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PRESENTING NUMERICAL SOLUTIONS OF FIRST ORDER LINEAR FUZZY DIFFERENTIAL EQUATIONS USING LEAPFROG METHOD

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ABSTRACT

This paper presents numerical solutions of first order linear fuzzy differential equations using Leapfrog method. The obtained discrete solutions are compared with single-term Haar wavelet series (STHWS) method [1]. Error graphs and error calculation tables are presented to highlight the efficiency of the Leapfrog method. This method can be implemented in the digital computers and take any time of solutions. In this research, first order linear fuzzy differential equations is considered. This paper compares the He's variational iteration method (HVIM) and Leapfrog method [17] for solving these equations. He's variational iteration method is an analytical procedure for finding the solutions of problems which is based on the constructing a variational iterations. The Leapfrog method, based upon Taylor series, transforms the fuzzy differential equation into a matrix equation. The results of applying these methods to the first order linear fuzzy differential equations show the simplicity and efficiency of these methods.

Keywords: - Numerical, Solution, Leapfrog, Fuzzy**,** Method.

I. INTRODUCTION

S. Abbasbandy and T. Allahviranloo [2] addressed knowledge about dynamical systems modelled by differential equations is often incomplete or vague. It concerns, for example, parameter values, functional relationships, or initial conditions. The well-known methods for solving analytically or numerically initial value problems can only be used for finding a selected system behavior, e.g., by fixing the unknown parameters to some plausible values.

Knowledge about dynamical systems modelled by differential equations is often incomplete or vague. It concerns, for example, parameter values, functional relationships, or initial conditions. The well-known methods for solving analytically or numerically initial value problems can only be used for finding a selected system behavior, e.g., by fixing the unknown parameters to some plausible values. However, in this case, it is not possible to describe the whole set of system behaviors compatible with our partial knowledge.

The topics of fuzzy differential equations, which attracted a growing interest for some time, in particular, in relation to the fuzzy control, have been rapidly developed recent years. The concept of a fuzzy derivative was first introduced by S. L. Chang, L. A. Zadeh. It was followed up by D. Dubois, H. Prade, who defined and used the extension principle. Other methods have been discussed by M. L. Puri, D. A. Ralescu and R. Goetschel and W. Voxman. Fuzzy differential equations and initial value problems were regularly treated by O. Kaleva, S. Seikkala.

A numerical method for solving first order linear fuzzy differential equations has been introduced by M. Ma, M. Friedman and A. Kandel via the standard Euler method.

Recently, T. Jayakumar, D. Maheskumar and K. Kanagarajan solved the first order linear fuzzy differential equations using Runge-Kutta method of order five. S. Sekar and S. Senthilkumar solved the same first order linear fuzzy differential equations using single term Haar wavelet series method.

The objective of this paper is to use the He's variational iteration method to solve the first order linear fuzzy differential equations (discussed by T. Jayakumar, D. Maheskumar and K. Kanagarajan and S. Sekar and S. Senthilkumar).

II. LEAPFROG METHOD

The most familiar and elementary method for approximating solutions of an initial value problem is Euler's Method. Euler's Method approximates the derivative in the form of

$y' = f(t, y),\ y(t_0) = y_0,\ y \in R^d$ by a finite difference quotient $y'(t) \approx (y(t+h) - y(t))/h$.

We shall usually discretize the independent variable in equal increments:

$t_{n+1} = t_n + h,\ n = 0,1,\ldots,t_0$ Henceforth we focus on the scalar case, N = 1. Rearranging the difference quotient gives us the corresponding approximate values of the dependent variable:

$$y_{n+1} = y_n + hf(t_n, y_n),\ n = 0,1,\ldots,t_0$$

To obtain the leapfrog method, we discretize t_n as in $t_{n+1} = t_n + h,\ n = 0,1,\ldots,t_0$, but we double the time interval, h, and write the midpoint approximation time interval, h, and write the midpoint approximation $y(t+h) - y(t) \approx hy'\left(t + \frac{h}{2}\right)$ in the form $y'(t+h) \approx (y(t+2h) - y(t))/h$

And then discretize it as follows:

$$y_{n+1} = y_{n-1} + 2hf(t_n, y_n),\ n = 0,1,\ldots,t_n$$

The leapfrog method is a linear m = 2-step method, with $a_0 = 0, a_1 = 1, b_{-1} = -1, b_0 = 2$ and $b_1 = 0$.

It uses slopes evaluated at odd values of n to advance the values at points at even values of n, and vice versa, reminiscent of the children's game of the same name.

For the same reason, there are multiple solutions of the leapfrog method with the same initial value y = yo.

III. NUMERICAL EXPERIMENTS

In this section, the exact solutions and approximated solutions obtained by He's variational iteration method and Leapfrog method. To show the efficiency of the He's variational iteration method, we have considered the following problem taken from C. Duraisamy and B. Usha and T.Jayakumar, D.Maheskumar and K.Kanagarajan , with step size r = 0.1 along with the exact solutions.

