International Journal of Engineering, Science and Mathematics
Vol. 11 Issue 09, September 2022,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijmra.us, Email: editorijmie@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

A NEW LUCAS OPERATIONAL MATRIX OF FRACTIONAL DERIVATIVES PROVIDES SPECTRAL SOLUTIONS FOR FRACTIONAL DIFFERENTIAL EQUATIONS

Shipra, Department of Mathematics, Pt. NRS Govt. College, Rohtak.

Emailid:shipra.kadyan2502@gmail.com

Abstract

This exposition is partitioned into two sections. The partial subordinate of the functional grid of the Lucas polynomials is processed in the primary area. A ghastly procedure is developed and exhibited utilizing this network to address a couple of partial request starting worth issues. The subsequent segment centres around a three-sided lattice whose parts are joined likewise to frame the main request subordinate of the old style Fibonacci numbers at x=1, and whose coefficients are relating in the development of the subsidiary of Fibonacci polynomials.

Keyword: Matrix, Lucas polynomial's, fractional differential equation, Fractional Calculus.

1. Introduction

Partial math is a fundamental part of numerical examination. Partial math has been the subject of various hypothetical and applied investigations. The utilization of the partial math with integrals and subordinates of any request is notable (perplexing or genuine). Various enterprises use partial analytics in different ways. For instance, fragmentary differential conditions can be utilized to demonstrate a large number of issues in mechanics (the hypothesis of viscos-elasticity), electrical designing (the transmission of ultrasound waves), (bio-) science (the displaying of proteins and polymers), medication (the displaying of human tissue under the mechanical burdens), and different fields. It is essential to foster mathematical answers for these issues utilizing various strategies in light of the fact that scientific answers for fragmentary differential conditions are not accessible 100% of the time. Various scholastics are exceptionally keen on mathematically analyzing different fragmentary differential condition structures around here. Common differential conditions with fragmentary request subsidiaries are utilized to reproduce a wide assortment of frameworks, and viscos-elastic damping is one of these pivotal designing applications. [1]. The straight and nonlinear normal and fragmentary differential conditions can be settled utilizing the subordinate functional networks. [2]. Finding a fragmentary request subordinate of the Fibonacci polynomial functional framework means a lot to the creator of [3]. The LUCAS POLYNOMIALs and their speculations are innately intriguing. The Lucas Polynomials arrangement can be changed over into the notable Lucas numbers grouping by setting x = 1. The notable Lucas numbers and the brilliant proportion have many purposes in different disciplines, including chart hypothesis, measurements, software engineering, material science, and science. Just a little part of these applications are canvassed in Koshy's significant work. There is no sign of any genuine exploration of Lucas polynomials and the connected numbers in the writing, regardless of the way that various scholastic authors have contributed essentially to the hypothetical conversation of Lucas polynomials. This gives us a solid motivation to examine these polynomials while talking about the arrangement of partial differential conditions.

This paper's primary goal is twofold:

- Creating new relations of Lucas polynomials to deliver another functional network for partial subordinates.
- Illustrating and incorporating two techniques for the utilization of tau, as well as utilizing collocation ghastly strategies to tackle multi-term partial request differential conditions.

Let the accompanying partial differential condition:

$$\begin{cases} D^{\alpha}x(y) + x^{(q)}(y) + x(y) = f(y) \\ x^{(r)}(0) = c_r \end{cases} \quad (1)$$

Where, $r = 0,1,2,\ldots,n-1, l = 0,1,2,\ldots,n, n-1 < \alpha \leq n$

1.1Preliminaries and notation

1.1.1 Fractional Calculus Properties

In this education, In the Caputo meaning, the fractional derivative of x(t) is unspoken [4]. The subsequent attribute for $m - 1 < \alpha < m$ for the caputo derivative.

$$D^{\alpha}t^{q} = \begin{cases} \frac{q!}{\Gamma(q-\alpha+1)} t^{k-\alpha}, q \geq \alpha \\ 0, \quad q < \alpha \end{cases} \quad (2)$$

