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STUDY OF SOME ASPECTS OF FLOWS OF VISCO ELASTIC FLUID IN TUBES

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ABSTRACT

It is investigated how a pulsating pressure gradient influences the unstable motion of Green-Rivlin fluids in cylindrical channels with arbitrary cross section. Limit of the initial flow and the non-linear constitutive structure are both perturbed simultaneously via a new approach of domain mapping. This method uses nested integrals to specify the structure of the non-linear constitutive element spanning semi-infinite time domains. Both the longitudinal field, which is the principal The principal component of the flow is the longitudinal field, which is generated in the first step of analysis, and the secondary component is the transverse field, which is generated in the second stage of analysis. The first order terms generated by the linearly viscoelastic longitudinal flow drive the secondary field. The aforementioned domain mapping technique produces a spectrum with shapes that each have their own distinct closed cross section. We offer longitudinal velocity profiles and transversal time-averaged, mean secondary flow streamline patterns for a specific fluid and the most widely-representative cross-sectional forms of the spectrum, the triangular, square, and hexagonal shapes.

1. INTRODUCTION Flow problems continue to capture the interest of academics in this day and age as a direct result of the real-world applications that can be found in sectors such as manufacturing and engineering. The flow of non-Newtonian fluids that occurs as a result of stretched surfaces has found significant application in a variety of industries, including the creation of papers, the production of plastics and food ingredients, the production of lubricants and organic fluids, and a great deal more. The manufacturing of lubricants, as well as a wide variety of flow difficulties involving complicated geometries and boundary conditions, all call for a fluid of a lower quality. Some of the physical concerns that fall into these categories are diluted extrusion solutions, slurry flows, and the production of lubricants. Because non-Newtonian fluids are so common in our everyday lives, there has been a lot of interest in them from the academic community.

examined the flow of a second-grade fluid with finite dimensions when it was only going in one direction temporarily. The researchers Mandapat and Gupta looked into the process of heat conduction in a non-Newtonian fluid as it moved over a stretching sheet. In addition to that, we discussed the mercurial nature of the fluid of second-grade quality that is found in Couette. Using a modified differential technique, a solution was found to the problem of heat transfer that occurred when a fluid of lower quality was passed over a medium that had the ability to absorb heat. We were successful in solving the problem of a viscoelastic fluid traveling across a sloping sheet by employing a solution that was only semi-analytical.

The relationship between shear stress and shear strain is the formula for calculating viscosity. The properties of a viscous fluid can be changed when a viscosity that is dependent on temperature is introduced to the fluid. Temperature shifts result in significant differences in the thickness (viscosity) of a variety of liquids, including water and oils, among others. Temperature and viscosity are two aspects of a substance's physical qualities that are related in opposite ways in the case of liquids but directly to one another in the case of gases. If there is a temperature variation in the system, there will be a considerable change in the viscosity of the fluid. This is because the temperature distribution inside the flow zone is not always the same in many thermal transport processes. If we wish to accurately predict flow activities, it is essential for us to make the assumption that the viscosity is temperature dependent. Elbashbeshy and a number of other researchers looked at the effects of varying the parameters on free convective fluid flow across a vertical plate. a non-Newtonian fluid with a non-constant viscosity was the issue that needed to be resolved. a third-grade level of fluid results were established by taking into account the thickness variations near the boundary layer. Seddeek did research on hydromagnetic flow that had a variable magnetic field as well as the viscosity of the fluid. math was utilized to help figure out the solution after looking at how the viscosity of a fluid affects how it flows through a viscous medium. We looked into a fluid of a lower grade that had varying viscosity as well as the analytical answer for it. Along with the impacts of non-uniform thickness and thermal conductivity, the topic of discussion was the convective heat transport in a viscous fluid that takes place over a spinning vertical cone. investigated how a shift in the surface's thickness and an alteration in the flow's internal energy would affect a flow in two dimensions via a stretched surface. Under the rotating disk, incompressible fluid flow was investigated in order to take into account the disc's different thicknesses as well as its temperature properties.

A viscoelastic material has the following properties:

- Hysteresis can be observed in the relationship that exists between stress and strain.

- A good illustration of stress relaxation would be the reduction of stress caused by step-constant strain.
- The phenomenon of creep happens when a stress is applied in a series of discrete steps at a constant rate.
- its stiffness is strain-rate-dependent. stress levels, etc.

