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Common Fixed Point Theorem of Four Maps in a Complete Menger Space

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Abstract: In this paper, I proved a common fixed point theorem of four maps in a complete Menger space using compatible maps of type(A) and continuity.

Keywords: Continuity, Completeness, Compatibility of type(A), Menger Space.

1. Introduction

The notion of Probabilistic Metric Space (or Statistical Metric Space) was initially introduced by Menger [5] in 1944, which is a generalization of metric space. The idea in probabilistic metric space is associated with a distribution function assigned to a pair of points, say (x, y), denoted by $\mathcal{F}_{x,y}(t)$ where t > 0 and is interpreted as the probability that distance between x and y is less than t, whereas in the metric space the distance function is a single positive number. Schweizer and Sklar [7] gave some basic results in this space. Many authors observed that contraction condition in metric space may be exactly translated into PM-Space endowed with minimum norm. A generalization of Banach contraction principle in Menger space is given by Sehgal and Bharucha [8]. Some basic definitions and theorems in Menger space which are used for proving the main result are as follows.

Definition 1.1 [7] "Let $\Delta : [0,1] \times [0,1] \rightarrow [0,1]$ be a mapping. Then Δ is said to be a triangular-norm (briefly, t-norm) if for all $\alpha, \beta, \gamma \in [0,1]$,

(i) $\Delta(\alpha, 1) = \alpha,\ \Delta(0, 0) = 0$;

(ii) $\Delta(\alpha, \beta) = \Delta(\beta, \alpha)$;

(iii) $\Delta(\alpha, \beta) \geq \Delta(\gamma, \delta)$ for $\alpha \geq \gamma,\ \beta \geq \delta$;

(iv) $\Delta(\Delta(\alpha, \beta), \gamma) = \Delta(\alpha, \Delta(\beta, \gamma))$."

Example 1.2 [7] "The four basic t-norms are as follows:

(i) The minimum t-norm: $\Delta_M(\alpha, \beta) = \min\{\alpha, \beta\}$.

(ii) The product t-norm: $\Delta_p(\alpha, \beta) = \alpha\beta$.

(iii) The Lukasiewicz t-norm: $V_L(\alpha, \beta) = \min\{\alpha + \beta - 1, 0\}$.

(iv) The weakest t-norm, the drastic product:

$$\Delta_D(\alpha,\beta) = \begin{cases} \min\{\alpha,\beta\} & \text{if } \max\{\alpha,\beta\} = 1, \\ 0, & \text{otherwise.} \end{cases}$$

We have the following ordering in the above stated norms:

$$\Delta_D < \Delta_L < \Delta_P < \Delta_M."$$

Definition 1.3 [7] "A mapping $\mathcal{F} : \mathbb{R} \to \mathbb{R}^+$ is a distribution function if it is left continuous and non-decreasing with $\inf \mathcal{F}(x) = 0$ and $\sup \mathcal{F}(x) = 1$ for all real x."

We shall denote the set of all distribution functions by $\mathcal{L}$ whereas $\mathcal{H}(t)$ be the Heaviside distribution function defined as

$$\mathcal{H}(t) = \begin{cases} 0, & \text{if } t \leq 0 \\ 1, & \text{if } t > 0. \end{cases}$$

Definition 1.4 [6] "The ordered pair $(\mathcal{K}, \mathcal{F})$ is called a PM space if $\mathcal{K}$ be a non-empty set and $\mathcal{F}: \mathcal{K} \times \mathcal{K} \to \mathcal{L}$ be a mapping satisfying:

(p_1) $\mathcal{F}_{x,y}(t) = 1$ for all $t > 0$, if and only if $x = y$;

(p_2) $\mathcal{F}_{x,y}(0) = 0$;

(p_3) $\mathcal{F}_{x,y}(t) = \mathcal{F}_{y,x}(t)$;

(p_4) $\mathcal{F}_{x,y}(t) = 1$ and $\mathcal{F}_{y,z}(s) = 1$, then $\mathcal{F}_{x,z}(t+s) = 1$,

for all x, y, z in $\mathcal{K}$ and $t, s \geq 0$.

