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IMPORTANCE OF INTERVAL IN REAL ANALYSIS

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INTRODUCTION

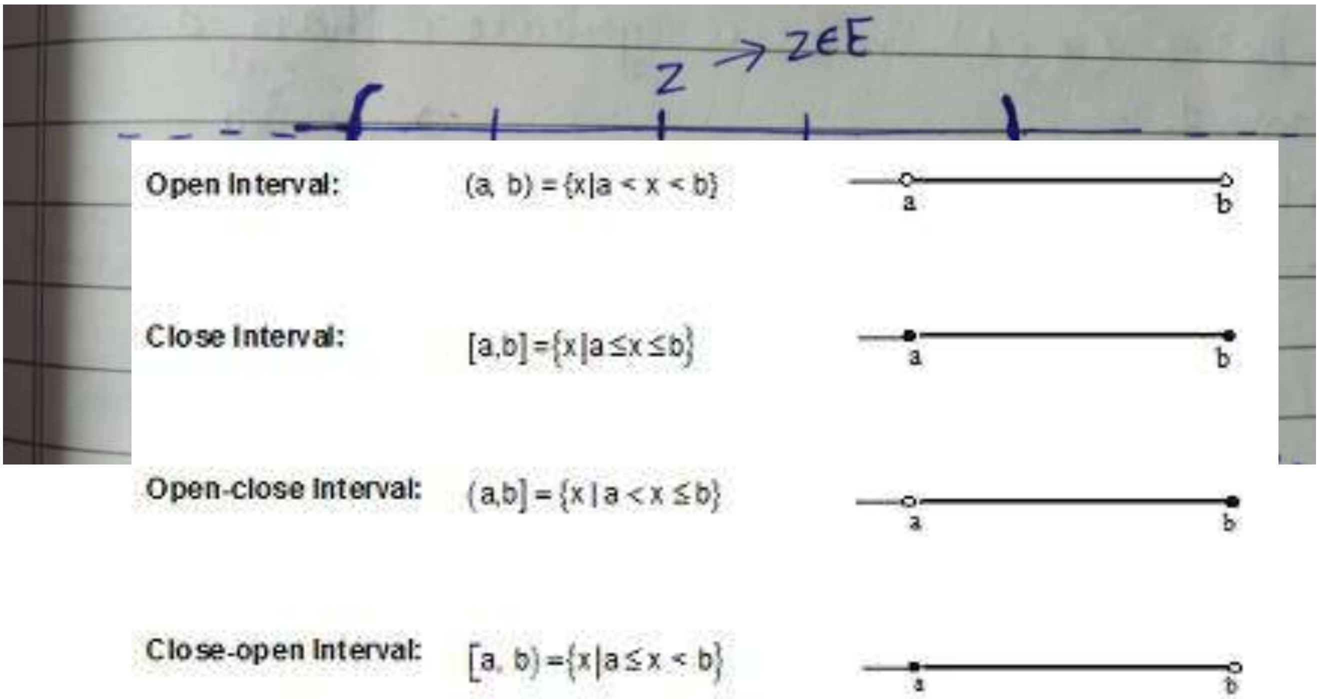
Here we will discuss the Intervals of Real Analysis. Interval is a Part of Set Theory. So, before understanding the interval, we will try to understand little Set theory. Set Theory, Branches of mathematics, deals with the properties of well defined collections of objects, which may or may not be of a mathematical nature, such as number or function. Germen Mathematician, Georg Cantor (Georg Ferdinand Ludwig Philipp Cantor) who founded set theory and meaning full concept of transfinite Numbers. In mathematics, a Set is a well defined Collection of district objects. By a well defined Collection of objects. We mean that there is a rule or rules by means of which it is possible to say, without ambiguity, whether a particular object belongs to the collection or not. The objects in a set are district means that we do not repeat an object again and again in a set. The objects belonging to the set are called elements or member of a set.

