

Differential Equations Use in the Medical Field

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Abstract: The medical field is increasingly important because it aids in the diagnosis and treatment of ailments, illnesses, and conditions and dissects the molecular basis of every disease. The work that is now in progress has applied Different Equations (DE) to the realm of medicine. DE stands for derivative expression, which denotes a mathematical statement involving one or more derivatives as well as the rate at which unchanging variables fluctuate. Since DE could be directly measured and perceived for systems that underwent changes, it has been used widely in science, engineering, and different quantitative study fields. The use of DEs in the treatment of cancer and dengue fever in the medical area has been highlighted in the paper, along with pathology, cardiology, and dengue fever.

Keywords: Different Equations, Cardiology, Medical Field, Pathology, Variable Order.

1. Introduction

A Differential Equation (DE) has been used in mathematics to express the connection between one or more functions and their derivatives. It uses derivatives to depict the rate of change and functions to represent physical quantities. DE has been used to model behaviour in complicated systems [1]. Some of the types of DE include normal or partial, homogeneous or heterogeneous, linear or non-linear. Simple non-linear partial differential equations (PDEs) have been found to be effective in solving more difficult issues and providing a solution. However, it necessitates spatiotemporal characteristics such as huge length and timescale, making the numerical solution for PDE difficult. It has been computationally interacted in order to resolve the finer characteristics shown in the solution [2]. Similar to this, a suitable deep learning solution for PDE has been found by reducing the PDE residual. By restricting the neural network, PDE residue have been decreased. Thus, by substituting a neural network for the conventional technique, an adequate solution for PDE has been found [3]. Variable Order (VO) Fractional Differential Equations (FDEs) have been used to examine dependent variables such as time and/or space. VO-FDE has been viewed as a more exact and effective approach in real-world phenomena. VO-FDE has shown useful for simulating a wide range of phenomena in a number of disciplines, including engineering, science, and medicine [4]. These days, stochastic differential equations (SDEs) are employed to assess the stability of the system. Despite being widely used, SDE applications may suffer from oscillations as a result of temporal delays. It has been applied in a variety of industries, including biology, medicine, engineering, and more [5]. Engineering and medical science both use fractional differential equations (FDE) constrained with Caputo-Fabrizio (CF) derivatives extensively [6]. Since fractional calculus may be used for a variety of topics and produces better findings, it has grown in favour among scientists. It is thought to be the area of mathematics that is expanding the quickest [7]. According to the proposed paper, numerous studies on the stability of Impulsive Fractional Differential Equations (IFDEs) under finite and infinite delay have been conducted. IFDEs are useful in a variety of industries and offer the right

solutions [8]. DE is therefore thought to be the most intriguing study area in recent years. PDEs may be used in medical imaging, nerve electrical signalling, appropriate oxygenation of healing tissues, and other applications. Therefore, developing numerical solutions for fractional PDEs has become more important [9]. For some complex problems, the fractional PDE may be more accurate than the integer-order PDE. The mathematical modelling of infectious diseases using fraction order differential equations has received increased attention recently in the field of biology. Non-linear ordinary differential equations have been used to depict a variety of illness mathematical models. Such a mathematical model has been discussed in the suggested paper [10].

2. Differential equation use in the medical area

The existing research [11] resolves the DE in Bio fluid Problems using a series-based method known as the DTM (Differential Transform Method). The formulation of the issue in the paper was based on the fact that individuals suffering from renal illnesses were undergoing hemodialyses when the semi-permeable membrane became blocked. Eigenvalues have been determined in the study using different Sherwood membranes, Peclet numbers, axial directions, angles, and radius within the realm of DTM. The results have shown that the angular displacement has been the crucial metric that may be kept as a marker for the purity of the blood in medical words.

2.1. Implementation of differential equation in cardiology

DE have been employed in cardiology to acquire the precise analytical data. The suggested paper reviewed that action potential dynamics in cardiac tissues can be better understood by looking at fractional, time, and PDE at different stages. The current paper's primary goal is to solve the models in an analytical manner. Recent consideration has been given to using a fractional calculus model to clarify biological and health-related processes, particularly the dynamics of cardiac events, based on mathematical predictions, stationary solution stability, and optimal control. It has been discovered by using the current paper that further data and information are needed to evaluate the performance of derived models, including clinical and experimental comparison with formal cardiac models. It may also be possible to explore how the model is implemented in cardiac pacemakers and rhythm control [12].

2.2. Employment of differential equation in pathology

A novel modular system that combines the best elements of continuum mechanics, agent-based modelling, and particle tracking techniques is what the current study's goal is to deliver. The multistage nature of the adaption phenomenon has been managed by the current investigation. The MM has been defined as the well-known PDE, according to a frequent justification of the approach. The paper used a methodology that was entirely dependent on fixed grid and PDE. The explicitation of the fluid-elastic interface interaction at the point where the model has been combined into a pair of coupled PDEs may be one of the method's pillars [13].