Fluids With High Viscosity That Are Confined To Narrow Channels

Stenoses are localized artery narrowing can occur in people or animals and is frequently brought on by intravascular plaques. Both humans and animals are susceptible to developing stenosis. In order to better understand how to detect, prevent, and diagnose cardiovascular issues, there has been a recent uptick in the use of numerical approaches to analyze blood flow in stenotic arteries. This curiosity stems, in large part, from the belief that studying blood flow in stenotic arteries is more challenging than studying blood flow in healthy arteries. Blood flow characteristics, such as high levels of shear stress at the wall or its modulation, may play an important role in the etiology and progression of a wide range of cardiovascular diseases, according to circumstantial evidence. This suggests that research into these topics is even more important than it already was. The majority of the research done in this field concentrates its attention on the flow through cylindrical pipes that have a uniform cross-section.

On the other hand, the idea put forth by Whitmore, which states that blood arteries branch off at regular intervals and that artery width changes with distance, is generally accepted. As a consequence of this, a considerable number of problems associated with blood flow can be simplified by focusing on the core idea of flow in a changeable cross-section. It was suggested that the majority of the vessels could be modeled as long, thin, gradually tapers shapes. Because various researchers have pointed to hydrodynamic factors as having a vital role in the development of stenosis, the mathematical modeling of blood flow through a stenosed tube is extremely important. There has been a lot of research done to understand blood rheology when the flow is constant, but relatively little research has been done to understand blood rheology when the flow is turbulent. In a living creature, the heart generates a pulsatile flow, and the major arteries' pliability eventually mutes the flow's oscillations. However, the periodic pattern of blood flow is observed in smaller capillaries and arterioles, where the distensibility of the walls is substantially reduced and the influence of pulse frequency becomes more prominent.

OBJECTIVE

1. To conduct research on the physical aspects of secondary flows of viscoelastic fluids in a straight tube.
2. To do research on the equation describing the flow of fluid through a viscoelastic tube.

EQUATION FOR FLUID FLOWING VISCOELASTIC TUBE PRESSURE AND RADIUS

In the event of an incompressible fluid moving through a tube featuring axial symmetry, the following conditions are presumed to hold true, The equilibrium radii are far smaller than the characteristic lengths of the wave processes, and the deformation of the tube wall and the wall thickness are negligible compared to the radius; Strain refers to the variation in tube radius that occurs as a function of both coordinates and elapsed time.

In light of the fact that the actual motion takes place at the spectrum's endpoints, we are able to formulate the equation of motion for the tube wall by applying the principle of least action.

$$J[R(x,t)] = \int_{t_0}^{t_1} L dt \rightarrow \min_{R(x,t)}$$

The state of the system can be graphically represented by the Lagrangian L, which is the sum of the system's kinetic energy and its potential energy minus one another.

$$L = T - U, \qquad U = U_{el} - A$$

The elastic potential energy, denoted by Uele, possessed by the tube is compared to the amount of work, denoted by A, that is carried out during expansion by the pressure and dissipation forces.

We are going to examine a section of the tube that has been severed using the cylindrical coordinate system (r, x ≡ z), and this section will contain a cutout. The following equation can be used to define the kinetic energy of a tube element with a length of l, where l is equivalent to the typical wave lengths in the flow of the fluid:

$$T = \int_0^l \pi \rho_w h_0 R_0 R_t \, dx$$

The elastic potential energy of a tube element of length l consists of two parts. The elastic energy of a wall is depicted in the first image as a collection of nonlinear elastic rings with no underlying connection.

$$U_1 = \int_0^l \left[\pi \kappa h (R - R_0)^2 + \frac{2\pi\kappa_1 h}{3}(R - R_0)^3 \right] dx$$

$$\kappa = \frac{E}{R_0(1-\sigma^2)}$$

Here, κ is known as the longitudinal young's modulus, and it describes the increase in the linear elasticity of a tube part as its length grows, σ is Poisson’s ratio, and κ1 possesses a quadratic adjustment to the law of Hooke, which is known as the nonlinear elasticity coefficient.

