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Developement of Effective Estimation Strategy for Population Mean in Two-Phase Sampling

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Abstract

This paper focuses on the problem of estimating the population mean in two-phase (double) sampling. We have proposed a class of chain exponential ratio and regression type estimators of population mean utilizing the information on two auxiliary variables. The optimum property of the estimators has been discussed. The dominance of the suggested estimator over some contemporary estimators of population mean has been established through empirical investigations carried over the data set of some natural population and artificially generated population. Proper recommendations are helped to the survey statistician in real field.

Keywords:

Double sampling;
study variable;
auxiliary variable;
chain-type-regression;
Efficiency.

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1. Introduction

Information on variables correlated with the variable under study is popularly known as auxiliary information. The application of information on auxiliary variable for estimating the population parameters played an eminent role in sample surveys. Auxiliary information may be used at planning, design and estimation stages to extend improved estimation procedures in sample surveys. Use of auxiliary information at estimation stage was introduced during the 1940's with a comprehensive theory provided by Cochran [4]. In many circumstances, auxiliary information may be easily available for all the units of population. For example, seat capacity of every vehicle is known in survey sampling of transportation and number of beds available in different hospitals may be known well in advance in health care surveys. When such information lacks, it is more convenient to take a large preliminary sample to estimate the parameters of the auxiliary variable alone. This preparation is applicable in two-phase (or double) sampling. Two-phase sampling is a very powerful and cost-effective technique for producing efficient estimates of the unknown population parameters of the study variable.

In order to construct an efficient estimator of the population mean of the auxiliary variable in first-phase (preliminary) sample, Chand [2] introduced a procedure of chaining second auxiliary variable with the first auxiliary variable in first phase sample by using the ratio estimator. This estimator is known as chain-type ratio estimator. Again this work was expanded by Kiregyera [8, 9], Tracy *et al.* [20], Singh and Espejo [14], Gupta and Shabbir [6], Shukla *et al.* [11], Choudhury and Singh [3] and among others where they proposed various chain-type ratio and regression estimators.

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Motivated with the above discussion, we have defined chain exponential ratio and regression type estimator of population mean and studied its properties. Performances of the proposed estimator have been examined through theoretical and numerical illustrations which presents the effectiveness of the proposed strategies. Suitable recommendations to the survey statistician are made for their practical applications.

2. Formulation of the Class of Estimators

2.1 Sample Structure and Some Existing Estimation Procedures

Let y_k, x_k and z_k be the values of the study variable y, first auxiliary variable x and second auxiliary variable z respectively associated with the k[th] unit of the finite population $U = (U_1, U_2, U_3, ..., U_N)$. We wish to estimate the population mean $\bar{Y}$ of the study variable y in presence of auxiliary variables x and z, when the population mean $\bar{X}$ of x is unknown but information on z is readily available for all the units of population.

Thus, to estimate $\bar{Y}$, a first phase sample $S'(S' \subset U)$ of size n is drawn by simple random sampling without replacement scheme (SRSWOR) from the entire population U and observed for the auxiliary variables x and z to furnish the estimate of $\bar{X}$. Again a second-phase sample S of size m $(m \le n)$ is drawn as a subsample of the first phase sample $(\text{i. e. } S \subset S')$ by simple random sampling without replacement (SRSWOR) to observe the study variable y.

Hence onwards, we use the following notations:

$\bar{Z}$: Population mean of the auxiliary variable z.

$\bar{x}', \bar{z}'$: Sample means of the respective variables based on the first phase sample of size n .

$\bar{x}, \bar{y}, \bar{z}$: Sample means of the respective variables based on the second phase sample of size m.

$b_{yx}(m), b_{xz}(n)$: Sample regression coefficients between the variables shown in subscripts and based on the sample sizes indicated in the braces.

$s_{yx}(n), s_{xz}(n), s_{yz}(n)$: Sample covariance between the variables shown in subscripts and based on the second phase sample of size n.

s_x^2 : Sample mean square of the variable x based on the second phase sample of size n.