Standard Notation used for some sets.
IN -A Set of all Natural Numbers
W -A set of all whole numbers.
Z- A set of all integers
$\mathbf{Z}^+/\mathbf{Z}^-$ -A set of all Positive / Negative integers
Q-A Set of all rational Numbers.
$\mathbf{Q}^+/\mathbf{Q}^-$ -A set of all +ve /-ve Rational Numbers
IR-A set of all Real Numbers
IR+ / IR - = A set of all +ve /-ve Real Number

* INTERVAL *

In Mathematics, a (real) interval is a setOf real number that contains all real no. lying between any two
numbers of the Set. If A subset E of IR (real no.) is said to be an interval if, whenever x, y ∈E and z ∈E

$$x < z < y \text{ then } z \in E.$$



For example:- The Set of numbers x satisfying 0< x < 1 is an interval which contains. 0,1 and all
numbers in between.
All contains in (0,1).
There are basically Eleven (11) Types interval.
Null set , {a}, (a,b), [a,b), (a,b], [a, b], (a,∞), [a, ∞) ,(-∞, α), (-∞, a],(-∞,∞) = R
Real intervals plays an important role in the theory of integration, because they are the Simplest Sets
whose "length" (Or "Size") is easy to define.

Key Points

1. An open interval does not include its endpoints and is indicated with parentheses ().

   For Example:- (0,1) means greater than 0 and Less than 1.

   (0,1) = { x / 0 < x < 1 } . And its also denoted by] 0,1 [

2. A closed Interval is an Interval which includes all its limit points and also includes Endpoints and its denoted with square Brackets []. For Ex. → [0,1] means greater than or equal to zero and less than or equal to 1.

3. A half open Interval includes only one of its endpoints and is denoted by mixing open and closed interval. the notations for open and closed interval.

4. A degenerate Interval is any -set Consisting of a single real number (an interval of the form [a,a]). Some authors include the empty set in this definition. A real interval that is neither empty nor degenerate is said to be proper and has infinitely many elements.

5. An interval is said to be left-bounded or Right bounded , if there is some real no. That is ,respectively Smaller than or larger than all its element. An Interval is said to be bounded if it is both left and right bounded; and is said to be unbounded otherwise .Intervals that are bounded at only one end are said to be half-bounded. The empty sets is bounded and the set of all reals is the only interval that Is Unbounded at both ends. Bounded Intervals are also Commonly known as finite intervals.

6. Bounded Intervals are bounded sets in the sense,that their diameter (which is equal to the absolute difference between the endpoints (a,b) , diameter → |a-b| = r) is finite.
The diameter may be called the length, width,measure,range or Size of the interval. The Size of unbounded interval is usually defined as $+\infty$ and the size of the empty Interval may be defined 0.

7. The Center (mid-point) of bounded interval Withendpts. a and b is (a+b)/2 and its radius is the half-length (|a-b|)/2. These Concepts are undefined for empty or unbounded intervals.

8. An Interval is said to be left-open if and only if
It contains no minimum (an element that is Smaller than all other elements); right open if it contains no maximum; and open if it has both properties. The interval $[0,1) = \{x / 0 \leq x < 1\}$, is left closed andRight open.

9. The interior of an interval I is the largest, open interval that is Contained in I; it is also the set of points in I which are not endpoints of I.

$I = [a,b]$, $I^\circ = (a,b)$

That implies (a,b) contained in [a,b].
The closure of I is the smallest closed Interval that contains I ,which is also the set I augmented

with its finite End points.

I = (a,b) closure(I) = I U I' where I' denotes limit point of I .

Cl(I) = (a,b) U [a,b]

CL(I) = [a,b]

(a,b) C [a,b]

Some Basic Concepts of Interval

If one of the endpoints is to be excluded from the set, the corresponding square bracket can be

either replaced with a parenthesis, or Converse is also true.