1.1.2 Some relevant properties of Lucas polynomials

The Lucas polynomials are characterized by the repeat association that follows,

$$K_{m+2}(y) = yK_{m+1}(y) + K_m(y), m \geq 0$$

With starting conditions $K_0(y) = 2, K_1(y) = y$, and they container be produced[5] by the subsequent procedure illustration:

$$M_m(y) = \Sigma_{L=0}^{\left[\frac{m}{2}\right]} \left(\frac{m}{m-l}\right) \binom{m-l}{l} y^{m-2l} (3)$$

Theorem1.1.2.1 The subsequent inversion formula to (3) in which the polynomials y_i is articulated in expressions of Lucas polynomials is,

$$y^m = \begin{cases} \Sigma_{j=0}^{\left[\frac{m}{2}\right]} (-1)^i \binom{m}{j} K_{m-2j}(y), & \text{if } m \text{ is odd} \\ \Sigma_{j=0}^{\left[\frac{m-1}{2}\right]} (-1)^i (j) K_{m-2j}(y) + (-1)^{\frac{m}{2}} \binom{m-1}{\frac{m}{2}} K_0(y), & \text{if } m \text{ is even} \end{cases}$$

(4)

Proof

CaseI: When$n$ is odd.

Clearly, outcomes hold for m=1. Let the outcome holds pointed atn, so,

$$y^m = \Sigma_{j=0}^{\left[\frac{m}{2}\right]} (-1)^j \binom{m}{j} K_{m-2j}(y)$$

Take a note, when there is an expansion in the two sides of the equality by y and by the depiction of the Lucas polynomials produces,

$$y^{m+1} = \Sigma_{j=0}^{\left[\frac{m}{2}\right]} (-1)^j \binom{m}{j} K_{m-2j+1}(y) - \Sigma_{j=0}^{\left[\frac{m}{2}\right]} (-1)^j \binom{m}{j} K_{m-2j-1}(y)$$

These two summations will eventually be joined into one. Note that we have accomplished this by raising the records of the second summation by one.

$$y^{m+1} = \Sigma_{j=0}^{\left[\frac{m}{2}\right]} (-1)^j \binom{m}{j} K_{m-2j+1}(y) - \Sigma_{j=1}^{\left[\frac{m}{2}\right]+1} (-1)^{j-1} \binom{m}{j-1} K_{m-2j+1}(y) =$$

$$\Sigma_{j=0}^{\left[\frac{m}{2}\right]} (-1)^j \binom{m}{j} K_{m-2j+1}(y) - \Sigma_{j=1}^{\left[\frac{m}{2}\right]+1} (-1)^{j-1} \binom{m}{m-1} K_{m-2j+1}(y) \qquad (4^*)$$

For our theory to be right, it does to exhibition that,

$$y^{m+1} = \Sigma_{j=0}^{\left[\frac{m}{2}\right]} (-1)^j \binom{m+1}{j} M_{m-2j+1}(y) + (-1)^{\frac{m+1}{2}} \binom{m}{\frac{m+1}{2}} K_0(y)$$

To exhibit this, we should from (4*),

$$y^{m+1} = K_{m+1} + \Sigma_{j=1}^{\left[\frac{m}{2}\right]} (-1)^j \binom{m}{j} K_{m-2j+1}(y) - \Sigma_{j=1}^{\left[\frac{m}{2}\right]} (-1)^{j-1} \binom{m}{j-1} K_{m-2j+1}(y) -$$

$$(-1)^{\frac{m}{2}} \binom{m}{\frac{m-1}{2}} K_0(y) \qquad (5^*)$$

Currently, the coefficient of $K_{m-2j+1}(y) = \binom{m}{j} + \binom{m}{j-1} = \binom{m+1}{j}$, consequently equation (5*) develops

$$y^{m+1} = \Sigma_{j=0}^{\left[\frac{m}{2}\right]} (-1)^j \binom{m+1}{j} K_{m-2j+1}(y) + (-1)^{\frac{m+1}{2}} \binom{m}{\frac{m+1}{2}} K_0(y)$$

We get as per our assumptions.