2.3. Mathematical analysis of dengue fever

The understanding of the mechanisms regulating dengue and the kinetics of transmission has been greatly aided by mathematical models. Compartmental models, which are controlled by Ordinary Differential Equations (ODE), have been used mathematically to explain the spread of dengue fever and to regulate the interaction between people and mosquitoes. The model has been theoretically examined to determine the related dengue-free equilibrium. The open space spraying of pesticide may be the most significant to

encompass the spread of dengue, according to a mathematical study of single-type control strategies that have been put into action. In order to determine the productive reproduction number, the matrix method was applied. The assumed outcomes have demonstrated that the adaptation of any of the control interventions which has been mentioned in the work could lead to the rejection of the predominant of Dengue among the population. [14].

2.4. Use of differential equation in cancer therapy

Reactive oxygen species (ROS) have proven important secondary messengers in both the development of cancer and the chemotherapy process. A mathematical model has been used to predict chemotherapy responses, improve medication dose procedures, and get a better understanding of the strong frameworks that govern cancer progression. The development of ODE was used in the current study to characterise the dynamics of N species. In order to target cancer cells, the majority of chemotherapeutic drugs around the world have elevated ROS to lethal levels, however long-term ROS exposure has been linked to decreased chemotherapy efficiency [15]. According to the suggested study, mathematical modelling using an infectious illness model will give biologists a new starting point in the field of medicine [16].

Ansarizadeh [17] recently proposed a system model that utilised partial differential equations and demonstrated the nature and behaviour of tumour, immune, and normal cells present in the tumour when employing chemotherapy drugs. This model was discussed in the publication. Additionally, they discovered that the use of biological factors resulted in a reduction in the number of cancer cells that were present in the dangerous and risky area of the tumour.

3. Recent trends applied in medical field with differential equations

Machine learning utilises vast design spaces to find connections, while multiscale modelling predicts system dynamics to find causation. According to recent developments, multiscale modelling and machine learning have improved our understanding of biological, behavioural, and biomedical systems. Based on the scale of interest, multiscale modelling approaches have been split into two categories: partial differential equation-based approaches and approaches based on ordinary differential equations.

In order to trigger the system integral reaction during sickness, development, medicinal intervention, or environmental changes, ordinary differential equations have been used extensively. While partial differential equations are generally employed to examine fundamentally heterogeneous dimensional patterns and variable fields at specific places, such as to research heart contraction, blood flow in the cardiovascular system, etc. [18].

Nowadays, DE and PDE, algorithms, and particular methodologies are integrated in Artificial Neural Networks (ANN) and Deep Learning (DL) in order to use computers to uncover challenging patterns in enormous amounts of data. Deep learning has recently been applied to MRI for the purpose of focussing medical images with the aid of differential and partial differential equations [19].

Backward stochastic differential equations using a single variable, or 1D BSDE, have avoided computing conditional expectation. Future attempts to combine the BSDE approach with the multidimensional case to create Markovian coupled forward-backward SDEs have been many [20].

4. Challenges in adopting differential equation in medical field

Establishing a clinical approach that identifies ROS in cancer in a spatiotemporal manner, within a living (vivo) human body, is the primary downside of employing DE in cancer biology. The present mathematical model, which integrates the knowledge and

experimental methods that have been required for ROS analysis, detection, and clinical translation, has necessitated multidisciplinary collaboration between modelling, experimental, and clinical sectors. New chemotherapeutic designs will be presented to treat cancer in the future, and future generation models will be built to better understand how cancer redox biology functions [15].

The current mathematical model for asthma has qualitatively described the development of the airway smooth muscles over both a short- and long-term time in an inflammatory and normal environment. According to the model, the long-term expansion of airway smooth muscle has been attributed to the speed at which inflammation resolves, the frequency with which inflammatory episodes occur, and the severity of infection. The problem with the current study is that it does not take into account the mechanical interaction between cells and extracellular matrix, which has an impact on the rate of apoptosis, growth, and overall capacity of the airway wall. Another challenge in using mathematical models in biological systems is that mathematical models are very hard and complex in analytical solving and it requires computational model applications in order to obtain the numerical value of the model solution [21].

5. Conclusion

The application of DE is more significant for the diagnosis, analysis, interpretation, and therapy of numerous disorders in the medical field, as is clear from this review. Data collected using mathematical models that include ODE, PDE, linear and non-linear differential equations, homogeneous and non-homogeneous differential equations, and other mathematical constructs are more accurate than data obtained using a classical model. Additionally, it has been found that applying computational models to mathematical findings makes them more effective when used in the medical area. This review paper will serve as a reference for further investigation into the use of differential equations in the realm of medicine.