$\beta_{yx}, \beta_{xz}, \beta_{yz}$: Population regression coefficients between the variables shown in subscripts.

To estimate the population mean $\bar{Y}$, the classical ratio estimator is presented as

$$\bar{y}_r = \frac{\bar{y}}{\bar{x}}\bar{X}. \qquad (1)$$

where $\bar{y}$ and $\bar{x}$ are the sample means of variables y and x respectively based on the second phase sample S.

If $\bar{X}$ is unknown, we estimate $\bar{Y}$ under two-phase sampling set up as

$$t_1 = \frac{\bar{y}}{\bar{x}}\bar{x}' \qquad (2)$$

Where $\bar{x}'$ is the sample mean of the auxiliary variable x based on the first-phase (preliminary) sample S'.

Srivastava [17] proposed the ratio method of estimation and its structure in two phase sampling is given as

$$t_2 = \bar{y}\left(\frac{\bar{x}'}{\bar{x}}\right)^{\alpha} \qquad (3)$$

Where α is a real scalar, which can be properly determined by minimizing the mean square error (M. S. E.) of the estimator t_2.

The method in which the estimate of $\bar{Y}$ is developed using the auxiliary information on x can also be extended to improve the estimate of $\bar{X}$ in the first-phase sample, if another auxiliary variable z closely related to x but remotely related to y is used. Thus, assuming that the population mean of the auxiliary variable z is known, Chand [2] suggested a chain-type ratio estimator as

$$t_3 = \frac{\bar{y}}{\bar{x}} \bar{x}'_{rd} \tag{4}$$

where $\bar{x}'_{rd} = \frac{\bar{x}'}{\bar{z}'} \bar{Z}$, $\bar{z}'$ and $\bar{Z}$ are the sample mean based on the first phase sample of size n and population mean of the auxiliary variable z respectively.

Kiregyera [9] considered chain-type regression and regression in regression estimators as

$$t_4 = \bar{y} + b_{yx}(n)(\bar{x}'_{ld} - \bar{x}) \tag{5}$$

where $\bar{x}'_{ld} = \bar{x}' + b'_{xz}\left(\bar{Z} - \bar{z}'\right)$ and b'_{xz} is the sample regression coefficient between the variables x and z and based on the sample S' of size n.

Singh and Espejo [14] proposed a ratio - product type estimator in double sampling as

$$t_5 = \bar{y}\left[p\frac{\bar{x}'}{\bar{x}} + (1-p)\frac{\bar{x}}{\bar{x}'}\right] \tag{6}$$

where p is a real scalar which may be properly determined to minimize the mean square error of the estimator t_5.

Singh and Vishwakarma [15] constructed exponential ratio and product type estimator of population mean in two-phase sampling as

$$t_6 = \bar{y}\exp\left(\frac{\bar{x}' - \bar{x}}{\bar{x}' + \bar{x}}\right) \tag{7}$$

and

$$t_7 = \bar{y}\exp\left(\frac{\bar{x} - \bar{x}'}{\bar{x} + \bar{x}'}\right) \tag{8}$$

respectively.

2.2 Proposed Class of Estimators

Motivated with the earlier work, we have defined a class of chain exponential ratio and regression type estimators as

$$t_p = \bar{y}\left(\frac{\bar{x}'_{ld}}{\bar{x}}\right)^{\alpha_1} \exp\left(\frac{\bar{x}'_{ld} - \bar{x}}{\bar{x}'_{ld} + \bar{x}}\right) \tag{9}$$

Where α_1 is a real constant which can be suitably determined by minimizing the MSE of the class of estimators t_p and $\bar{x}'_{ld} = \bar{x}' + b^{(n)}_{xz}\left(\bar{Z} - \bar{z}'\right)$.