The second phase is described by the elastic energy of longitudinal wall fibers. This energy is defined as the increase in area of a wall element of length l that is generated by bending along the x axis:

$$U_2 = \int_0^l 2\pi khR\sqrt{1 + R_x^2}\,dx - \int_0^l 2\pi khR\,dx$$

The k-factor provides a quantitative measure of the longitudinal forces that are exerted on a wall. According to it, the value xx should be used to represent the normal axial component of the wall stress tensor. This is comparable to the stress that is continuously being applied along the axis of a blood vessel.

We obtain larger strains from the smaller ones.

$$U_2 = \int_0^l \pi khRR_x^2\,dx$$

The total elastic potential energy of a l-meter-long tube section can be calculated using the formula given below.

$$U_{el} = \int_0^l \left(\pi\kappa h(R - R_0)^2 + \frac{2}{3}\pi\kappa_1 h(R - R_0)^3 + \pi khRR_x^2\right) dx$$

Both the pressure inside the vessel, P(x, t), and the pressure outside the vessel, Pe, are assumed to be constant across the whole cross section of the vessel. The viscosity of the fluid will stand in for the forces that are being applied by the vessel walls in this experiment. Using the formula, one can account for the fundamental effort that is exerted by a variety of viscous forces. Some examples of these forces are those that oppose the motion of the wall, the pressure of the fluid, and the pressure of the environment.

$$\delta A = \int_0^l 2\pi R\left(hf - \mu\frac{\partial R}{\partial t}\right)\sqrt{1 + R_x^2}\,dx\,\delta R + \int_0^l 2\pi R(P - P_e)\,dx\,\delta R$$

$$f = \chi\frac{\partial^3 R}{\partial x^2 \partial t}$$

Because the value of the force f can be deduced from the radial component of the stress tensor that is caused by viscosity, we can state the relationship between the two in a more straightforward manner as follow have

$$\delta A = \int_0^l 2\pi R\left[\chi h\frac{\partial^3 R}{\partial x^2 \partial t} - \mu\frac{\partial R}{\partial t} + P - P_e\right] dx\,\delta R$$

Here, χ is the material's viscosity derived from an analogy with the fluid's dynamic viscosity, or does it exist independently μ represents the resistance of the medium to the movement of the tube wall and is written as a proportionality coefficient.

When we take into account the expressions, we are provided with a Lagrangian:

$$L = \int_0^l \left(\pi \rho_w h_0 R_0 R_t^2 - \pi \kappa h (R - R_0)^2 - \frac{2}{3} \pi \kappa_1 h (R - R_0)^3 - \pi k h R R_x^2 \right) dx + A$$

Where A represents the result of an expression.

By minimizing the functional on the class of smooth functions R (x, t) considered on the time interval [t 0, t 1], we derive the Euler equation and the transversality requirements in the form. Following the principle of doing as little as necessary, we proceed in this manner.

$$R(P - P_e + \chi h R_{txx} - \mu R_t) = \rho_w h_0 R_0 R_{tt} - khRR_{xx} - \frac{kh}{2} R_x^2 + \kappa h (R - R_0) + \kappa_1 h (R - R_0)^2$$

$$\left. \frac{\partial R(x,t)}{\partial x} \right|_{\substack{x=0 \\ x=l}} = 0, \qquad \left. \frac{\partial R(x,t)}{\partial t} \right|_{t=t_1} = 0$$

Taking into consideration the fact that the radius of the wall has slightly altered η(x, t)

$$R(x,t) = R_0 + \eta(x,t), \quad h = h_0 \frac{R_0}{R}, \quad R_0 = \text{const}, \quad h_0 = \text{const}$$

and neglecting the term kh0η2 x /2R0 the longwave perturbation approximation yields higher-order terms

$$P - P_e = \rho_w h_0 \eta_{tt} - k h_0 \eta_{xx} - \chi h_0 \eta_{txx} + \mu \eta_t + \frac{\kappa h_0}{R_0} \eta + \frac{\kappa_2 h_0}{R_0^2} \eta^2$$

$$\kappa_2 \equiv \kappa_1 R_0 - 2\kappa$$

In its simplest form, the equation of state that we have here describes the movement of fluid as it passes through the tube. When the tube is at rest, the fluid pressure that is contained within it has a relationship that is linear to the radius of the tube.