Case II When m is taken to be even,

Consider the consequence holds for ‘m’ i.e.,

$$y^m = \Sigma_{j=0}^{\left[\frac{m-1}{2}\right]}(-1)^j \binom{m}{j} K_{m-2j}(y) + (-1)^{\frac{m}{2}} \binom{m-1}{\frac{m}{2}} K_0(y)$$

By increasing the two sides of the fairness by y and applying the Lucas polynomials' definition, we acquire

$$y^{m+1} = \Sigma_{j=0}^{\left[\frac{m-1}{2}\right]}(-1)^j \binom{m}{j} yK_{m-2j}(y) + (-1)^{\frac{m}{2}} \binom{m-1}{\frac{m}{2}} yK_0(y)$$

$$= \Sigma_{j=0}^{\left[\frac{m-1}{2}\right]}(-1)^j \binom{m}{j} K_{m-2j+1}(y) - \Sigma_{j=0}^{\left[\frac{m-1}{2}\right]}(-1)^j \binom{m}{j} K_{m-2j+1}(y) \ +(-1)^{\frac{m}{2}} \binom{m-1}{\frac{m}{2}} K_1(y)$$

These two summations will eventually be joined into one. Note that we have accomplished this by raising the files of the second summation by one.

$$y^{m+1} = \Sigma_{j=0}^{\left[\frac{m-1}{2}\right]}(-1)^j \binom{m}{j} K_{m-2j+1}(y) - \Sigma_{j=0}^{\left[\frac{m-1}{2}\right]}(-1)^{j-1} \binom{m}{j-1} K_{m-2j+1}(y) \ +(-1)^{\frac{m}{2}} \binom{m-1}{\frac{m}{2}} K_1(y)$$

Since m is even, soup steps condition creates

$$y^{m+1} = \Sigma_{j=0}^{\left[\frac{m-1}{2}\right]}(-1)^j \binom{m}{j} K_{m-2j+1}(y) - \Sigma_{j=0}^{\left[\frac{m-1}{2}\right]}(-1)^{j-1} \binom{m}{j-1} K_{m-2j+1}(y) \ +(-1)^{\frac{m}{2}} \binom{m-1}{\frac{m}{2}} K_1(y)$$

$$= \Sigma_{j=0}^{\left[\frac{m-1}{2}\right]}(-1)^j \binom{m+1}{j} k_{m-2j+1}(y) + (-1)^{\left[\frac{m}{2}\right]}\left(\binom{m}{\left[\frac{m}{2}\right]-1} + \binom{m-1}{\left[\frac{m}{2}\right]}\right)$$

$$= \Sigma_{j=0}^{\left[\frac{m}{2}\right]}(-1)^j \binom{m+1}{j} K_{m-2j+1}(y)$$

Which is what is desired.

1.2Operational matrix of Derivative

Supposing the equation first has an incessant function answer that will be articulated in the Lucas polynomials $x(y) = \Sigma_{l=1}^{\infty} a_l K_l(y)$, and let $x_m(y)$ be an estimate to $x(y)$,that is

$$x(y) \sim x_m(y) = \Sigma_{l=1}^{m+1} a_l K_l(y) = A^T K(y),$$

There$A^T = [a_1 a_2 \dots a_{m+1}], K(y) = [K_1(y) K_2(y) \dots K_{m+1}(y)]^T$

1.2.1 Lucas working matrix of derivative

The r[th] order derivative of first equation can be rewritten as [6]

$$x^{(r)}(y) = A^T D^r K(y), D = [d_{ji}] = \begin{cases} j \sin\dfrac{(i-j)\pi}{2}, & i > j \\ 0, & i \le j \end{cases}, r = 0,1,2, \dots m \qquad (5)$$

Where$D$ is $m \times m$working matrix for derivative,

The Caputo derivative (CD) of the vector F can be stated by

$$D^{\alpha} K(y) = B^{(\alpha)} K(y) \qquad (6)$$

Where $B^{(\alpha)}$ is the $(m+1) \times (m+1)$operative matrix of fractional derivative,

For this unit, we derive matrix $B^{(\alpha)}$.