3. Research Method

3.1 Mean Square Errors of the Proposed Class of Estimators t_p

It can be show that the suggested class of estimators t_p defined in equations (9) is chain exponential ratio and regression type estimator. Therefore, it is biased for population mean $\bar{Y}$. Hence, we obtained its mean square error (MSE) under large sample approximations using the following transformations:

$\bar{y} = \bar{Y}(1+e_1)$, $\bar{x} = \bar{X}(1+e_2)$, $\bar{x}' = \bar{X}(1+e_3)$, $\bar{z}' = \bar{Z}(1+e_4)$, $\bar{z} = \bar{Z}(1+e_5)$,

$s'_{xz} = S_{xz}(1+e_6)$, $s_z^2 = S_z^2(1+e_7)$, $s_{z'}^2 = S_z^2(1+e_8)$, $s_{yx} = S_{yx}(1+e_9)$, $s_x^2 = S_x^2(1+e_{10})$, $s_{yz} = S_{yz}(1+e_{11})$, $s_{xz} = S_{xz}(1+e_{12})$,

and $E(e_i) = 0$ for (i = 1, 2, . . ., 12), e_i for (i = 1, 2, . . ., 12) are relative error terms.

Under the above transformations the class of estimators t_p can be represented as

$$t_p = \bar{Y}(1+e_1)\left[\left\{\left(1+e_3-\frac{\beta_{xz}\bar{Z}}{\bar{X}}e_4\right)(1+e_2)^{-1}\right\}^{\alpha_1}\exp\left\{\frac{\bar{X}\left(e_3-\beta_{xz}\frac{\bar{Z}}{\bar{X}}e_4-e_2\right)}{\bar{X}\left(2+e_2+e_3-\beta_{xz}\frac{\bar{Z}}{\bar{X}}e_4\right)}\right\}\right]$$

$$= \bar{Y}\left[\left(1+e_1+e_3-e_2-\beta_{xz}\frac{\bar{Z}}{\bar{X}}e_4\right)\left(\alpha_1+\frac{1}{2}\right)\right] \tag{10}$$

We have derived the expressions for bias and mean square error of the proposed class of estimators t_p under two - phase sampling structure and presented them below.

We have obtained the following expectations of the sample statistics as

$$\left.\begin{aligned}
&E(e_1^2) = f_1C_y^2,\ E(e_2^2) = f_1C_x^2,\ E(e_3^2) = f_2C_x^2,\ E(e_4^2) = f_2C_z^2\\
&E(e_1e_2) = f_1\rho_{yx}C_yC_x,\ E(e_1e_3) = f_2\rho_{yx}C_yC_x,\\
&E(e_2e_3) = f_2C_x^2,\ E(e_2e_4) = E(e_3e_4) = f_2\rho_{xz}C_xC_z,\\
&E(e_4e_5) = f_2\frac{\mu_{102}}{\bar{Z}S_{xz}},\ E(e_4e_6) = f_2\frac{\mu_{003}}{\bar{Z}S_z^2},\\
&E(e_2e_5) = f_2\frac{\mu_{201}}{\bar{X}S_{xz}},\ E(e_2e_6) = f_2\frac{\mu_{102}}{\bar{X}S_z^2},\\
&E(e_1e_4) = f_2\rho_{yz}C_yC_z.
\end{aligned}\right\} \tag{11}$$

where

$$f_1 = \frac{1}{n}-\frac{1}{N},\ f_2 = \frac{1}{m}-\frac{1}{N},\ f_3 = \frac{1}{n}-\frac{1}{m},$$

$$\mu_{pqr} = \frac{1}{N}\sum_{i=1}^{N}(x_i-\bar{X})^p(y_i-\bar{Y})^q(z_i-\bar{Z})^r;\ (p, q, r \geq 0)$$