Every metric space can always be realized as a probabilistic metric space by putting the relation $\mathcal{F}_{x,y}(t) = \mathcal{H}\big(t - d(x,y)\big)$ for allx, y in $\mathcal{K}$."

Definition 1.5 [6] "The ordered triplet $(\mathcal{K}, \mathcal{F}, \Delta)$ is called a Menger space if $(\mathcal{K}, \mathcal{F})$is a probabilistic metric space, Δ is a t-norm and satisfies for all x, y, z in $\mathcal{K}$ and $t, s \geq 0$,

(p_5) $\mathcal{F}_{x,z}(t+s) \geq \Delta\big(\mathcal{F}_{x,y}(t), \mathcal{F}_{y,z}(s)\big)$."

Definition 1.6 [6] "A sequence $\{x_n\}$ in a Menger space $(\mathcal{K}, \mathcal{F}, \Delta)$ is said to be:

(i) Cauchy sequence in $\mathcal{K}$ if for every $\epsilon > 0$ and $\lambda > 0$, we can find a positive integer $N_{\epsilon,\lambda}$ satisfying $\mathcal{F}_{x_n,x_m}(\epsilon) > 1 - \lambda$, for all $n, m \geq N_{\epsilon,\lambda}$.

(ii) Convergent at a point $x \in \mathcal{K}$ if for every $\epsilon > 0$ and $\lambda > 0$, there exists a positive integer $N_{\epsilon,\lambda}$ satisfying $\mathcal{F}_{x_n,x}(\epsilon) > 1 - \lambda$, for all $n \geq N_{\epsilon,\lambda}$."

The space $\mathcal{K}$is said to becomplete if every Cauchy sequence is convergent in $\mathcal{K}$.

Definition 1.7 [6] "Let S and T be two self-mappings of a Menger space $(\mathcal{K}, \mathcal{F}, \Delta)$. Then S and T are said to be compatible if $\lim_{n\to\infty} \mathcal{F}_{STx_n,TSx_n}(t) = 1$ for all $t > 0$ where $\{x_n\}$ is a sequence in $\mathcal{K}$ satisfying

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = u, \text{ where } u \in \mathcal{K}."$$

Definition 1.8 [10] "Two self-mappings A and S of a non-empty set $\mathcal{K}$ are said to be weakly compatible (or coincidentally commuting) if they commute at their coincidence points i.e. if Az = Sz for some z ∈ $\mathcal{K}$, then ASz = SAz."

Theorem 1.9 [10] "If two self-mappings A and S of a Menger space $(\mathcal{K}, \mathcal{F}, \Delta)$ are compatible, then they are weakly compatible."

Definition 1.10 [2] "Let S and T be two self-mappings of a Menger space $(\mathcal{K}, \mathcal{F}, \Delta)$. Then S and T are said to be compatible of type (A) if we can find a sequence $\{x_n\}$ in $\mathcal{K}$ satisfying $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = u$, where $u \in \mathcal{K}$ and $\lim_{n\to\infty} \mathcal{F}_{STx_n, TTx_n}(t) = 1$ and $\lim_{n\to\infty} \mathcal{F}_{TSx_n, SSx_n}(t) = 1$ for all $t > 0$."

Definition 1.11 [2] "Let S and T be two self-mappings of a Menger space $(\mathcal{K}, \mathcal{F}, \Delta)$. Then S and T are said to be compatible of type (β) if we can find a sequence $\{x_n\}$ in $\mathcal{K}$ satisfying $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = u$, where $u \in \mathcal{K}$ and $\lim_{n\to\infty} \mathcal{F}_{SSx_n, TTx_n}(t) = 1$ for all $t > 0$."

Definition 1.12 [1] "Two self-maps S and T of a set $\mathcal{K}$ are occasionally weakly compatible maps (shortly owc) if and only if we can find a point x in $\mathcal{K}$ satisfying Sx = Tx and STx = TSx."

Theorem 1.13 [3] "Let S and T be compatible maps of type (A) in a Menger space $(\mathcal{K}, \mathcal{F}, \Delta)$ and $Sx_n, Tx_n \to u$ for some u in $\mathcal{K}$. Then

(i) $TSx_n \to Su$ if S is continuous.

(ii) STu = TSu and Su = Tu if S and T are continuous."