By$(2) and$ (4) for $j = 0,1,2,, \dots, m$

$$D^{\alpha} K_j(y) = \Sigma_{i=0}^{\left[\frac{j}{2}\right]} \left(\frac{j}{j-i}\right) \binom{j-i}{j} D^{\alpha} y^{j-2i}$$

$$= \begin{cases} \Sigma_{i=0}^{\left[\frac{j}{2}\right]} \left(\frac{j}{j-i}\right) \binom{j-i}{j} \dfrac{\Gamma(j+1-2i)}{\Gamma(j+1-2i-\alpha)} x^{j-2i-\alpha}, & j-2i \ge \alpha \\ 0, & j-2i < \alpha \end{cases}$$

If $j - 2j \ge \alpha$, and $j - 2i$ is odd, then by (4),we obligate.

$$D^{\alpha} K_i(x) = y^{-\alpha} \Sigma_{k=0}^{\left[\frac{j}{2}\right]} \left(\frac{j}{j-i}\right) \binom{j-i}{i} \frac{\Gamma(j+1-2i)}{\Gamma(j+1-2i-\alpha)} x^{j-2i} = y^{-\alpha} \Sigma_{j=0}^{\left[\frac{j}{2}\right]} B_{jil} K_{j-2i-2l}$$

Somewhere,

$$A_{jil} = \Sigma_{l=0}^{\left[\frac{j-2i}{2}\right]} (-1)^l \binom{i-2i}{l}, B_{jil} = \left(\frac{j}{j-i}\right) \binom{j-i}{k} \frac{\Gamma(j+1-2i)}{\Gamma(j+1-2i-\alpha)} A_{jil}$$

If $j - 2i \ge \alpha$, and $j - 2i$ is even, thenby (4),we have

$$D^{\alpha} K_j(y) = y^{-\alpha} \Sigma_{k=0}^{\left[\frac{j}{2}\right]} \left(\frac{j}{j-i}\right) \binom{j-i}{i} \frac{\Gamma(j+1-2i)}{\Gamma(j+1-2i-\alpha)} y^{j-2i}$$

$$= y^{-\alpha} \Sigma_{i=0}^{\left[\frac{j}{2}\right]} B_{jil} K_{j-2i-2l}$$

$$+ y^{-\alpha} \Sigma_{k=0}^{\left[\frac{j}{2}\right]} \left(\frac{j}{j-i}\right) \binom{j-i}{i} \frac{\Gamma(j+1-2i)}{\Gamma(j+1-2i-\alpha)} (-1)^{\frac{i-2i}{2}} \binom{j-2i-1}{\frac{j-2i}{2}} K_0(x)$$

Where,

$$A'_{jil} = \Sigma_{l=0}^{\left[\frac{j-2i-1}{2}\right]} (-1)^l \binom{j-2i}{l}, B'_{jil} = \left(\frac{j}{j-i}\right) (i) \frac{\Gamma(j+1-2i)}{\Gamma(j+1-2i-\alpha)} A'_{jil}$$

1.3Numerical Method

We cover the utilization of the fragmentary Lucas Polynomials functional network to determine partial differential conditions exhaustively in this meeting. This part offers a few mathematical outcomes upheld by correlations with different information from the writing to guarantee the viability, relevance, and higher exactness of the two proposed strategies in this examination. The trials that follow are assessed for their most noteworthy standard mistake, especially

$$\begin{cases} B^{(\alpha)}K(y) + A^T D^l K(y) + A^T K(y) = f(y) \\ \quad A^T D^r K(0) = c_r, r = 0,1, \ldots n-1 \end{cases} \quad (7)$$

Example 1: Let the following inhomogeneous Bagley-Trovik equation:

$$\begin{cases} D^2\mu(y) + D^{\left(\frac{3}{2}\right)}\mu(y) + \mu(y) = f(y), y\epsilon(0,1), \\ \quad Subject\ to\ condition \\ \quad \mu(0) = 1 \\ \quad \mu'(0) = \alpha \end{cases} \quad 7(A)$$

Some place f (y) is selected. So, the specific clarification of higher up equationis u(y) = sin(αy).

