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STUDY ON ROLE OF FUZZY NUMBERS IN DECISION MAKING PROBLEMS

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KEYWORDS:

fuzzy-based, fuzzy numbers

ABSTRACT

In most cases, quantitative and qualitative input components are included throughout the human-based decision making process of industrial settings. The decision-making process will always involve some element of subjectivity due to the nature of human judgement. In an environment for making decisions in industry that is fuzzy-based and in which fuzzy numbers are used to evaluate subjective variables, ranking fuzzy numbers becomes one of the most important steps that must be taken before a final choice can be made. In this article, we suggest a strategy for sorting fuzzy numbers that is based on their similarities. The suggested method's relevance in resolving selected industrial-related decision making issues, including the risk assessment and pattern identification issues, is presented below.

1. INTRODUCTION

Atanassov presented the Intuitionistic Fuzzy Sets (IFSs) in 1986. In contrast to the fuzzy set by Zadeh (1965) IFS is described by both enrollment and non-participation capacities to such an extent that the amount of the two qualities is not exactly or equivalent to one. IFSs can delineate fuzzy characters in more definite and in thorough way so the ambiguity and uncertainty of data can be demonstrated totally when contrasted with fuzzy sets. As though data has become progressively significant, numerous specialists concentrated on IFSs.

Intuitionistic Fuzzy Set: (Atanassov, 1986)

An intuitionistic fuzzy set over universe of discourse E is defined as,

$$P = \{ \langle x, \mu_P(x), \upsilon_P(x) \rangle / x \in E \}$$

described by a membership function $\mu_P : E \to [0,1]$, a non membership function $\upsilon_P : E \to [0,1]$ such that $0 \le \mu_P(x) + \upsilon_P(x) \le 1,\ \mu_P(x), \upsilon_P(x) \in [0,1]\ \forall\ x \in E.$

$\mu_P(x)$ and $\upsilon_P(x)$ denotes membership and non membership degrees of x to P respectively.

Moreover, $1 - \mu_P(x) - \upsilon_P(x),\ \ \forall\ x \in E.$ is called an indeterminacy degree of x to P denoted by $\pi_P(x)$. In particular, if $\pi_P(x) = 0, x \in E$ then, at that point the non-enrollment is only the supplement of participation and subsequently the IFS „P' diminishes to fuzzy set. Henceforth, fuzzy sets are specific instances of IFSs.

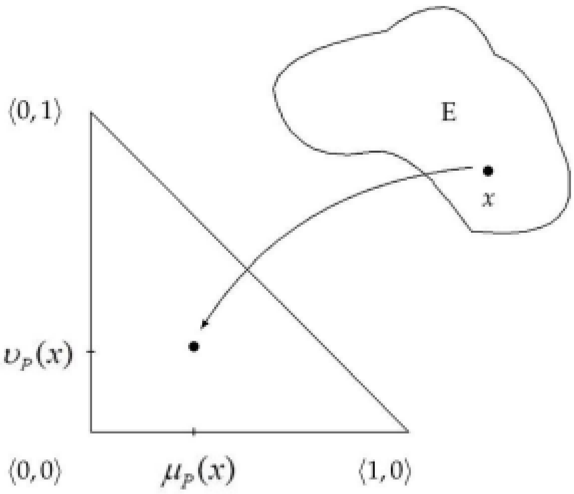

Fig. 1.1: Intuitionistic Fuzzy Set

Intuitionistic Fuzzy Number (Atanassov, 1983)

An Intuitionistic Fuzzy Number (IFN) $\tilde{P}$ is

(1) intuitonistic fuzzy sub set of the real line

(2) convex for the membership function $\mu_{\tilde{P}}(x)$ i.e.,

$$\mu_{\tilde{P}}(\lambda x + (1-\lambda)y) \ge \min(\mu_{\tilde{P}}(x), \mu_{\tilde{P}}(y))\ \ \forall\ x, y \in R, \lambda \in [0,1].$$

(3) concave for the non-membership function, $\upsilon_{\tilde{P}}(x)$ i.e.,

$$\upsilon_{\tilde{P}}(\lambda x+(1-\lambda)y)\leq \max(\upsilon_{\tilde{P}}(x),\upsilon_{\tilde{P}}(y)) \quad \forall x,y\in R,\lambda\in[0,1].$$

(4) normal, that is, there is some 0 ∈x R such that $\mu_{\tilde{P}}(x_0)=1,\upsilon_{\tilde{P}}(x_0)=0.$

INTERVAL VALUED INTUITIONISTIC FUZZY SET CONCEPT

Often it is turning into an entanglement for specialists to precisely measure their perspective as careful numbers in the stretch [0, 1]. Henceforth it is more reasonable to address them in span structure. Atanassov and Gargov (1989) summed up the concept of IFSs and presented stretch esteemed intuitionistic fuzzy sets (IVIFSs) by joining IFS concept with span esteemed fuzzy set concept.