Theorem 1.14 [11] "Let $(\mathcal{K}, \mathcal{F}, \Delta)$ be a Menger space. If there exists a constant $k \in (0, 1)$ such that $\mathcal{F}_{x_{n+1}, x_n}(kt) \geq \mathcal{F}_{x_n, x_{n-1}}(t)$ for all x, y in $\mathcal{K}$ and $t > 0$, then $\{x_n\}$ is a Cauchy sequence in $\mathcal{K}$."

Theorem 1.15 [10] "Let $(\mathcal{K}, \mathcal{F}, \Delta)$ be a Menger space. If there exists a constant $k \in (0, 1)$ such that $\mathcal{F}_{x,y}(kt) \geq \mathcal{F}_{x,y}(t)$ for all x, y in $\mathcal{K}$ and $t > 0$, then x = y."

Theorem 1.16 [10] "In a Menger space $(\mathcal{K}, \mathcal{F}, \Delta)$ if $\Delta(a, a) \geq a$, for all $a \in [0, 1]$, then $\Delta(a,b) = \text{Min}\{a, b\}$ for a, b ∈ [0, 1]."

2. Main Result

Theorem 2.1 Let A, S, L and M be self-maps on a complete Mengerspace $(\mathcal{K}, \mathcal{F}, \Delta)$ with $\Delta(a, a) \geq a$, for all $a \in [0, 1]$ and satisfying :

(i) $L(\mathcal{K}) \subseteq S(\mathcal{K})$, $M(\mathcal{K}) \subseteq A(\mathcal{K})$;

(ii) the pairs (L, A) and (M, S) are compatible maps of type (A);

(iii) either A or L is continuous;

(iv) there exists $k \in (0, 1)$ such that

$\mathcal{F}_{Lx,My}(kt) \geq \text{Min}\,\{\mathcal{F}_{Ax,Lx}(t), \mathcal{F}_{Sy,My}(t), \mathcal{F}_{Sy,Lx}(1-\alpha q)t,$

$\mathcal{F}_{Ax,My}\big((1+\alpha q)t\big), \mathcal{F}_{Ax,Sy}(t)\},$

for all $x, y \in \mathcal{K}$, $\alpha \in [0,1]$, $q \in (0,1)$ and $t > 0$.

Then A, S, L and M have a unique common fixed point in $\mathcal{K}$.

Proof. Let $x_0 \in \mathcal{K}$. From condition (i) there exists $x_1, x_2 \in \mathcal{K}$ such that $Lx_0 = Sx_1 = y_0$ and $Mx_1 = Ax_2 = y_1$. Inductively, we can make sequences $\{x_n\}$ and $\{y_n\}$ in $\mathcal{K}$ such that $Lx_{2n} = Sx_{2n+1} = y_{2n}$ and $Mx_{2n+1} = Ax_{2n+2} = y_{2n+1}$ for $n = 0, 1, 2, \ldots$.

Taking $x = x_{2n}$ and $y = x_{2n+1}$ in (iv), we get

$\mathcal{F}_{Lx_{2n},Mx_{2n+1}}(kt) \geq \text{Min}\,\{\mathcal{F}_{Ax_{2n},Lx_{2n}}(t), \mathcal{F}_{Sx_{2n+1},Mx_{2n+1}}(t), \mathcal{F}_{Sx_{2n+1},Lx_{2n}}((1-\alpha q)t),$

$\mathcal{F}_{Ax_{2n},Mx_{2n+1}}\big((1+\alpha q)t\big), \mathcal{F}_{Ax_{2n},Sx_{2n+1}}(t)\},$

that is, $\mathcal{F}_{y_{2n},y_{2n+1}}(kt) \geq \text{Min}\{\mathcal{F}_{y_{2n-1},y_{2n}}(t), \mathcal{F}_{y_{2n},y_{2n+1}}(t), \mathcal{F}_{y_{2n-1},y_{2n+1}}\big((1+\alpha q)t\big),$

$\mathcal{F}_{y_{2n-1},y_{2n}}(t)\}$

$\geq \text{Min}\,\{\mathcal{F}_{y_{2n-1},y_{2n}}(t), \mathcal{F}_{y_{2n},y_{2n+1}}(t), \mathcal{F}_{y_{2n-1},y_{2n}}(t), \mathcal{F}_{y_{2n},y_{2n+1}}(\alpha q t)\}$