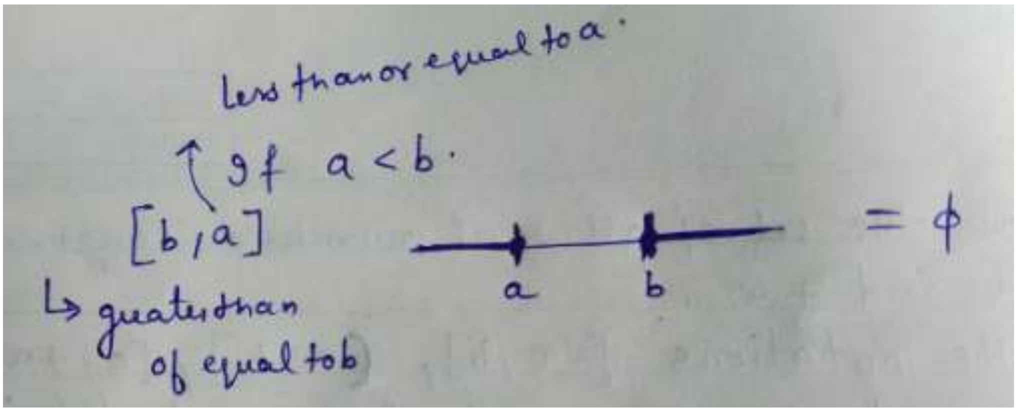
$(a,b) =]a,b[= \{x \in R \mid a<x<b\}$,

$[a,b) = [a,b[= \{x \in R \mid a<x<b\}$,

$(a,b] =]a,b] = \{x \in R \mid a<x\leq b\}$,

$[a,b] = [a,b] = \{x \in R \mid a\leq x\leq b\}$.

Each Interval (a,a), [a,a) and (a,a] represent the empty set, whereas [a,a] denotes the Singleton



set{a}, when a>b, all four notations are usually taken to represent the empty sets.[a,a] = {a} is

also an Interval. Because we take two elements from that Singleton set to prove the interval and take one element in between them and if that middle element is also in the same set then we called it is interval. But if we cannot take two elements from the Singleton set then there will be no contradiction, so the Singleton set will also be an Interval.

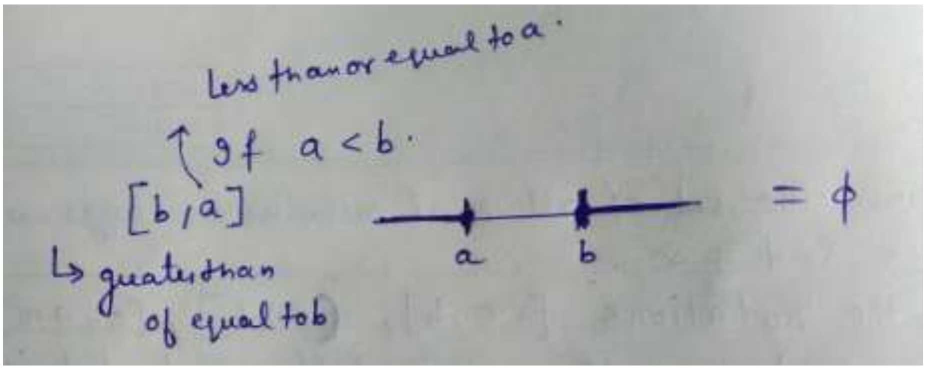
- **Infinite Endpoints:-** An Interval may be defined as a subset of the extended real Numbers; the set of all real numbers augmented with -∞ and +∞.Here, the notations (-∞,b], [-∞,b] ,[a,+∞) and [a,+∞] are all meaningful and district. In particular (-∞,+∞) denotes the set of all ordinary real no. ; while [-∞,+∞] denotes the extended real number.

In the ordinary reals, one may use an infinite endpoints to indicate that there is no bound in that direction. For ex. (0, +∞) is the set of +ve real numbers, also written as R+.

Types of Interval

- The interval of real no. Can be classified into the eleven different types, where a and b arereal numbers and a <b

1) Empty :- [b,a] = (b,a) = [b,a) = (b,a] = (a,a)= [a,a) = (a,a]= { }



2) Degenerate: [a,a] = {a}

- Proper and Bounded:

  Open :(a,b) = {x /a<x<b }
  Closed :[a b] = {x / a≤ x ≤ b }
  Left closed ,right open.
  Left open, right closed.

- Left bounded and right unbounded

  Left open : (a,+∞) = {x | x>a }
  left closed : - [a,+∞) = {x | x≥ a}

- Left Unbounded and right bounded :-

Right-open: $(-\infty,b) = \{x/x<b\}$

Right-closed:-$(-\infty,b] = \{x/x\leq b\}$

- Unbounded at Both ends (Simultaneously open and closed) :-$(-\infty,+\infty)$ =R

- **Some Important Properties of Interval.**

1. The interval are precisely the connected subset of IR.I
2. Image of an Interval by any Continuous function is also an Interval.
3. The intersection of any collection of intervals is always an interval.
4. The Union of two interval is an Interval if and only if they have a non-empty intersection or an open end- point of one interval is a closed end point of the other.