VISCOELASTIC TUBE FLUID-FLOW EQUATION

We employ the continuity equation in conjunction with the axial component of the two-dimensional Navier-Stokes equation to describe the flow of fluid inside an axially symmetric viscoelastic tube of arbitrary cross-section.

$$\frac{\partial (vr)}{\partial r} + \frac{\partial (ur)}{\partial x} = 0$$

$$\frac{\partial u}{\partial t} + v \frac{\partial u}{\partial r} + u \frac{\partial u}{\partial x} + \frac{1}{\rho} \frac{\partial P}{\partial x} = \nu_0 \left[\frac{\partial^2 u}{\partial x^2} + \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right) \right]$$

where radial velocity, denoted by v, and axial velocity, denoted by u, are the two components of flow velocity that make up total velocity, ν0 is the density of the fluid and the kinematic viscosity.

Let's say the axial velocity component has this radial profile:

$$u(r,x,t) = \frac{s+2}{s}\left[1 - \left(\frac{r}{R}\right)^{s}\right]u_a(x,t), \quad u_a(x,t) = \frac{2}{R^2}\int_0^{R(x,t)} u(r,x,t)r\,dr$$

The value of the exponent s gives an indication of how steep the profile is. The one-dimensional equations for the conservation of fluid mass and momentum can be found by doing an average calculation over the cross section of the tube equations

$$\frac{\partial S}{\partial t} + \frac{\partial (Su_a)}{\partial x} = 0$$

$$\frac{\partial u_a}{\partial t} + u_a\frac{\partial u_a}{\partial x} + \frac{1}{\rho}\frac{\partial P}{\partial x} = \nu_0\frac{\partial^2 u_a}{\partial x^2} - 2\nu_0(s+2)\frac{u_a}{R^2}$$

Here, $u_a = u_a(x,t)$ is the typical axial velocity measured across a cross section and $S = S(x,t)$ is the area that constitutes the tube's cross-section. The following will not include the subscript of ua because it has been omitted.

Since $S(x,t) = \pi R(x,t)^2$, This equation simplifies to the following form:

$$R_t + uR_x + \frac{1}{2}Ru_x = 0$$

Taking into consideration the little differences in the tube's radius $R = R_0 + \eta$, we obtain the equation

$$\eta_t + \frac{1}{2}R_0u_x + \frac{1}{2}\eta u_x + u\eta_x = 0$$

In order to get the most out of this examination, we are going to zero in exclusively on the longwave approximation for high Reynolds numbers and how it can be used to the study of nonlinear waves. This basic estimation of the sizes of the blood vessels in the body is applicable to both large and medium-sized arteries.

Because of this, we are able to derive a set of equations in the form to describe the flow of fluid in a single dimension through an axially symmetric viscoelastic tube when the Reynolds numbers are high:

$$\eta_t + \frac{1}{2}R_0u_x + \frac{1}{2}\eta u_x + u\eta_x = 0$$

$$u_t + uu_x + \frac{1}{\rho}P_x = 0$$

$$P = \rho_w h_0\eta_{tt} - kh_0\eta_{xx} - \chi h_0\eta_{txx} + \mu\eta_t + \frac{\kappa h_0}{R_0}\eta + \frac{\kappa_2 h_0}{R_0^2}\eta^2 + P_e$$

Fluid flow in an elastic tube can be described by a simple linearized system constructed by assuming the flow pressure is proportional to the radius perturbation and eliminating the nonlinear parts from the system of equations for the inviscid fluid. This deterministic setup describes the flow of the fluid.

$$\eta_t + \frac{R_0}{2}u_x = 0, \qquad u_t + \frac{1}{\rho}P_x = 0, \qquad P = \frac{\kappa h_0}{R_0}\eta + P_e$$

It is possible to write system in the form:

$$\eta_t + \frac{R_0}{2}u_x = 0, \qquad u_t + \frac{\kappa h_0}{\rho R_0}\eta_x = 0$$

The derivative of the first equation with respect to velocity yields the following linear wave equations for the perturbations in velocity to x and the second equation with respect to t. This allows us to obtain the desired results.

$$u_{tt} = \frac{\kappa h_0}{2\rho}u_{xx}$$

The radius perturbations and the pressure perturbations are described by the same equation.