$\geq \text{Min}\,\{\mathcal{F}_{y_{2n-1},y_{2n}}(t), \mathcal{F}_{y_{2n},y_{2n+1}}(t), \mathcal{F}_{y_{2n},y_{2n+1}}(\alpha q t)\}.$

As t-norm is continuous, letting $\alpha q \to 1$ we get

$\mathcal{F}_{y_{2n},y_{2n+1}}(kt) \geq \text{Min}\,\{\mathcal{F}_{y_{2n-1},y_{2n}}(t), \mathcal{F}_{y_{2n},y_{2n+1}}(t), \mathcal{F}_{y_{2n},y_{2n+1}}(t)\}$

$= \text{Min}\,\{\mathcal{F}_{y_{2n-1},y_{2n}}(t), \mathcal{F}_{y_{2n},y_{2n+1}}(t)\}.$

Hence, $\mathcal{F}_{y_{2n},y_{2n+1}}(kt) \geq \text{Min}\,\{\mathcal{F}_{y_{2n-1},y_{2n}}(t), \mathcal{F}_{y_{2n},y_{2n+1}}(t)\}.$

Similarly, $\mathcal{F}_{y_{2n+1},y_{2n+2}}(kt) \geq \text{Min}\,\{\mathcal{F}_{y_{2n},y_{2n+1}}(t), \mathcal{F}_{y_{2n+1},y_{2n+2}}(t)\}.$

Therefore for all n we have

$$\mathcal{F}_{y_n,y_{n+1}}(kt) \geq \text{Min}\,\{\mathcal{F}_{y_{n-1},y_n}(t), \mathcal{F}_{y_n,y_{n+1}}(t)\}.$$

Consequently,

$$\mathcal{F}_{y_n,y_{n+1}}(t) \geq \text{Min}\,\{\mathcal{F}_{y_{n-1},y_n}(k^{-1}t), \mathcal{F}_{y_n,y_{n+1}}(k^{-1}t)\}.$$

Applying the above inequality repeatedly, we get

$$\mathcal{F}_{y_n,y_{n+1}}(t) \geq \text{Min}\,\{\mathcal{F}_{y_{n-1},y_n}(k^{-1}t), \mathcal{F}_{y_n,y_{n+1}}(k^{-m}t)\}.$$

Since $\mathcal{F}_{y_n,y_{n+1}}(k^{-m}t) \to 1$ as $m \to \infty$, it follows that

$$\mathcal{F}_{y_n,y_{n+1}}(kt) \geq \{\mathcal{F}_{y_{n-1},y_n}(t)\} \text{ for all } n \in N \text{ and for all } x > 0.$$

Therefore, by Theorem 1.14, $\{y_n\}$ is a Cauchy sequence in $\mathcal{K}$, which is complete.

Hence $\{y_n\} \to z \in \mathcal{K}$. Also its sub-sequences,

$$\{Lx_{2n}\} \to z, \ \{Sx_{2n+1}\} \to z, \qquad (2.1)$$

$$\{Mx_{2n+1}\} \to z, \{Ax_{2n}\} \to z. \quad (2.2)$$

Case I. When A is continuous, $(A)^2x_{2n} \to Az$ and $ALx_{2n} \to Az$. Also L and A are compatible maps of type (A), we have $LAx_{2n} \to Az$.

Take $x = Ax_{2n}$ and $y = x_{2n+1}$ with $\alpha = 0$ in (iv), we get

$\mathcal{F}_{LA\,x_{2n}, Mx_{2n+1}}(kt) \geq \text{Min}\,\{\mathcal{F}_{A^2x_{2n}, LA\,x_{2n}}(t), \mathcal{F}_{Sx_{2n+1}, Mx_{2n+1}}(t),$

$\quad \mathcal{F}_{Sx_{2n+1}, LAx_{2n}}(t), \mathcal{F}_{A^2x_{2n}, Mx_{2n+1}}(t), \mathcal{F}_{A^2x_{2n}, Sx_{2n+1}}(t)\}.$

As $n \to \infty$, we have

$\mathcal{F}_{Az,z}(kt) \geq \text{Min}\,\{\mathcal{F}_{Az,Az}(t), \mathcal{F}_{z,z}(t), \mathcal{F}_{z,Az}(t), \mathcal{F}_{Az,z}(t), \mathcal{F}_{Az,z}(t), \mathcal{F}_{Az,z}(t)\},$

that is $\mathcal{F}_{Az,z}(kt) \geq \mathcal{F}_{Az,z}(t)$.