Example - (a, b) U [b,c] = (a,c]

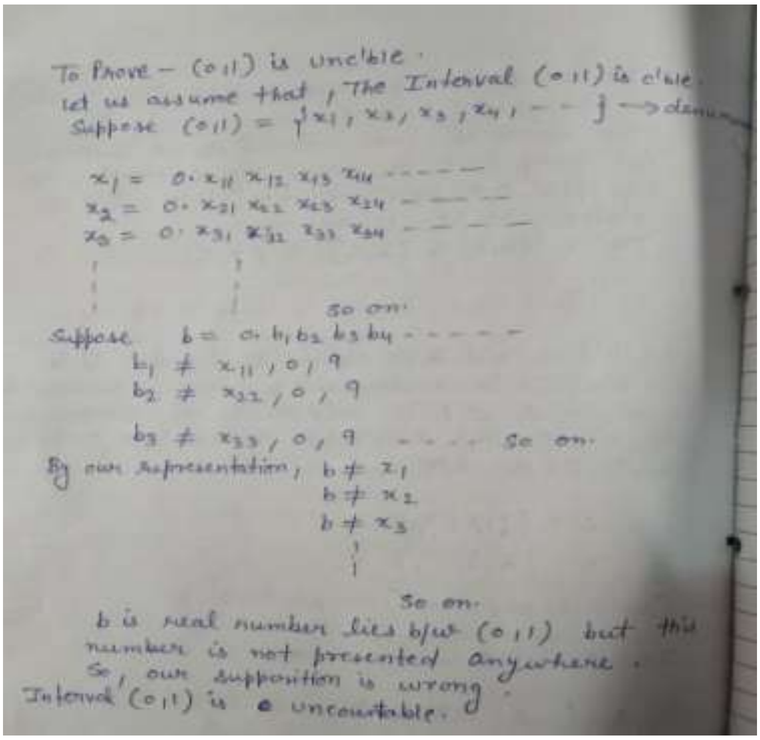
- **Countability or uncountability of Interval**

A Set **S** is said to be countable if either it is finite or it is denumerable where denumerable means- A set K is said to be denumerable if K~N i.e, there exist a one one onto function **f :N → K**. Then the set k is denumerable.

Empty set = **(a,a)**→finite. →countable
Closed set = **[a,a]** → finite → countable

Let we check an Interval **(0,1)** subset of IR.

To Prove- **(0,1)** is uncountable. Let us assume that The Interval (0,1) is countable.
Suppose **(0,1) = {x1, x2, …………}**


To Prove – (0,1) is unc'ble.
let us assume that, The Interval (0,1) is c'ble.
Suppose $(0,1) = \{x_1, x_2, x_3, x_4, \ldots\}$ → denum...

$x_1 = 0.x_{11}\, x_{12}\, x_{13}\, x_{14} \ldots$
$x_2 = 0.x_{21}\, x_{22}\, x_{23}\, x_{24} \ldots$
$x_3 = 0.x_{31}\, x_{32}\, x_{33}\, x_{34} \ldots$
so on.

Suppose $b = 0.b_1 b_2 b_3 b_4 \ldots$
$b_1 \neq x_{11}, 0, 9$
$b_2 \neq x_{22}, 0, 9$
$b_3 \neq x_{33}, 0, 9$ ... so on.
By our representation, $b \neq x_1$
$b \neq x_2$
$b \neq x_3$
so on.
b is real number lies b/w (0,1) but this number is not presented anywhere.
So, our supposition is wrong.
Interval (0,1) is uncountable.

Superset of uncountable sets uncountable.
IR→ Real number is also uncountable.
And we know that every two Interval are equivalent by their cardinality.

(a,b) ~ R

(c,d) ~ R

That implies,

(a,b) ~ (c,d)

And by Hilbert paradox,

[a,b] ~ (a,b)

So, by conclusion , we get every non empty, non singleton Interval is Uncountable.

Check some sets are Interval or not ?

Ø = (a,a) c R → Interval
{a} = [a,a] c R → Interval
N(set of natural number) = { 1,2,3,4,5,…………}
Let 2,3 ∈ N and take 2.5 between them 2 < 2.5 < 3 but 2.5 does not belong to natural number. So, N is not an Interval .
And by same way we know that , Q , Z ,I , N are not an Interval.

Conclusion

Real Intervals plays an important role in mathematics. Intervals are very useful when describing domain and range. We can use interval notation to show that a falls between two
endpoints. Intervals plays an important role in the theory of integration ,topology.

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