References

- [1] S. Dwivedi, "Differential Equations and its Applications," 2022.
- [2] Y. Bar-Sinai, S. Hoyer, J. Hickey, and M. P. Brenner, "Learning data-driven discretizations for partial differential equations," *Proceedings of the National Academy of Sciences*, vol. 116, pp. 15344-15349, 2019.
- [3] L. Lu, X. Meng, Z. Mao, and G. E. Karniadakis, "DeepXDE: A deep learning library for solving differential equations," *SIAM Review*, vol. 63, pp. 208-228, 2021.
- [4] H. Sun, A. Chang, Y. Zhang, and W. Chen, "A review on variable-order fractional differential equations: mathematical foundations, physical models, numerical methods and applications," *Fractional Calculus and Applied Analysis*, vol. 22, pp. 27-59, 2019.
- [5] Q. Zhu, "Stability analysis of stochastic delay differential equations with Lévy noise," *Systems & Control Letters*, vol. 118, pp. 62-68, 2018.
- [6] X. Zheng, H. Wang, and H. Fu, "Well-posedness of fractional differential equations with variable-order Caputo-Fabrizio derivative," *Chaos, Solitons & Fractals*, vol. 138, p. 109966, 2020.
- [7] F. Jarad, T. Abdeljawad, and Z. Hammouch, "On a class of ordinary differential equations in the frame of Atangana-Baleanu fractional derivative," *Chaos, Solitons & Fractals*, vol. 117, pp. 16-20, 2018.

- [8] X. Li, J. Shen, and R. Rakkiyappan, "Persistent impulsive effects on stability of functional differential equations with finite or infinite delay," *Applied Mathematics and Computation*, vol.329, pp.14-22, 2018.
- [9] H. Ahmad, A. Akgül, T. A. Khan, P. S. Stanimirović, and Y.-M. Chu, "New perspective on the conventional solutions of the nonlinear time-fractional partial differential equations," *Complexity*, vol.2020, 2020.
- [10] B. Ghanbari, S. Kumar, and R. Kumar, "A study of behaviour for immune and tumor cells in immunogenetic tumour model with non-singular fractional derivative," *Chaos, Solitons & Fractals*, vol.133, p.109619, 2020.
- [11] P.K.Singh and P.Sharma, "A Comparative Study of Fluid Flow in Hemodialyzer using Differential Transform Method," in *Macromolecular Symposia*, 2021, p.2000338.
- [12] S.A.David, C.A.Valentim, and A.Debbouche, "Fractional modeling applied to the dynamics of the action potential in cardiac tissue," *Fractal and Fractional*, vol.6, p.149, 2022.
- [13] M.Garbey, S.Casarin, and S.A.Berceli, "A versatile hybrid agent-based, particle and partial differential equations method to analyze vascular adaptation," *Biomechanics and modeling in mechanobiology*, vol.18, pp.29-44, 2019.
- [14] A.Abidemi, H.O.Fatoyinbo, and J.K.K.Asamoah, "Analysis of dengue fever transmission dynamics with multiple controls: a mathematical approach," in *2020 International Conference on Decision Aid Sciences and Application (DASA)*, 2020, pp.971-978.
- [15] H.Yang, R.M. Villani, H. Wang, M. J.Simpson, M. S.Roberts, M.Tang, et al., "The role of cellular reactive oxygen species in cancer chemotherapy," *Journal of Experimental & Clinical Cancer Research*, vol.37, pp.1-10, 2018.
- [16] S. Kumar, A. Kumar, B. Samet, J. Gómez-Aguilar, and M. Osman, "A chaos study of tumor and effector cells in fractional tumor-immune model for cancer treatment," *Chaos, Solitons & Fractals*, vol.141, p.110321, 2020.
- [17] P. Veerasha, D. Prakasha, and H. M. Baskonus, "New numerical surfaces to the mathematical model of cancer chemotherapy effect in Caputo fractional derivatives," *Chaos: An Interdisciplinary Journal of Nonlinear Science*, vol. 29, p. 013119, 2019.
- [18] M. Alber, A. Buganza Tepole, W. R. Cannon, S. De, S. Dura-Bernal, K. Garikipati, et al., "Integrating machine learning and multiscale modeling—perspectives, challenges, and opportunities in the biological, biomedical, and behavioral sciences," *NPJ digital medicine*, vol.2, pp.1-11, 2019.
- [19] A. S. Lundervold and A. Lundervold, "An overview of deep learning in medical imaging focusing on MRI," *Zeitschrift für Medizinische Physik*, vol.29, pp.102-127, 2019.
- [20] A.Sghir and S.Hadiri, "A new numerical method for 1-D backward stochastic differential equations without using conditional expectations," *Random Operators and Stochastic Equations*, vol.28, pp.79-91, 2020.
- [21] P.Aghasafari, U.George, and R.Pidaparti, "A review of inflammatory mechanisms in airway diseases," *Inflammation research*, vol. 68, pp. 59-74, 2019.