Using Theorem 1.15, we obtain

$$Az = z \quad (2.3)$$

Taking $x = z$ and $y = x_{2n+1}$ with $\alpha = 0$ in (iv), we get

$\mathcal{F}_{Lz, Mx_{2n+1}}(kt) \geq \text{Min}\,\{\mathcal{F}_{Az,Lz}(t), \mathcal{F}_{Sx_{2n+1}, Mx_{2n+1}}(t), \mathcal{F}_{Sx_{2n+1}, Lz}(t),$

$\mathcal{F}_{Az, Mx_{2n+1}}(t), \mathcal{F}_{Az, Sx_{2n+1}}(t)\}.$

Taking $n \to \infty$, we get

$\mathcal{F}_{Lz,z}(kt) \geq \text{Min}\,\{\mathcal{F}_{z,Lz}(t), \mathcal{F}_{z,z}(t), \mathcal{F}_{z,Lz}(t), \mathcal{F}_{Lz,z}(t), \mathcal{F}_{Lz,z}(t)\},$

$\qquad = \mathcal{F}_{Lz,z}(t).$

By Theorem 1.15, we get $Lz = z$. So, $z = Lz = Az$.

Since $L(\mathcal{K}) \subseteq S(\mathcal{K})$, there exists $v \in \mathcal{K}$ such that $z = Lz = Sv$.

Taking $x = x_{2n}$ and $y = v$ with $\alpha = 0$ in (iv), we get

$\mathcal{F}_{Lx_{2n}, Mv}(kt) \geq \text{Min}\,\{\mathcal{F}_{Ax_{2n}, Lx_{2n}}(t), \mathcal{F}_{Sv,Mv}(t), \mathcal{F}_{Sv, Lx_{2n}}(t),$

$\mathcal{F}_{Ax_{2n}, Mv}(t), \mathcal{F}_{Ax_{2n}, Sv}(t)\}.$

Letting $n \to \infty$ and using (2.2), we have

$\mathcal{F}_{z,Mv}(kt) \geq \text{Min}\,\{\mathcal{F}_{z,z}(t), \mathcal{F}_{z,Mv}(t), \mathcal{F}_{z,z}(t), \mathcal{F}_{z,Mv}(t), \mathcal{F}_{z,z}(t)\},$

$\qquad = \mathcal{F}_{z,Mv}(t).$

Therefore, by Theorem 1.15, $Mv = z$ and so $z = Mv = Sv$.

Thus, v is a coincidence point of M and S. Since M and S are compatible maps of type (A), we have $MSv = SMv$. Thus, $Sz = Mz$.

By taking $x = x_{2n}$ and $y = z$ with $\alpha = 0$ in (iv), we get

$\mathcal{F}_{Lx_{2n}, Mz}(kt) \geq \text{Min}\,\{\mathcal{F}_{Ax_{2n}, Lx_{2n}}(t), \mathcal{F}_{Sz,Mz}(t), \mathcal{F}_{Sz, Lx_{2n}}(t),$

$\mathcal{F}_{Ax_{2n}, Mz}(t), \mathcal{F}_{Ax_{2n}, Sz}(t)\}.$

Taking $n \to \infty$, and using equation (2.1), we get

$\mathcal{F}_{z,Mz}(kt) \geq \text{Min}\ \{\mathcal{F}_{z,z}(t), \mathcal{F}_{Mz,z}(t), \mathcal{F}_{Mz,z}(t), \mathcal{F}_{z,Mz}(t), \mathcal{F}_{z,Mz}(t)\},$

$= \mathcal{F}_{z,Mz}(t).$

Therefore, by Theorem 1.15, $Mz = z$ and so $z = Az = Lz = Mz = Sz$.

i.e. z is a common fixed point of four maps.