We relate αμLucas framework strategy (TLMM) for divergent upsides of α and M. We show in Table 1, anassessment

Table 1: Comparison between TLMM and collocation Lucas matrix method (CSM)

	α=1		α= 4π	
N	TLMM	CSM [46]	TLMM	CSM [46]
4	1.0 .10−4	3.4 .10−4	2.1 .10−2	3.9 .100
8	2.3 .10−7	4.3 .10−7	5.2 .10−6	4.7 .10−1
16	7.5 .10−11	1.8 .10−8	3.9 .10−10	3.5 .10−5
32	3.7 .10−13	7.1 .10−10	6.1 .10−13	1.4 .10−6

Amid the consequences gained by the submission of TLMM with those attained by the Chebyshev Spectral Method (CSM). The showed results in this table demonstration that our algorithm stretches a small ererror in almost all cases.

Example 2.*Let the following linear fractional IVP:*

$D^p\mu(y) + \mu^2(y) = 1, y\epsilon(0,1), p\epsilon(0,1]$

$|e_{N(y)}| \leq 2e^{\mu}\cosh(2A)\left(1 - \frac{\Gamma(N+1,\mu)}{\Gamma(N+1)!}\right)$ 7(B)

Table 2: Comparison between CLMM and [7(B)]

P=0.7			P=0.8		P=0.9		
y	7(B)	CLMM	7(B)	CLMM	7(B)	CLMM	Exact
0.1	0.219	0.181	0.155	0.148	0.139	0.121	0.089
0.2	0.345	0.286	0.295	0.263	0.248	0.210	0.187
0.3	0.419	0.383	0.393	0.361	0.346	0.285	0.281
0.4	0.510	0.462	0.473	0.447	0.432	0.373	0.389
0.5	0.556	0.547	0.540	0.520	0.488	0.475	0.472
0.6	0.613	0.581	0.597	0.583	0.575	0.549	0.547
0.7	0.653	0.646	0.647	0.638	0.614	0.616	0.614
0.8	0.689	0.664	0.688	0.664	0.684	0.655	0.654
0.9	0.710	0.715	0.724	0.723	0.727	0.706	0.726

Subjectto the initial condition $\mu(0) = 0$.

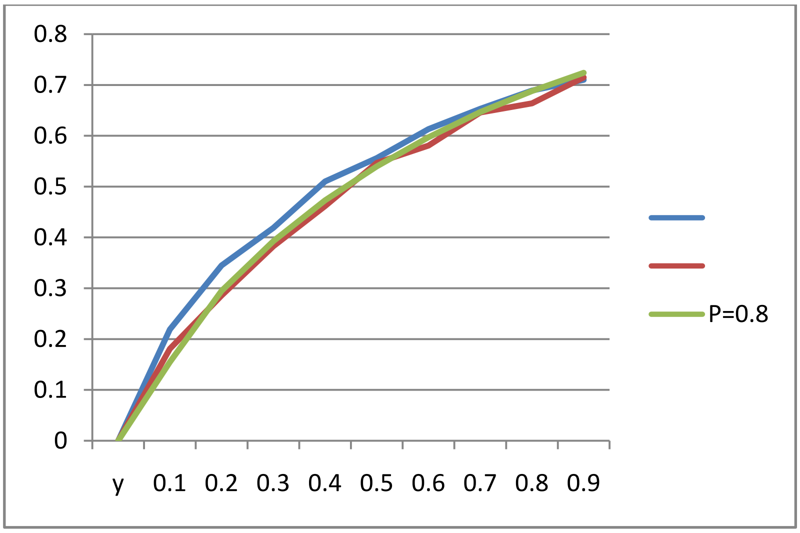

Figure1: Different solutions

The exact solution of (7(B)) in case p= 1 is u(y) = tanhy. In Table 2, we associate our consequences with individuals gained in [7(B)]. Figure 1 exemplifies that the estimated solutions for numerous values of p near the value 1, have a comparable behavior.