Moence and Korteweg came to the conclusion that the following is how the speed of a pressure pulse looks as it moves through an elastic tube:

$$c_0 = \sqrt{\frac{\kappa h_0}{2\rho}} = \sqrt{\frac{Eh_0}{2\rho R_0(1-\sigma^2)}}$$

The variables that do not depend on any dimensions come first.

$$t = \frac{l}{c_0}t', \qquad x = lx', \qquad u = c_0 u', \qquad \eta = \frac{R_0}{2}\eta'$$

$$P = P_0 P', \qquad P_0 = P_e$$

To begin, there are the variables that are independent of the dimensions.

$$\eta_t + u_x + \frac{1}{2}\eta u_x + u\eta_x = 0$$

$$u_t + uu_x + \frac{1}{\alpha}P_x = 0$$

$$P = \gamma\eta_{tt} - \beta\eta_{xx} + \lambda\eta_t - \delta\eta_{txx} + \alpha\eta + \alpha_1\eta^2 + 1$$

$$\alpha = \frac{\rho c_0^2}{P_0}, \qquad \beta = \frac{kh_0R_0}{2P_0l^2}, \qquad \gamma = \frac{\rho_w h_0 R_0 c_0^2}{2P_0 l^2}$$

$$\delta = \frac{\chi h_0 R_0 c_0}{2P_0 l^3}, \qquad \lambda = \frac{\mu R_0 c_0}{2P_0 l}, \qquad \alpha_1 = \frac{\kappa_1 h_0 R_0}{4P_0} - \alpha$$

NONLINEAR EVOLUTIONARY EQUATIONS DESCRIBE VISCOELASTIC TUBE FLUID FLOW PERTURBATIONS

The multiscale and perturbation methods are currently well-represented in the research literature, and they are able to be utilized to get most evolutionary nonlinear equations. It would appear that this strategy was first used in the location where the extremely minute parameters are located in the system of equations.

$$\varepsilon \ll 1 \quad \left(\varepsilon_1 = \frac{a_0}{R_0}, \quad \varepsilon_2 = \frac{R_0}{l}, \quad \varepsilon_3 = \frac{h_0}{R_0}\right)$$

where a0 is the amplitude of the perturbation that is being applied to the radius. Arteries have values of 0.1, 0.4, and 0.2 respectively for these characteristics, which are considered to be usual. It is convenient to switch to "slow" time variables after identifying the direction of wave propagation in order to study the evolution of the perturbations in the low-amplitude long wave approximation. This is because the characteristic velocities of pressure waves (pulse waves) are large in comparison to the flow velocities. Our value will be determined by the most insignificant of the aforementioned attributes (ε ~ 0.1). We will then seek the solution of the system of equations using the variables.

$$\eta = \varepsilon^2 \eta', \qquad u = \varepsilon^p u', \qquad P = 1 + \varepsilon^p P', \qquad p \in N$$
$$\xi = \varepsilon^m (x - t), \qquad \tau = \varepsilon^n t, \qquad m, n \in N, \qquad n > m$$
$$\frac{\partial}{\partial x} = \varepsilon^m \frac{\partial}{\partial \xi}, \qquad \frac{\partial}{\partial t} = \varepsilon^n \frac{\partial}{\partial \tau} - \varepsilon^m \frac{\partial}{\partial \xi}$$

Substitution carried out after the terms of the first two equations have been removed by εm+ p and in the last equation by εp, inevitably results in the equation system.

$$\varepsilon^{n-m} \eta'_\tau - \eta'_\xi + u'_\xi + \varepsilon^p \frac{1}{2} \eta' u'_\xi + \varepsilon^p u' \eta'_\xi = 0$$

$$\varepsilon^{n-m} u'_\tau - u'_\xi + \varepsilon^p u' u'_\xi + \frac{1}{\alpha} P'_\xi = 0$$

$$P' = \varepsilon^{2n} \gamma \eta'_{\tau\tau} - \varepsilon^{n+m} 2\gamma \eta'_{\tau\xi} + \varepsilon^{2m} (\gamma - \beta) \eta'_{\xi\xi} + \varepsilon^n \lambda \eta'_\tau - \varepsilon^m \lambda \eta'_\xi - \varepsilon^{n+2m} \delta \eta'_{\tau\xi\xi} + \varepsilon^{3m} \delta \eta'_{\xi\xi\xi} + \alpha \eta' + \varepsilon^p \alpha_1 \eta'^2$$

In order to uncover the answer to this system's problem, we will be looking for asymptotic expansions.