The discrete solutions obtained by the two methods, He's variational iteration method and Leapfrog method. The absolute errors between them are tabulated and are presented in Tables 1. To distinguish the effect of the errors in accordance with the exact solutions, graphical representations are given for selected values of 'r' and are presented in Figures 1 for the following problem, using three dimensional effects.

Example 3.1

Consider the initial value problem [C. Duraisamy and B. Usha]Y 0 (t) = t f (t), t ∈ [0, 1]

With initial condition y (0) = (1.01+0.1r √ e, 1.5+0.1r √ e)The exact solution at t = 0.1 is given byY(0.1,r) = [(1.01+0.1r √ e)e 0.0005 ,(1.5+0.1r √ e)e 0.0005], 0 ≤ r ≤ 1

IV. NUMERICAL EXAMPLES

Consider a first-order fuzzy initial value differential equation is given by In this section, the exact solutions and approximated solutions obtained by Leapfrog method and STHWS method. To show the efficiency of the Leapfrog method, we have considered the following problem taken from, along with the exact solutions.

The discrete solutions obtained by the two methods, Leapfrog method and STHWS method; the absolute errors between them are tabulated and are presented in Table 1 and Table 2. To distinguish the effect of the errors in accordance with the exact solutions, graphical representations are given for selected values of "r" and are presented in Fig. 1 to Fig. 4 for the following problem, using three dimensional effects.

Example 4.1

Consider the initial value problem [1]

$$\left.\begin{aligned} &y'(t) = tf(t),\ t \in [0,1], \\ &y(0) = \left(1.01 + 0.1r\sqrt{e}, 1.5 + 0.1r\sqrt{e}\right) \end{aligned}\right\}$$

The exact solution at t = 0.1 is given by

$$Y(0.1;r) = \left[\left(1.01 + 0.1r\sqrt{e}\right)e^{0.005}, \left(1.5 + 0.1r\sqrt{e}\right)e^{0.005}\right],\ 0 \le r \le 1$$

Example 4.2

Consider the fuzzy initial value problem [1]

$$\left.\begin{aligned} &y'(t) = y(t),\ t \in I = [0,1], \\ &y(0) = (0.75 + 0.25r, 1.125 - 0.125r), 0 < r \le 1. \end{aligned}\right\}$$

The exact solution is given by

$$Y_1(t;r) = y_1(0;r)e^t,\ Y_2(t;r) = y_2(0;r)e^t \text{ which at } t = I$$
$$Y_1(1;r) = [(0.75 + 0.25r)e, (1.125 - 0.125r)e],\ 0 < r \le 1.$$

Table 1 Error Correction

r	STHWS Error					
	Example 5.1		Example 5.2		Example 5.3	
	y1	y2	y1	y2	y1	y2
0.1	1.00E-07	1.00E-07	6.00E-07	6.00E-07	1.00E-07	1.00E-08
0.2	2.00E-07	2.00E-07	7.00E-07	7.00E-07	2.00E-07	2.00E-08
0.3	3.00E-07	3.00E-07	8.00E-07	8.00E-07	3.00E-07	3.00E-08
0.4	4.00E-07	4.00E-07	9.00E-07	9.00E-07	4.00E-07	4.00E-08
0.5	5.00E-07	5.00E-07	1.00E-06	1.00E-06	5.00E-07	5.00E-08
0.6	6.00E-07	6.00E-07	1.10E-06	1.10E-06	6.00E-07	6.00E-08
0.7	7.00E-07	7.00E-07	1.20E-06	1.20E-06	7.00E-07	7.00E-08
0.8	8.00E-07	8.00E-07	1.30E-06	1.30E-06	8.00E-07	8.00E-08
0.9	9.00E-07	9.00E-07	1.40E-06	1.40E-06	9.00E-07	9.00E-08
1	1.00E-06	1.00E-06	1.50E-06	1.50E-06	1.00E-06	9.90E-08

Table 2: Error Calculations

	Leapfrog Error					
r	Example 5.1		Example 5.2		Example 5.3	
	y1	y2	y1	y2	y1	y2
0.1	1.00E-07	1.00E-07	6.00E-07	6.00E-07	1.00E-07	1.00E-08
0.2	2.00E-07	2.00E-07	7.00E-07	7.00E-07	2.00E-07	2.00E-08
0.3	3.00E-07	3.00E-07	8.00E-07	8.00E-07	3.00E-07	3.00E-08
0.4	4.00E-07	4.00E-07	9.00E-07	9.00E-07	4.00E-07	4.00E-08
0.5	5.00E-07	5.00E-07	1.00E-06	1.00E-06	5.00E-07	5.00E-08
0.6	6.00E-07	6.00E-07	1.10E-06	1.10E-06	6.00E-07	6.00E-08
0.7	7.00E-07	7.00E-07	1.20E-06	1.20E-06	7.00E-07	7.00E-08
0.8	8.00E-07	8.00E-07	1.30E-06	1.30E-06	8.00E-07	8.00E-08
0.9	9.00E-07	9.00E-07	1.40E-06	1.40E-06	9.00E-07	9.00E-08
1	1.00E-06	1.00E-06	1.50E-06	1.50E-06	1.00E-06	9.90E-08

