensional parameters and potential functions and applying normal mode analysis described in equations (13), (15) and (22), respectively, the Eqs. (15), (16) and (17) convert as follows

$$[u^*, v^*, \sigma_{yy}^*, \sigma_{yx}^*](y) = \sum_{n=0}^{3} [\gamma_{1n}, \gamma_{2n}, H_{3n}, H_{4n}] M_n e^{-k_n y}, \tag{29}$$

where $H_{qn}, (n = 1,2,3)$ and $(q = 1,2,3)$ are given in "appendix **A** ".

5. Boundary conditions

We consider a homogeneous isotropic thermoelastic half-space occupying the region $y \geq 0$ with quiescent initial state. The half-space is subjected to a thermal load. Also, no mechanical load is imposed to the considered medium. The suitable boundary conditions on $y = 0$ may be given as

(i) The normal stress condition (stress free), so that

$$\sigma_{yy}(x, 0, t) = p_1, \tag{30}$$

(ii) The tangential stress condition (stress free), then

$$\sigma_{yx}(x, 0, t) = 0, \tag{31}$$

(iii) Mathematically, boundary conditions on conductive temperature θ in terms of insulated boundary can be written as

$$\theta(x, 0, t) = 0, \tag{32}$$

Using the expressions of the field variables from equations (39) to (43) into the above boundary conditions, one can obtain

$$\begin{bmatrix} H_{31} & H_{32} & H_{33} \\ H_{41} & H_{42} & H_{43} \\ H_{21} & H_{22} & H_{23} \end{bmatrix} \begin{bmatrix} M_1 \\ M_2 \\ M_3 \end{bmatrix} = \begin{bmatrix} p_1 \\ 0 \\ 0 \end{bmatrix}, \qquad (33)$$

The expressions for $M_n\,(n = 1,2,3)$ procured by solving the system (33) are given as

$$M_1 = \frac{\Delta_1}{\Delta}, \quad M_2 = \frac{\Delta_2}{\Delta}, \quad M_3 = \frac{\Delta_3}{\Delta}, \qquad (34)$$

where Δ and $\Delta_n\,(n = 1,2,3)$ are defined in "appendix **A**".

Substituting expressions of Eq. (34) into Eqs. (28)-(29) along with Eq.(22), provide us the following expressions of field variables

$$[\Pi, \Psi, \theta](y) = \frac{1}{\Delta}\sum_{n=0}^{3} [1, H_{1n}, H_{2n}]\Delta_n e^{-k_n y}, \qquad (35)$$

$$[u, v, \sigma_{yy}, \sigma_{yx}](y) = \frac{1}{\Delta}\sum_{n=0}^{3} [\gamma_{1n}, \gamma_{2n}, H_{3n}, H_{4n}]\Delta_n e^{-k_n y}. \qquad (36)$$

6. Numerical results and discussion

The following material constants are used to carry out the numerical computations:

$\lambda = 3.64 \times 10^{10}\text{Nm}^{-2}, \quad \mu = 5.46 \times 10^{10}\text{Nm}^{-2}, \quad \rho = 2330\text{Kgm}^{-3}, \quad \tau = 5 \times 10^{-5}\text{s}$
$T_0 = 300KQ_0 = 1, \quad \alpha_t = 1.98m^3\text{kg}^{-1} \times 10^{-4}, K = 300\text{wm}^{-1}\text{k}^{-1},$
$\mu_0 = 12.56 \times 10^{-7}\text{Hm}^{-1}, \ \tau_0 = 0.5, C_E = 695\text{JKgg}^{-1}\text{K}^{-1}, \quad t = 0.01, \quad \omega = 1.4.$

The computations are carried out for non-dimensional variables. The variation of field variables with respect to distance y, are presented graphically.