Case II. When L is continuous, $L^2x_{2n} \rightarrow Lz$ and $LAx_{2n} \rightarrow Lz$. Also L and A are compatible maps of type (A), we have $ALx_{2n} \rightarrow Lz$.

Taking $x = Lx_{2n}$ and $y = x_{2n+1}$ with $\alpha = 0$ in (iv), we get

$\mathcal{F}_{LLx_{2n},Mx_{2n+1}}(kt) \geq \text{Min}\{\mathcal{F}_{ALx_{2n},LLx_{2n}}(t),\ \mathcal{F}_{Sx_{2n+1},Mx_{2n+1}}(t),$

$\mathcal{F}_{Sx_{2n+1},LLx_{2n}}(t),\ \mathcal{F}_{ALx_{2n},Mx_{2n+1}}(t),\ \mathcal{F}_{ALx_{2n},Sx_{2n+1}}(t)\}.$

Taking $n \rightarrow \infty$, we get

$\mathcal{F}_{Lz,z}(kt) \geq \text{Min}\ \{\mathcal{F}_{Lz,Lz}(t), \mathcal{F}_{z,z}(t), \mathcal{F}_{z,Lz}(t), \mathcal{F}_{Lz,z}(t), \mathcal{F}_{Lz,z}(t)\}$

$= \mathcal{F}_{Lz,z}(t).$

Therefore, by Theorem 1.15, $Lz = z$.

Similarly, we get $Mz = Sz = z$.

By the hypothesis of the theorem $M(\mathcal{K}) \subseteq A(\mathcal{K})$, there exists $w \in \mathcal{K}$ such that

$z = Mz = Aw$. Taking $x = w$, $y = x_{2n+1}$ with $\alpha = 0$ in (iv), we get

$\mathcal{F}_{Lw,Mx_{2n+1}}(kt) \geq \text{Min}\ \{\mathcal{F}_{Aw,Lw}(t), \mathcal{F}_{Sx_{2n+1},Mx_{2n+1}}(t),$

$\mathcal{F}_{Sx_{2n+1},Lw}(t), \mathcal{F}_{Aw,Mx_{2n+1}}(t), \mathcal{F}_{Aw,Sx_{2n+1}}(t)\}.$

Taking $n \rightarrow \infty$, we get

$\mathcal{F}_{Lw,z}(kt) \geq \text{Min}\ \{\mathcal{F}_{z,Lw}(t), \mathcal{F}_{z,z}(t), \mathcal{F}_{z,Lw}(t), \mathcal{F}_{Lz,z}(t), \mathcal{F}_{z,z}(t)\},$

$= \mathcal{F}_{z,Lw}(t).$

Therefore, by Theorem 1.15, $Lw = z = Aw$, and since L and A are compatible maps of type (A), we get $Lz = Az$. Therefore, $Az = Sz = Lz = Mz = z$ and hence z is a common fixed point of four maps.

For uniqueness, let $z_1 (z_1 \neq z)$ be another common fixed point of the given self-maps. Then $z_1 = Az_1 = Lz_1 = Mz_1 = Sz_1$.

By taking $x = z$ and $y = z_1$ with $\alpha = 0$ in (iv), we get

$\mathcal{F}_{Lz,Mz_1}(kt) \geq \text{Min}\ \{\mathcal{F}_{Az,Lz}(t), \mathcal{F}_{Sz_1,Mz_1}(t), \mathcal{F}_{Sz_1,Lz}(t),$

$\mathcal{F}_{Az,Mz_1}(t), \mathcal{F}_{Az,Sz_1}(t)\},$

that is,

$\mathcal{F}_{z,z_1}(kt) \geq \text{Min}\ \{\mathcal{F}_{z,z_1}(t), \mathcal{F}_{z,z}(t), \mathcal{F}_{z_1,z_1}(t), \mathcal{F}_{z,z_1}(t), \mathcal{F}_{z_1,z}(t)\}$

which gives

$\mathcal{F}_{z,z_1}(kt) \geq \mathcal{F}_{z,z_1}(t)$. Therefore, by Theorem 1.15 $z_1 = z$.

Hence, z is a unique common fixed point of self-maps A, S, L and M.

This completes the proof.

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