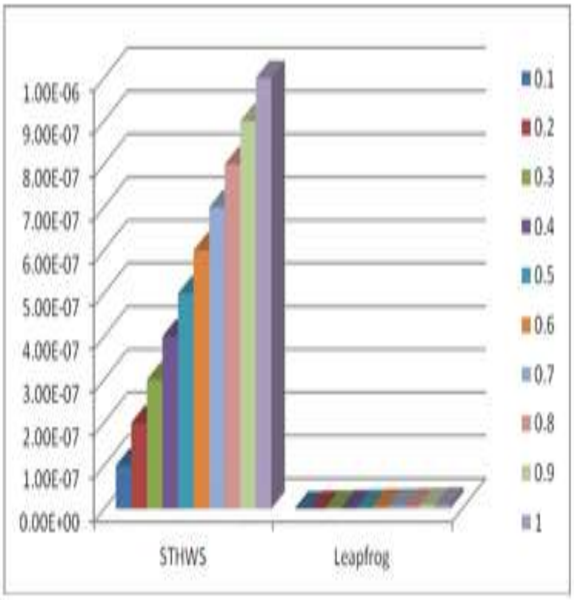


Fig. 1 Error estimation of Example 4.1 at y1

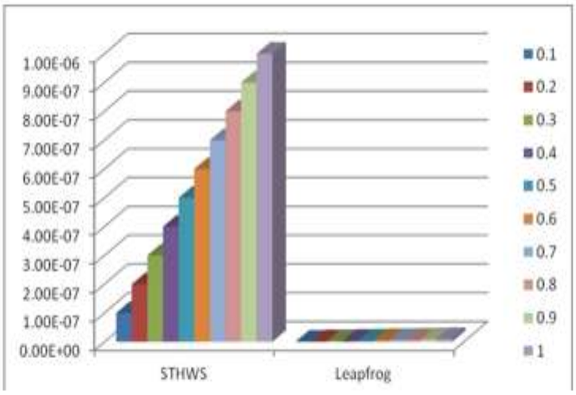
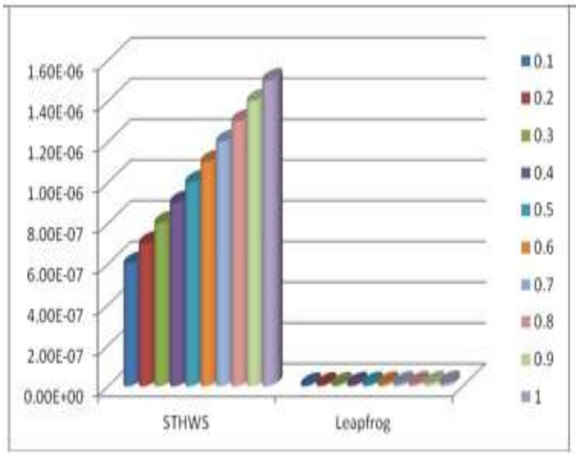


Fig. 2 Error estimation of Example 4.1 at y2



Error estimation of Example 4.2 at y1

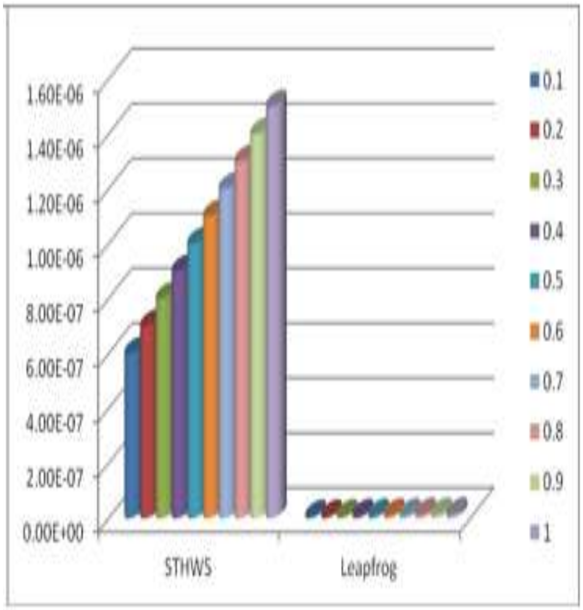


Fig. 4 Error estimation of Example 4.2 at y2

V. CONCLUSION

He's variational iteration method is a powerful, accurate, and flexible tool for solving many types of fuzzy differential equations (problems) in scientific computation.

The obtained approximate solutions of the first order linear fuzzy differential equations are compared with exact solutions and it reveals that the He's variational iteration method works well for finding the approximate solutions.

In this paper, the Leapfrog method has been successfully employed to obtain the approximate analytical solutions of the first order linear fuzzy differential equations. Compare to STHWS method, Leapfrog method gives less error from the Table 1 and Table 2. Also it is clear that from the Fig. 1 to Fig. 4 the Leapfrog method introduced in Section 2 performs better than STHWS method.

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