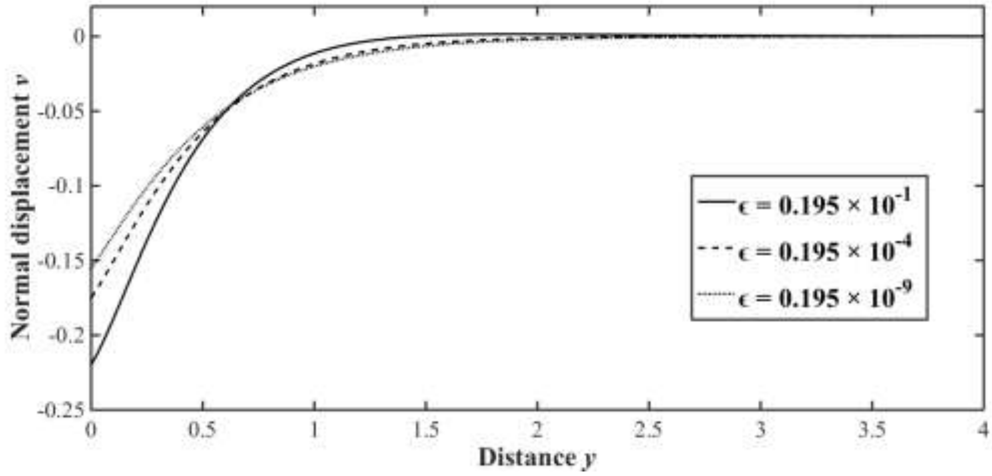

Fig.1: Variation of normal displacement with non local parameter

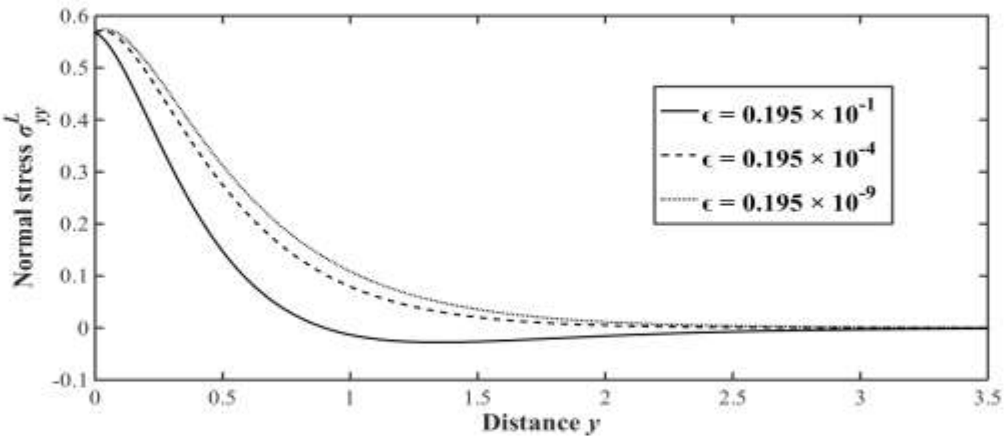

Fig.2: Variation of normal stress with non local parameter

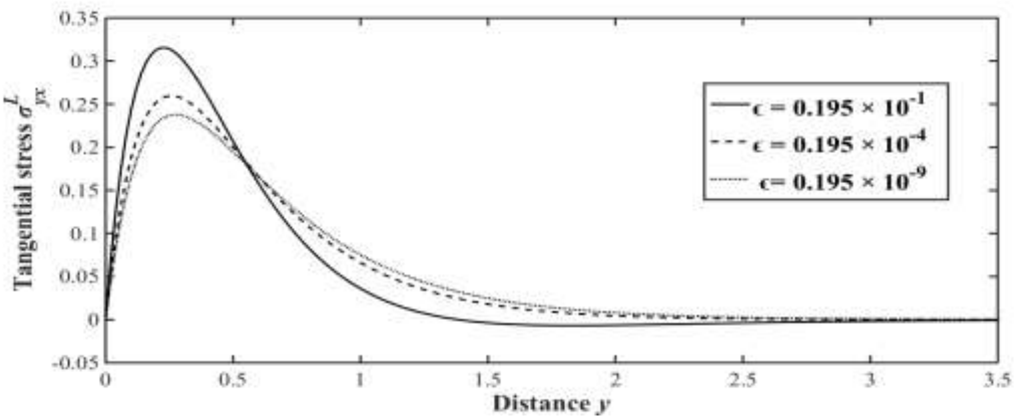

Fig.3: Variation of tangential stress with non local parameter

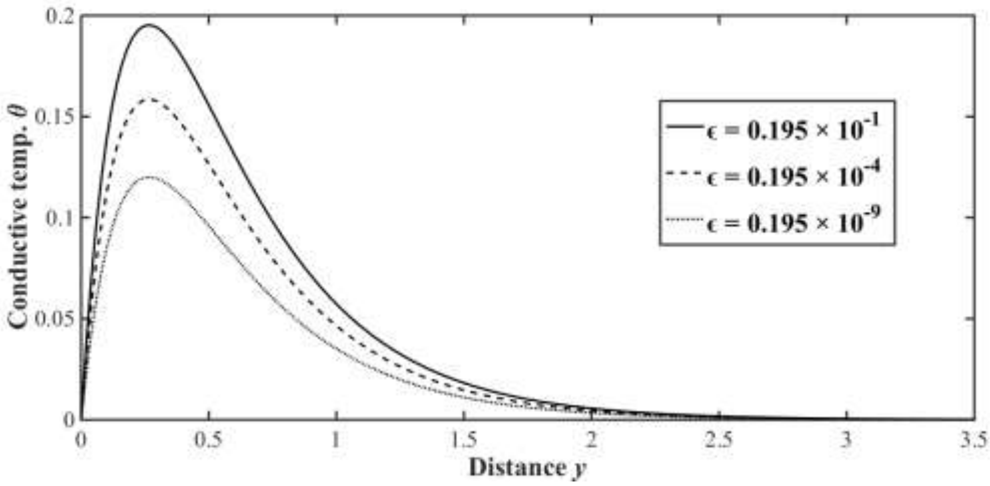

Fig.4: Variation of conductive temperature with no local parameter

Category 1: In this category, we have examined the effects of non local parameter $\epsilon(= 0.195 \times 10^{-1}, 0.195 \times 10^{-4}, 0.195 \times 10^{-9})$ on the variations of different physical variables. Figure 1 indicates the distribution of the normal displacement v against distance y for different values of non local parameter. It is found that non local parameter has increasing and decreasing effect the profile of normal displacement. Figure 2 explains the spatial variation of the normal stress distribution σ_{yy} against distance$y$ for different values of non local parameter. The figure shows that the distribution of normal stress follows a similar trend for all the values of non local parameter and dissimilarity lies on the ground of magnitudes. It is also manifested that decrement in the values ofnon local parameter is responsible for an increment in magnitude of normal stress. Figure 3 explains the spatial variation of the tangential stress distribution σ_{yy} against distance$y$ for non local parameter. The figure shows that the distribution of tangential stress follows a similar trend for all the values of non local parameterand in agreement with the boundary condition. Figure 4 depicts the distribution of the temperature θagainst distance y for different values of non local parameter. It is observed that all the curves of the temperature starting from with zero value which is consistent with the boundary condition assumed. It is also manifested that adecrement in the value of non local parameter is responsible for adecrement in magnitude of temperature.