Expanding binomially, exponentially, using results from equation (10) and retaining the terms up to first order of sample size; we have derived the expressions of mean square error M.S.E. of the class of estimators t_p as

$$M(t_p) = E(t_p-\bar{Y})^2$$

$$= \bar{y}^2\left\{f_1c^2{}_y+\frac{(2\alpha_1+1)^2}{4}A_1-(2\alpha_1+1)B_1\right\} \tag{12}$$

where

$A_1 = \left(f_3+f_2\rho_{xz}^2\right)C_x^2$ and $B_1 = f_2\rho_{yz}\rho_{xz}C_yC_x+f_3\rho_{yx}C_yC_x$

3.2 Minimum MSE OF Proposed Class of Estimators

It is obvious from the equations (10) that the variances of the proposed class of estimators t_p depend on the value of the constant α_1. Therefore, we desire to minimize their variances and discussed them below.

The optimality condition under which proposed class of estimators t_p have minimum variance is obtained as

$$2\alpha_1 + 1 = \frac{2\left(f_3\rho_{yx} + f_2\rho_{yz}\rho_{xz}\right)c_y c_x}{\left(f_3 + f_2\rho_{xz}^2\right)c_x^2} \qquad (13)$$

$$\Rightarrow \alpha_1 = \frac{B_1}{A_1} - 0.5 \qquad (14)$$

Substituting the optimum value of the constant α_1 in equation (12), we have the minimum variance of the class of estimators t_p as

$$\text{Min. } M(t_p) = \bar{Y}^2\left[f_1 c_y^2 - \frac{\left(f_3\rho_{yx} + f_2\rho_{yz}\rho_{xz}\right)^2 c_y^2}{\left(f_3 + f_2\rho_{xz}^2\right)}\right] \qquad (15)$$

Remark 3.2: It is to be mentioned that the optimum value of α_1 depend on unknown population parameters such as C_x, C_y, C_z, ρ_{yx} and ρ_{xz}. Thus, to make the class of estimators practicable, these unknown population parameters may be estimated with their respective sample estimates or from past data or guessed from experience gathered over time. Such problems are also considered by Reddy [10], Tracy *et al.* [20] and Singh *et al.* [16].

4. Results and Analysis

4.1 Efficiency comparisons of the proposed class of estimator t_p

To examine the performances of the proposed class of estimators, we have compared their efficiencies with some existing estimators of population mean such as $\bar{y}$ (sample mean estimator) and $t_i\ (i = 1, 2, \ldots, 7)$. The Variance/ MSEs/ minimum MSEs of the existing estimators t_i are obtained up to the first order of approximations as presented below.

$$M(t_1) = \bar{Y}^2\left[f_1C_y^2 + f_2C_x^2 - 2f_2\rho_{yx}C_yC_x\right]$$

$$\text{Min. } M(t_2) = S_y^2\left[f_1 - f_3\rho_{yx}^2\right]$$

$$M(t_3) = \bar{Y}^2\left[f_1C_y^2 + f_2C_z^2 + f_3C_x^2 - 2f_2\rho_{yz}C_yC_z - 2f_3\rho_{yx}C_yC_x\right]$$

$$M(t_4) = \left[f_1\left(1 - \rho_{yx}^2\right) + f_2\left\{\left(1 + \rho_{xz}^2\right)\rho_{yx}^2 - 2\rho_{yx}\rho_{xz}\rho_{yz}\right\}\right]S_y^2$$

$$\text{Min. } M(t_5) = \left[f_1\left(1 - \rho_{yx}^2\right) + f_2\rho_{yx}^2\right]S_y^2$$

$$M(t_6) = \bar{Y}^2\left[f_1C_y^2 + \frac{f_3}{4}C_x^2 - f_3\rho_{yx}C_yC_x\right]$$

$$M(t_7) = \bar{Y}^2\left[f_1C_y^2 + \frac{f_3}{4}C_x^2 + f_3\rho_{yx}C_yC_x\right]$$

The variance of $\bar{y}$ is obtained as

$V(\bar{y}) = f_1 S_y^2$.