$$u' = u_1 + \varepsilon^q u_2 + o(\varepsilon^q); \qquad \eta' = \eta_1 + \varepsilon^q \eta_2 + o(\varepsilon^q)$$
$$P' = P_1 + \varepsilon^q P_2 + o(\varepsilon^q), \qquad q \in N$$

By substituting and equating the coefficients of ε0, we are able to establish the following relations in the zeroth approximation:

$$-\eta_{1\xi} + u_{1\xi} = 0, \qquad -u_{1\xi} + \frac{1}{\alpha} P_{1\xi} = 0, \qquad P_1 = \alpha \eta_1$$

As a result, we have

$$u_1(\xi, \tau) = \eta_1(\varepsilon, \tau) + \psi(\tau), \qquad P_1(\xi, \tau) = \alpha \eta_1(\xi, \tau)$$

where ψ(τ) is a fictitious number that derives its significance from the boundaries of u1 and η1.

The derivative with regard to the evolutionary equations was derived using to τ and the nonlinear term uuξ , with allowance for the first approximation (∼ εq) we need to set n − m = p = q.

In this case we neglect the terms of higher order than n − m (n ≥ m) and, after eliminating P , obtain system (3.3) in the form:

$$\varepsilon^{n-m}\eta'_\tau - \eta'_\xi + u'_\xi + \varepsilon^p \frac{1}{2}\eta' u'_\xi + \varepsilon^p u'\eta'_\xi = 0$$
$$\varepsilon^{n-m}u'_\tau - u'_\xi + \varepsilon^p u' u'_\xi + \varepsilon^{2m}\frac{\gamma-\beta}{\alpha}\eta'_{\xi\xi\xi} + \eta'_\xi + \varepsilon^{3m}\frac{\delta}{\alpha}\eta'_{\xi\xi\xi\xi} + \varepsilon^p \frac{2\alpha'}{\alpha}\eta'\eta'_\xi = \varepsilon^m \frac{\lambda}{\alpha}\eta'_{\xi\xi}$$

From this we obtain the equation

$$\varepsilon^{n-m}(\eta'_\tau + u'_\tau) + \varepsilon^p\left(u'u'_\xi + \frac{1}{2}\eta' u'_\xi + u'\eta'_\xi + \frac{2\alpha_1}{\alpha}\eta'\eta'_\xi\right) + \varepsilon^{2m}\left(\frac{\gamma-\beta}{\alpha}\eta'_{\xi\xi\xi}\right) + \varepsilon^{3m}\left(\frac{\delta}{\alpha}\eta'_{\xi\xi\xi\xi}\right) = \varepsilon^m\left(\frac{\lambda}{\alpha}\eta'_{\xi\xi}\right)$$

Setting m = 1 and taking the relation n−m = p = q into account, we will consider three cases for p, q, and n: (1) p = q = 1 and n = 2; (2) p = q = 2 and n = 3; and (3) p = q = 3 and n = 4

The higher derivative terms become more important in the mathematical model as time progresses, which is reflected by the increasing value of the n parameter.

The equations of evolution that describe changes in the flow through a viscoelastic tube will now be derived.

We'll start with the simple case ,(m = p = q = 1 and n = 2). Substituting (3.4) in (3.6) and equating the coefficients of ε^1, we obtain the equation.

$$\eta_{1\tau} + u_{1\tau} + u_1 u_{1\xi} + \frac{1}{2}\eta_1 u_{1\xi} + u_1\eta_{1\xi} + \frac{2\alpha_1}{\alpha}\eta_1\eta_{1\xi} = \frac{\lambda}{\alpha}\eta_{1\xi\xi}$$

As a direct result of this, we have arrived at the evolutionary equation thanks to the relational allowances.

$$\eta_{1\tau} = \left[\left(\frac{5}{4} + \frac{\alpha_1}{\alpha}\right)\eta_1 + \psi(\tau)\right]\eta_{1\xi} = \frac{\lambda}{2\alpha}\eta_{1\xi\xi} - \frac{\psi'(\tau)}{2}$$

Here, $\psi(\tau)$ enables a modification to be made to the rate at which waves move and when they occur. $\psi(\tau) \neq$ Const is a representation of an origin, and its objective is to $\psi(\tau)$ can be determined from relations .Let

$$\psi(\tau) = \eta_1|_{\xi=\xi_0} - u_1|_{\xi=\xi_0} = 0,$$
,

After that, it is referred to as a Burgers equation.