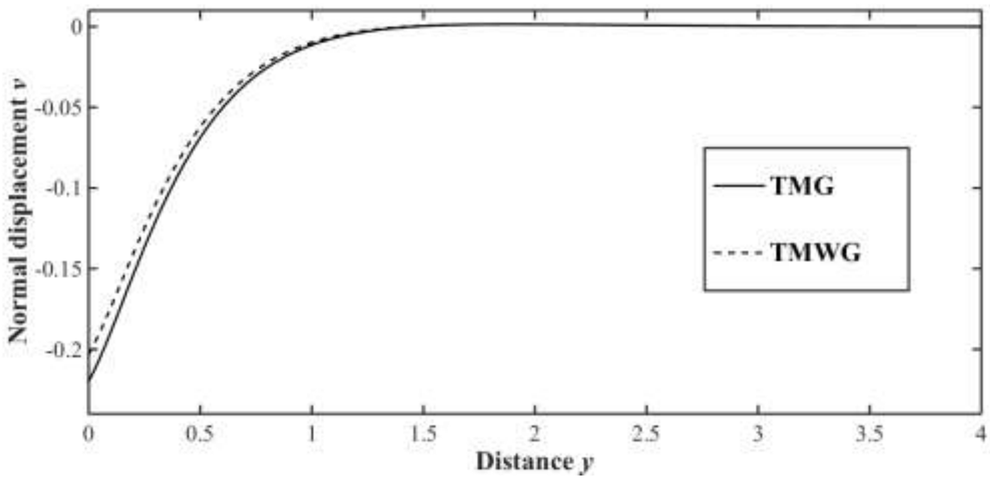

Fig.5: Variation of normal displacement with gravity

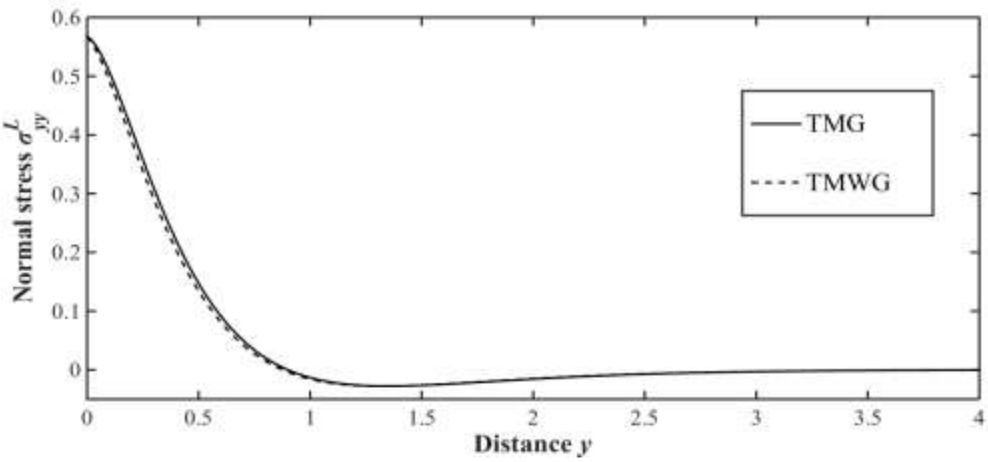

Fig.6: Variation of normal stress with hall current parameter

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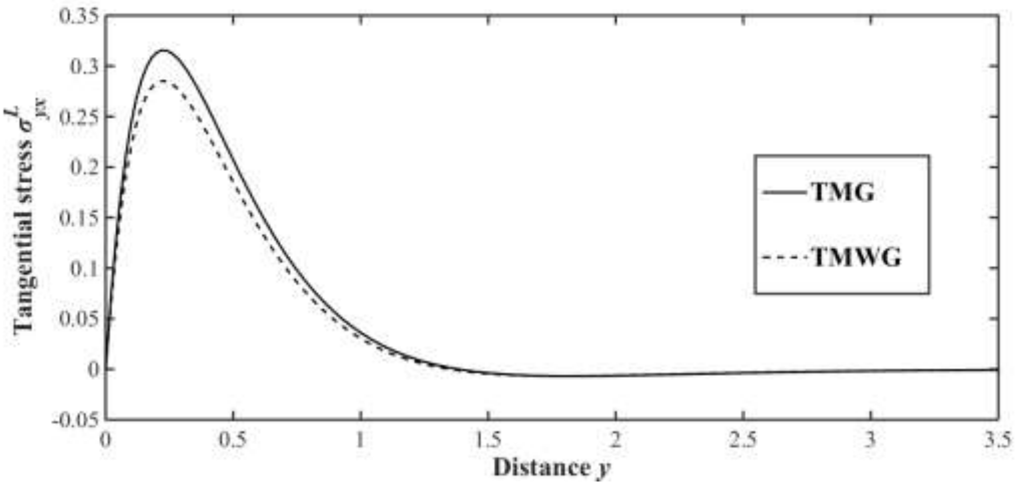