We have demonstrated the superiority of the suggested estimator t_p over the estimators t_i (i = 1, 2, . . ., 7) through numerical illustration.

4.2 Empirical Investigations through Natural Population

We have chosen five natural populations to demonstrate the efficacious performance of our proposed classes of estimators. The source of the populations, the nature of the variables y, x, z and the values of the various parameters are as follows.

Population I-Source: Cochran [5]

y: Number of ‘placebo’ children.

x: Number of paralytic polio cases in the placebo group.

z: Number of paralytic polio cases in the ‘not inoculated’ group.

$N = 34$, $n = 15$, $m = 10$, $\bar{Y} = 4.92$, $\bar{X} = 2.59$, $\bar{Z} = 2.91$, $C_y = 1.0123$, $C_x = 1.2318$, $C_z = 1.0720$,

$\rho_{yx} = 0.7326$, $\rho_{yz} = 0.6430$ and $\rho_{xz} = 0.6837$.

Population II- Source: Anderson [1]

y: Head length of second son.

x: Head length of first son.

z: Head breadth of first son.

$N = 25$, $n = 10$, $m = 7$, $\bar{Y} = 183.84$, $\bar{X} = 185.72$, $\bar{Z} = 151.12$, $C_y = 0.0546$, $C_x = 0.0526$,

$C_z = 0.0488$, $\rho_{yx} = 0.7108$, $\rho_{yz} = 0.6932$ and $\rho_{xz} = 0.7346$.

Population III- Source: Srivastava *et al.* [18]

y: Measurement of weight of children.

x: Mid - arm circumference of children.

z: Skull circumference of children.

$N = 120$, $n = 60$, $m = 30$, $\bar{Y} = 2934.58$, $\bar{X} = 1031.82$, $\bar{Z} = 3651.49$, $C_y = 2.005004$, $C_x = 1.5977$,

$C_z = 1.4469$, $\rho_{yx} = 0.93$, $\rho_{yz} = 0.84$ and $\rho_{xz} = 0.77$.

Population IV- Source: Handique *et al.* [7]

y: Forest timber volume in cubic meter (Cum) in 0.1 ha sample plot.

x: Average tree height in the sample plot in meter (m).

z: Average crown diameter in the sample plot in meter (m).

$N = 2500$, $n = 700$, $m = 80$, $\bar{Y} = 4.63$, $\bar{X} = 21.09$, $\bar{Z} = 13.55$, $C_y = 0.95$, $C_x = 0.98$, $C_z = 0.64$,

$\rho_{yx} = 0.79$, $\rho_{yz} = 0.72$ and $\rho_{xz} = 0.66$.

Population V- Source: Sukhatme and Sukhatme [19]

y: Area (acres) under wheat in 1937.

x: Area (acres) under wheat in 1936.

z: Total cultivated area (acres) in 1931.

$N = 34$, $n = 10$, $m = 7$, $\bar{Y} = 201.41$, $\bar{X} = 218.41$, $\bar{Z} = 765.35$, $C_y = 0.74$, $C_x = 0.76$, $C_z = 0.61$,

$\rho_{yx} = 0.93$, $\rho_{yz} = 0.9$ and $\rho_{xz} = 0.83$.

To have a tangible idea about the performance of the proposed class of estimators t_p we have compute percent relative efficiencies (PREs) of them and the existing estimators t_i (i = 1, 2, . . ., 7) under similar realistic situations and the findings are displayed in Table 1 – 2 where PREs are designated as PRE $= \frac{V(\bar{y})}{M(T)} \times 100$ and $M(T)$ denote variance/minimum MSE of an estimator T.