Interval Valued Intuitionistic Fuzzy Set (Atanassov and Gargov, 1989)

An interval valued intuitionistic fuzzy set in P over E is of the form,

$$\tilde{P}=\left\{\left(x,\left[\mu_{\tilde{P}}^{L}(x),\mu_{\tilde{P}}^{U}(x)\right],\left[\upsilon_{\tilde{P}}^{L}(x),\upsilon_{\tilde{P}}^{U}(x)\right]\right)/\,x\in E\right\}$$

Where $\mu_{\tilde{P}}^{L},\mu_{\tilde{P}}^{U},\upsilon_{\tilde{P}}^{L},\upsilon_{\tilde{P}}^{U}:E\rightarrow[0,1]$ such that

$$\mu_{\tilde{P}}^{L}\leq\mu_{\tilde{P}}^{U},\upsilon_{\tilde{P}}^{L}\leq\upsilon_{\tilde{P}}^{U},0\leq\mu_{\tilde{P}}^{L}+\upsilon_{\tilde{P}}^{L}\leq1,0\leq\mu_{\tilde{P}}^{U}+\upsilon_{\tilde{P}}^{U}\leq1$$

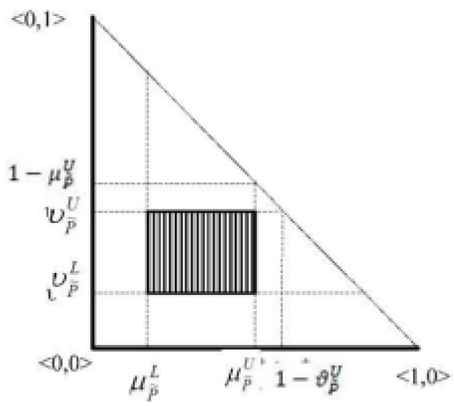

Fig 1.2. Interval Valued Intuitionistic Fuzzy Set

IVIFNs Score Function (Xu and Chen, 2007)

The score function of $\tilde{P} = ([\mu_{\tilde{P}}^L, \mu_{\tilde{P}}^U], [\upsilon_{\tilde{P}}^L, \upsilon_{\tilde{P}}^U])$, $S_E(\tilde{P})$ is defined as;

$$S_E(\tilde{P}) = \frac{\mu_{\tilde{P}}^L + \mu_{\tilde{P}}^U - \upsilon_{\tilde{P}}^L - \upsilon_{\tilde{P}}^U}{2}, \quad S_E(\tilde{P}) \in [-1,1].$$

. IVIFNs Accuracy Function (Xu and Chen, 2007)

The accuracy function of is represented as;

$$H_E(\tilde{P}) = \frac{\mu_{\tilde{P}}^L + \mu_{\tilde{P}}^U + \upsilon_{\tilde{P}}^L + \upsilon_{\tilde{P}}^U}{2}, \; H_E(\tilde{P}) \in [0,1].$$

INTERVAL VALUED TRAPEZOIDAL INTUITIONISTIC FUZZY SET CONCEPT

As the space of IFSs and IVIFSs is a discrete set, their participation and non enrollment degrees can just communicate fuzzy idea as far as "superb" or "great". To defeat this impediment Shu et.al., (2006) characterized three-sided intuitionistic fuzzy numbers (TIFNs) with the end goal that the area is a back to back set. Later on, Wang (2008) broadened the concept of three-sided intuitionistic fuzzy number to trapezoidal intuitionistic fuzzy number. Wan (2011) presented the span esteemed trapezoidal intuitionistic fuzzy numbers (IVTIFNs) in which the participation and non-enrollment esteems are stretches instead of exact numbers.