Fig.7: Variation of tangential stress with gravity

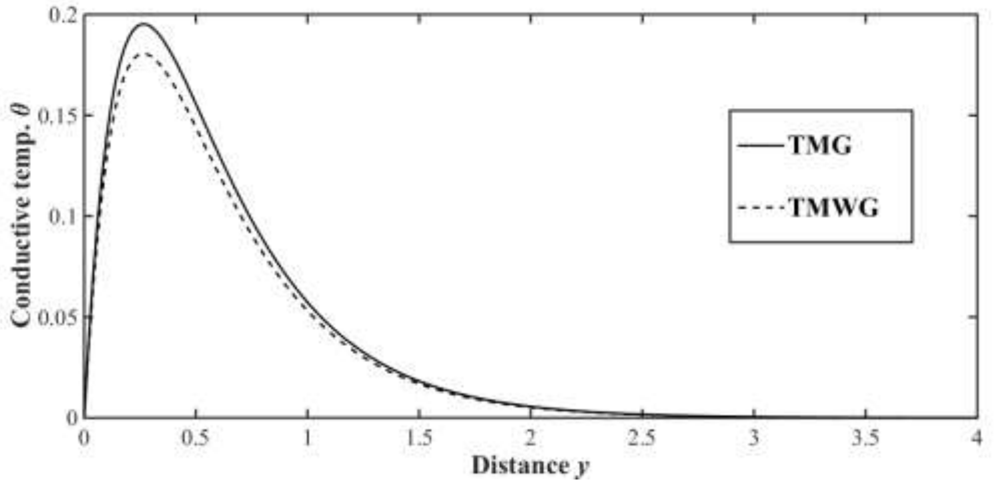

Fig.8: Variation of conductive temperature with gravity

Category 2: Figures (5)–(8) represent the variations of physical quantities in the frame of two different models: (i) thermoelastic medium with gravity (TMG), (ii) thermoelastic medium without gravity (TMWG). Figure 5 elucidates the space variation of normal displacement component *v*against distance *y*. Figure reveals that gravity acts to increase the magnitude of normal displacement distribution. Figure 6 is displayed to show the variation of normal stress with location *y*under the effect of gravity. It is found that the profiles of normal stress start with some positive value for both the cases and thereafter decrease to attain zero value, which is in accordance with the boundary condition. Moreover, gravity increases the values of normal stress distribution. Figure 7 illustrates the variation of tangential stress against distance y. As expected, tangential stress distribution is having a coincident starting point of zero magnitude for all the cases, which is in accordance with the boundary condition. Tangential stress field exhibits remarkable sensitivity towards gravity and shows increasing effect. Effects of gravity on temperature distribution are presented in Figure 8. It is shown that all the curves show similar trends. It can also be analyzed from the plot that gravity has an increasing effect on the profile of temperature field.