Example 3: *Consider the following nonlinear Lane-Emden equation*

$D^p\mu(y) + \frac{2}{y}\mu'(y) + \mu^3(y) = 2 + y^6, y\epsilon(0,1), p\epsilon(0,1]$ 7(C)

Subjectto the initial conditions

$\mu(0) = \mu'(0) = 0$ 7(A)

Table 3: Comparison between CLMM and the method in [7(A)]

P= 1.25			P= 1.5		P= 1.75		
y	BWM [7(A)]	CLMM	BMW	CLMM	BMW	CLMM	Exact
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.2	0.029	0.029	0.029	0.031	0.032	0.034	0.04
0.4	0.168	0.151	0.166	0.180	0.175	0.150	0.17
0.6	0.368	0.366	0.362	0.355	0.355	0.351	0.35
0.8	0.645	0.642	0.636	0.652	0.654	0.652	0.65
1.0	0.982	1.012	0.988	1.016	1.013	1.012	1

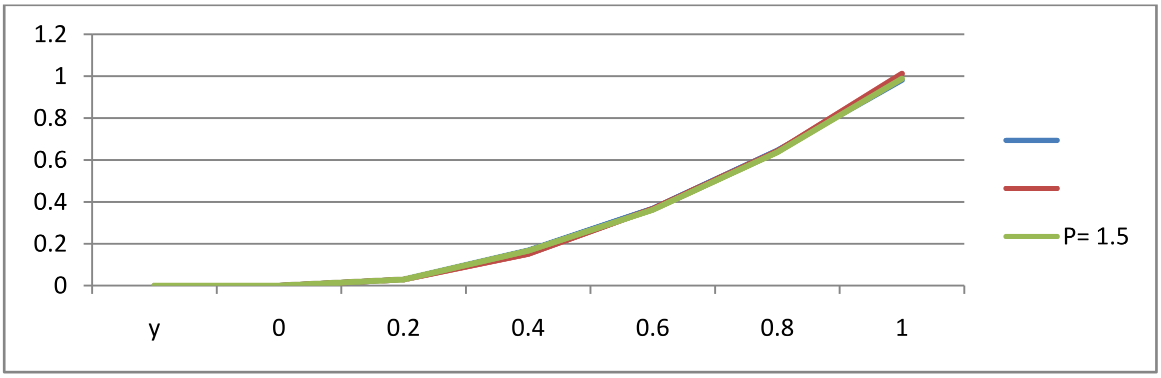

Figure2: Comparison between CLMM and the method in [7(A)]

The exact solution of (7©) in case p = 2 is u(y) = y2. In Table 3, we associate our results with those gained in [7(A)] using Bernoulli wavelets method (BWM). Figure 2 establish that the approximate answers for various values of p have a similar behavior.

1.3.1 Illustrative Test Examples

In this subdivision we relate the method accessible to explain the subsequent test examples.

Example 4. Contemplate the subsequent equation,

$$\begin{cases} x'''(y) + D^{(\frac{3}{2})}x(y) + x(y) = 1 + y \\ x(0) = 1 \\ x'(0) = 1 \\ x''(0) = 0 \end{cases} \quad (8)$$

ThefunctionalLucas polynomialmethod to explain (8) by $m = 2$. In this instance we have

$$L = [L_0(y), L_1(y), L_2(y)] = [2\,, y\,, y^2 + 2\ \ ], \qquad A = [a_1, a_2\,, a_3]$$

$$D^3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, B^{\frac{3}{2}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -\frac{\Gamma(3)}{\Gamma\left(\frac{3}{2}\right)} x^{-\frac{3}{2}} & 0 & \frac{\Gamma(3)}{\Gamma\left(\frac{3}{2}\right)} x^{-\frac{3}{2}} \end{bmatrix}$$

And

$$A^T D^3 L + A^T B^{\left(\frac{3}{2}\right)} K + A^T K = f(y),$$

$$x(0) = \sum_{l=0}^{3} a_l K_l(0) = 1, y'(0) = \sum_{l=0}^{3} a_l D K_l(0) = 1, y"(0) = \sum_{l=0}^{3} a_l D^2 K_l(0) = 0$$

On abridging these, we attain$a_i's$ and we have $x(y) = y + 1$, which is the preciseexplanation

2. Derivative of the Fibonacci polynomial

2.1 In this part, we'll take a gander at a network that makes the groupings that the FPSlead tothe creator of [16] showed various discoveries on the subordinate of the FP.