Table 1. PREs of various estimators with respect to $\bar{y}$ using natural population data set

Estimators	PRE				
	Pop-I	Pop-II	Pop-III	Pop-IV	Pop-V
$\bar{y}$	100	100	100	100	100
t_1	118.9748	134.6783	139.3521	105.0521	213.8133
t_2	133.9482	126.6649	236.1833	233.1262	148.5310
t_3	136.9096	178.8188	488.5686	228.0674	566.9582
t_4	185.5260	190.0258	517.2849	257.5740	557.1949
t_5	133.9482	126.6649	236.1833	233.1262	148.5310
t_6	132.6546	123.2513	163.4600	200.9033	135.3565
t_7	62.6649	72.3575	62.5038	50.2741	68.4722
t_p	186.79	192.72	520.43	257.66	574.57

4.3 Empirical Investigations through Artificially Generated Population

An important aspect of simulation is that one builds a simulation model to replicate the actual system. Simulation allows comparison of analytical techniques and helps in concluding whether a newly developed technique is better than the existing ones. Motivated by Singh and Deo [13] and Singh *et al.* [12] who have been adopted the artificial population generation techniques, we have generated five sets of independent random samples of size N (N = 100) namely $x'_{1_k}, y'_{1_k}, x'_{2_k}, y'_{2_k}$ and z'_k $(k = 1, 2, 3, \ldots, N)$ from a standard normal distribution with the help of R-software. By varying the correlation coefficients ρ_{yx} and ρ_{xz}, we have generated the following transformed variables of the population U with the values of $\sigma_y^2 = 50,\ \mu_y = 40,\ \sigma_x^2 = 25,\ \mu_x = 50,\ \sigma_z^2 = 9$ and $\mu_z = 30$ as

$$y_{1_k} = \mu_y + \sigma_y\left[\rho_{xy}x'_{1_k} + \left(\sqrt{1-\rho_{yx}^2}\right)y'_{1_k}\right],\ x_{1_k} = \mu_x + \sigma_x x'_{1_k} \text{ and } z_k = \mu_z + \sigma_z\left[\rho_{xz}x'_{1_k} + \left(\sqrt{1-\rho_{xz}^2}\right)z'_k\right].$$

Thus, we have derived following efficiency comparisons of our proposed strategy with the recent relevant ones with the above artificially generated population techniques as:

Table 2. PREs of various estimators using artificially generated population data set

Estimators	PRE			
	$\rho_{yx}=0.7$, $\rho_{xz}=0.6$	$\rho_{yx}=0.9$, $\rho_{xz}=0.6$	$\rho_{yx}=0.7$, $\rho_{xz}=0.8$	$\rho_{yx}=0.9$, $\rho_{xz}=0.8$
$\bar{y}$	100	100	100	100
t_1	120.94	149.20	109.67	157.01
t_2	136.82	172.32	125.53	183.10

t_3	133.32	167.45	102.90	280.47
t_4	157.94	213.28	140.45	303.11
t_5	136.82	172.32	125.53	183.10
t_6	133.01	151.74	124.70	151.78
t_7	65.44	62.08	67.39	63.71
t_p	158.03	213.34	140.48	303.13

5. Conclusion

The following conclusions may be read-out from the present study.
(a) Table - 1, shows that for high positive values of the correlation coefficients, the proposed classes of estimators t_p yield impressive gains in efficiency over the existing estimators $t_i (i = 1, 2, \ldots, 7)$. This pattern indicates that proposed class of estimators is more efficient than the existing ones which enhances their recommendations to survey statistician for their usage in real life problem.
(b) From Table - 2, it is observed that for fixed values of ρ_{xz}, the percent relative efficiencies of the classes of estimators t_p are increasing with the increasing values of ρ_{yx}. This behaviour indicates that our proposed class of estimators performs satisfactorily if highly correlated auxiliary variable is present in population.

These phenomenons indicate that the proposed class of estimators could perform more precisely, if information on high positively correlated auxiliary variables is available. Thus, it is clear that the use of auxiliary variables are highly rewarding in terms of the proposed classes of estimators. Therefore, the proposed class of estimator t_p is more justified as compared with previous work of similar situation. Hence, this proposed work may be suggested to the survey practitioner for their use in real world problems.

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