7. Conclusion

In this article, a systematic investigation of the two-dimension non local thermoelastic model with gravity is governed by a system of the linear differential equation. Normal mode analysis technique is adopted in this study is a quite efficient approach in handling such problems. The obtained theoretical and numerical results reveal that the considered parameters namely non local parameter and gravity have prominent effects on the physical quantities. The following conclusion can be drawn from the above study:

i. All the physical variables are continuous functions and tends to zero as y approaches infinity. Hence, all physical parameters maintain the phenomena of the finite speed of propagation, and all findings are consistent with the generalized theory of thermoelasticity.

ii. The existence of a nonlocal parameter has a significant impact on all the physical quantities under consideration. Non local parameter has amix kind of impacton the considered physical quantities.

iii. Gravity plays an important role in the variations of all the field quantities. From the profiles of normal displacement, normal stress, tangential stress and temperature, it is observed that the presence of gravity causes a increment in the numerical values of these field variables.

iv. We have discovered wave-type heat propagation in the medium from the distribution of temperature. The fact that the heat wave front travels in the medium with a finite speed as time passes shows how closely the generalized thermoelasticity theory describes physical behavior.

The research is of fundamental importance and finds its applications for experimental researchers /engineers working in the field of geophysics, seismology, material science and earthquake engineering. Applications of this problem are also found in the fields of seismology, geomechanics, earthquake engineering and soil dynamics etc., where the interest is about various phenomena occurring in earthquakes and measuring of displacement, stresses and temperature field due to the presence of certain sources.

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9. Appendix A

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$$P_1^{'} = \frac{P_1}{H_1}, P_2^{'} = \frac{P_2}{H_1}, P_3^{'} = \frac{P_3}{H_1},$$

$$H_1 = +A_1A_5 - A_3A_7$$

$$P_1 = +A_1A_6 + A_2A_5 - A_3A_8 - A_4A_7 - A_5A_{10} - A_1A_5A_9 + A_3A_7A_9$$

$$P_2 = -A_4A_8 + A_2A_6 - A_6A_{10} + A_5A_{10}m^2 - A_1A_6A_9 - A_2A_5A_9 + A_3A_8A_9 + A_4A_7A_9$$

$$P_3 = A_4A_8A_9 - A_2A_6A_9 + A_6A_{10}m^2$$

$$U_{1n} = (A_1k_n^2 + A_2); U_{2n} = (A_3k_n^2 + A_4); U_{3n} = (A_5k_n^2 + A_6); U_{41} = (A_7k_n^2 + A_8);$$

$$U_{5n} = -(k_n^2 - A_9), U_{6n} = -A_{10}(k_n^2 - m^2); T_{1n} = \left(1 - \epsilon^2(k_n^2 - m^2)\right); H_{1n} = \frac{-U_{4n}}{U_{3n}};$$

$$H_{2n} = (U_{1n} + U_{2n}H_{1n}); H_{3n} = \frac{\beta^2\left((k_n^2 - m^2 - H_{2n})\right) + 2(m^2 + \iota m k_n H_{1n})}{\left(1 - \epsilon^2(k_n^2 - m^2)\right)};$$

$$H_{4n} = \frac{-(k_nY_{1n} + \iota mY_{2n})}{\left(1 - \epsilon^2(k_n^2 - m^2)\right)}; \beta_2 = (3\lambda + 2\mu)\alpha_c, C_T^2 = \frac{(\lambda + 2\mu)}{\rho}, Y_{1n} = \iota m - k_nH_{1n}$$

$$Y_{2n} = (k_n + \iota mH_{1n});$$

$$\Delta = \begin{vmatrix} H_{31} & H_{32} & H_{33} \\ H_{41} & H_{42} & H_{43} \\ H_{21} & H_{22} & H_{23} \end{vmatrix}; \Delta_1 = \begin{vmatrix} p_1 & H_{32} & H_{33} \\ 0 & H_{42} & H_{43} \\ 0 & H_{22} & H_{23} \end{vmatrix}; \Delta_2 = \begin{vmatrix} H_{31} & p_1 & H_{33} \\ H_{41} & 0 & H_{43} \\ p_1 & 0 & H_{23} \end{vmatrix}; \Delta_3 = \begin{vmatrix} H_{31} & H_{32} & p_1 \\ H_{41} & H_{42} & 0 \\ H_{21} & H_{22} & 0 \end{vmatrix};$$