$$\eta_{1\tau} + \left(\frac{5}{4} + \frac{\alpha_1}{\alpha}\right)\eta_1\eta_{1\xi} = \frac{\lambda}{2\alpha}\eta_{1\xi\xi}$$

Because of the dimensionless coefficient, we are able to articulate our thoughts in terms of the underlying physics of the model. It is possible to calculate the coefficient of the nonlinear component by applying the formula.

$$\frac{5}{4} + \frac{\alpha_1}{\alpha} = \frac{1}{4} + \frac{R_0}{2}\frac{\kappa_1}{\kappa}$$

Therefore, the ratio of the nonlinear elastic modulus to the linear elastic modulus is directly proportional to the steepness of a nonlinear wave front. A linear elastic wall formula can be described as, i.e., $\kappa_1 = 0,$, then $\alpha_1 = -\alpha$ and equation takes the form:

$$\eta_{1\tau} + \frac{1}{4}\eta_1\eta_{1\xi} = \frac{\lambda}{2\alpha}\eta_{1\xi\xi}$$

These values serve as the parameters for the coefficient of the second derivative:

$$\frac{\lambda}{2\alpha} = \frac{R_0}{4\rho l c_0}\mu$$

As a direct consequence of this, the wave's intensity is reduced in a manner that is inversely proportional to the resistance coefficient of the medium.

When $\psi(\tau) = \text{const} = \psi_0$, which can be reached via a transformation of the variables that is not degenerate

$$\theta = \xi - \psi_0\tau, \qquad \tau' = \tau$$

The following is the solution to the initial equations, which can be written as:

$$\eta(x,t) = \varepsilon\eta'(\xi,\tau) \simeq \varepsilon\eta_1(\xi,\tau); \quad u(x,t) = \varepsilon u'(\xi,\tau) \simeq \varepsilon u_1(\xi,\tau) \simeq \varepsilon\eta_1(\xi,\tau)$$

$$P(x,t) = \varepsilon P'(\xi,\tau) \simeq \varepsilon P_1(\xi,\tau) = \varepsilon\alpha\eta_1(\varepsilon,\tau), \quad \xi = \varepsilon(x-t), \quad \tau = \varepsilon^2 t$$

We'll jump right into the second case. (m = 1, p = q = 2, and n = 3). Coefficients are substituted and equalized by ε2, we obtain the equation

$$\eta_{1\tau} + u_{1\tau} + u_1u_{1\xi} + \frac{1}{2}\eta_1u_{1\xi} + u_1\eta_{1\xi} + \frac{2\alpha_1}{\alpha}\eta_1\eta_{1\xi} + \frac{\gamma-\beta}{\alpha}\eta_{1\xi\xi\xi} = 0$$

It is possible to derive the equation of evolution by beginning with and working backwards from.

$$\eta_{1\tau} + \left[\left(\frac{5}{4} + \frac{\alpha_1}{\alpha}\right)\eta_1 + \psi(\tau)\right]\eta_{1\xi} + \frac{\gamma - \beta}{2\alpha}\eta_{1\xi\xi\xi} = -\frac{\psi'(\tau)}{2}$$

Setting ψ ≡ 0, The Korteweg-de Vries equation is what we get as a result:

$$\eta_{1\tau} + \left(\frac{5}{4} + \frac{\alpha_1}{\alpha}\right)\eta_1\eta_{1\xi} + \frac{\gamma - \beta}{2\alpha}\eta_{1\xi\xi\xi} = 0$$

Using The coefficient of the dispersion term can be expressed using this form if you want to write it down:

$$\frac{\gamma - \beta}{2\alpha} = \frac{1}{4}\left(\frac{R_0}{l}\right)^2\left[\frac{h_0}{R_0}\frac{\rho_w}{\rho} - 2(1 - \sigma^2)\frac{\sigma_{xx}}{E}\right]$$

Because of this, the dispersion coefficient is calculated using a ratio that takes into account both the longitudinal wall stress and the elasticity modulus of the wall, in addition to the difference in density that exists between the wall and the fluid.