2.2 Polynomials gained by originating the Fibonacci polynomial

Through deriving the Fibonacci polynomial, it is attained:

$F_1'(y) = 0$

$F_2'(y) = 1$

$F_3'(y) = 2y$

$F_4'(y) = 3y^2 + 2$

$F_5'(y) = 4y^3 + 6y$

$F_6'(y) = 5y^4 + 12y^2 + 3$

$F_7'(y) = 6y^5 + 20y^3 + 12y$

$F_8(y) = 7y^6 + 30y^4 + 30y^2 + 4$

$F_9'(y) = 8y^7 + 42y^5 + 60y^3 + 20y$

$F_{10}'(y) = 9y^8 + 56y^6 + 105y^4 + 60y^2 + 5$

In the first place, remember that the Fibonacci polynomial subsidiary conditions can be addressed in grids as$F' = BY$, where $F' = [F_1'(y), F_2'(y), F_2'(y), \ldots]^t, Y = [1, y, y^2, \ldots]^t$The lower three–sided networks t and B contain the coefficients that show up in the extension of the Fibonacci polynomial subordinates in rising powers of x:

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 6 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 0 & 12 & 0 & 5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 12 & 0 & 20 & 0 & 6 & 0 & 0 & 0 & 0 \\ 4 & 0 & 30 & 0 & 30 & 0 & 7 & 0 & 0 & 0 \\ 0 & 20 & 0 & 60 & 0 & 42 & 0 & 8 & 0 & 0 \\ 5 & 0 & 60 & 0 & 105 & 0 & 56 & 0 & 9 & 0 \\ 0 & 30 & 0 & 140 & 0 & 168 & 0 & 72 & 0 & 10 \end{bmatrix}$$

Note that matrix B is invertible.

Conclusion: An altogether new functional network for the subsidiaries of partial Lucas polynomials is created in this review. The multi-term fragmentary request differential conditions are settled as quickly resolvable logarithmic conditions utilizing the framework, tau, and collocation unearthly procedures. We think the technique portrayed in this study can be utilized to tackle further sorts of partial differential conditions. The analyzed mathematical models show that the two proposed techniques perform astoundingly well and are profoundly compelling.

References

[1] Bagley, R. L., &Torvik, P. J. (1983). Fractional calculus-a different approach to the analysis of visco-elastically damped structures. *AIAA journal*, *21*(5), 741-748.

[2] Wang, W., & Wang, H. (2015). Some results on convolved (p, q)-Fibonacci polynomials. *Integral Transforms and Special Functions*, *26*(5), 340-356.

[3] Ordokhani, Y. (2019, June). A Matrix Method Based on the FPs for solving fractional differential equations. In *This paper is presented at The 6th seminar on numerical analysis and its applications: SedighehSabermahani, YadollahOrdokhani.(2016) A matrix method based on the Fibonacci polynomialsfor solving fractional differential equations. Zenodo. doi/10.5281/zenodo* (Vol. 2652437).

[4] Keshavarz, E., Ordokhani, Y., &Razzaghi, M. (2014). Bernoulli wavelet operational matrix of fractional order integration and its applications in solving the fractional order differential equations. *Applied Mathematical Modelling*, *38*(24), 6038-6051.

[5] Özkan, E., &Altun, İ. (2019). Generalized Lucas polynomials and relationships between the Fibonacci polynomialsand Lucas polynomials. *Communications in Algebra*, *47*(10), 4020-4030.

[6]Koç, A. B., Çakmak, M., &Kurnaz, A. (2014). A matrix method based on the Fibonacci polynomialto the generalized pantograph equations with functional arguments. *Advances in Mathematical Physics*, *2014*.

[7] Falcon, S., & Plaza, A. (2009). On k-Fibonacci sequences and polynomials and their derivatives. *Chaos, Solitons & Fractals*, *39*(3), 1005-1019.