By applying the relations to the solutions of the Korteweg-de Vries equation, it is possible to obtain an approximation of the solution to the system.

$$\begin{aligned}
&\eta(x, t) = \varepsilon^2\eta'(\xi, \tau) \simeq \varepsilon^2\eta_1(\xi, \tau);\\
&u(x, t) = \varepsilon^2 u'(\xi, \tau) \simeq \varepsilon^2 u_1(\xi, \tau) \simeq \varepsilon^2\eta_1(\xi, \tau)\\
&P(x, t) = \varepsilon^2 P'(\xi, \tau) \simeq \varepsilon^2 P_1(\xi, \tau) = \varepsilon^2\alpha\eta_1(\varepsilon, \tau),\\
&\xi = \varepsilon(x - t), \quad \tau = \varepsilon^3 t
\end{aligned}$$

Now let's look at the third possible scenario (m = 1, p = q = 3, and n = 4). Substituting and equating the coefficients of ε3, we obtain the equation

$$\eta_{1\tau} + u_{1\tau} + u_1 u_{1\xi} + \frac{1}{2}\eta_1 u_{1\xi} + u_1\eta_{1\xi} + \frac{2\alpha_1}{\alpha}\eta_1\eta_{1\xi} + \frac{\delta}{\alpha}\eta_{1\xi\xi\xi\xi} = 0$$

It is possible to derive the equation of evolution by beginning with and working backwards from.

$$\eta_{1\tau} + \left[\left(\frac{5}{4} + \frac{\alpha_1}{\alpha}\right)\eta_1 + \psi(\tau)\right]\eta_{1\xi} + \frac{\delta}{2\alpha}\eta_{1\xi\xi\xi\xi} = -\frac{\psi'(\tau)}{2}$$

Setting ψ ≡ 0, It is possible to derive the nonlinear evolutionary equation of the fourth order by this process.

$$\eta_{1\tau} + \left(\frac{5}{4} + \frac{\alpha_1}{\alpha}\right)\eta_1\eta_{1\xi} + \frac{\delta}{2\alpha}\eta_{1\xi\xi\xi\xi} = 0$$

By utilizing this form, it can be demonstrated that the coefficient of the fourth derivative takes the form.

$$\frac{\delta}{2\alpha} = \frac{h_0 R_0}{4\rho l^3 c_0}\chi$$

Because of this, the amplitude of the wave that is described by is diminished by a factor that is equal to the tube's coefficient of viscosity.

The formulas offer a close estimate of the solution to the first set of equations that were presented.

$$\eta(x, t) = \varepsilon^3 \eta'(\xi, \tau) \simeq \varepsilon^3 \eta_1(\xi, \tau)$$
$$u(x, t) = \varepsilon^3 u'(\xi, \tau) \simeq \varepsilon^3 u_1(\xi, \tau) \simeq \varepsilon^3 \eta_1(\xi, \tau)$$
$$P(x, t) = \varepsilon^3 P'(\xi, \tau) \simeq \varepsilon^3 P_1(\xi, \tau) = \varepsilon^3 \alpha \eta_1(\varepsilon, \tau)$$
$$\xi = \varepsilon(x - t), \quad \tau = \varepsilon^4 t$$

If we take it for granted that the wave process that the Burgers equation represents has a characteristic time of one, then we get thus. $\varepsilon \sim 0.1$For a process represented The characteristic time for a process characterized by the Korteweg-de Vries equation is on the order of ten dimensionless units, while the characteristic time for a process defined by the nonlinear evolutionary equation is on the order of one hundred dimensionless unit.

CONCLUSION

Under the effect of a pulsing pressure gradient, the longitudinal and transverse flow fields of Green-Rivlin fluids in straight tubes with arbitrary cross-sections have been determined. Identical flow patterns can be achieved using a wide variety of forms and values for fluid and flow parameters, as demonstrated above. The structure that incorporates the shape component results in the emergence of several practical effects, the majority of which are of a kinematic nature $H(r, \theta)$ Playing about with the values of the parameters e1 and n enables easy modification of the model so that it can be used to characterize a diverse variety of cross-sectional shapes. Due to the fact that e1's value has a tendency to decrease as n grows, the greatest possible value is relatively low. Similar analyses have resulted in the same conclusions for a number of viscoelastic fluids, including those described by the Phan-Thien-Tanner model, with regard to constant flow in pipes that are not circular. In situations like these, it is possible to find a transverse stream function that has the same shape as the scenario in which e1 and n both have the same function.

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