Interval Valued Trapezoidal Intuitionistic Fuzzy Set: (Wan, 2011)

$\tilde{P} = \langle [p,q,r,s]; [\mu_{\tilde{P}}^L, \mu_{\tilde{P}}^U], [\upsilon_{\tilde{P}}^L, \upsilon_{\tilde{P}}^U] \rangle$ is called interval valued trapezoidal intuitionistic fuzzy set (IVTIFS), if its interval valued membership function is defined by,

$$\mu_{\tilde{P}}^{U}(x)=\begin{cases}\dfrac{x-p}{q-p}\mu_{\tilde{P}}^{U}, & p\le x<q\\ \mu_{\tilde{P}}^{U}, & q\le x\le r\\ \dfrac{s-x}{s-r}\mu_{\tilde{P}}^{U}, & r<x\le s\\ 0, & otherwise\end{cases}$$

And

$$\mu_{\tilde{P}}^{L}(x)=\begin{cases}\dfrac{x-p}{q-p}\mu_{\tilde{P}}^{L}, & p\le x<q\\ \mu_{\tilde{P}}^{L}, & q\le x\le r\\ \dfrac{s-x}{s-r}\mu_{\tilde{P}}^{L}, & r<x\le s\\ 0, & otherwise\end{cases}$$

interval valued non membership function is defined by,

$$\upsilon_{\tilde{P}}^{U}(x)=\begin{cases}\dfrac{q-x+\upsilon_{\tilde{P}}^{U}(x-p)}{q-p}, & p\le x<q\\ \upsilon_{\tilde{P}}^{U}, & q\le x\le r\\ \dfrac{x-r+\upsilon_{\tilde{P}}^{U}(s-x)}{s-r}, & r<x\le s\\ 0, & otherwise\end{cases}$$

And

$$\upsilon_{\tilde{P}}^{L}(x)=\begin{cases}\dfrac{q-x+\upsilon_{\tilde{P}}^{L}(x-p)}{q-p}, & p\le x<q\\ \upsilon_{\tilde{P}}^{L}, & q\le x\le r\\ \dfrac{x-r+\upsilon_{\tilde{P}}^{L}(s-x)}{s-r}, & r<x\le s\\ 0, & otherwise\end{cases}$$

Where $0 \le \mu_{\tilde{P}}^{L} \le \mu_{\tilde{P}}^{U} \le 1; 0 \le \upsilon_{\tilde{P}}^{L} \le \upsilon_{\tilde{P}}^{U} \le 1; 0 \le \mu_{\tilde{P}}^{U} + \upsilon_{\tilde{P}}^{U} \le 1; 0 \le \mu_{\tilde{P}}^{L} + \upsilon_{\tilde{P}}^{L} \le 1$; $p, q, r, s \in R$.

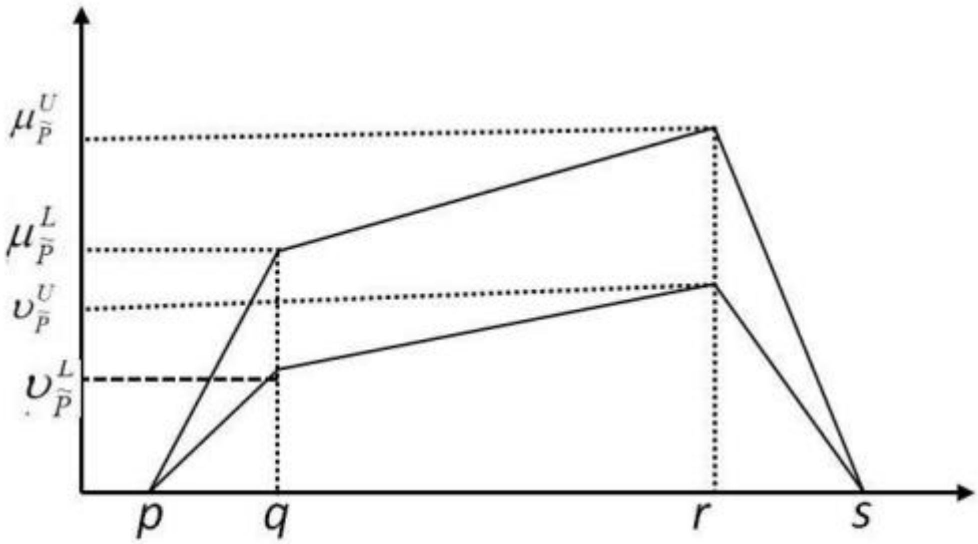

Fig. 1.3. Interval Valued Trapezoidal Intuitionistic Fuzzy Set

Example:

For an IVTIFN, $\tilde{P} = \langle [4,5,6,7]; [0.5,0.7]; [0.1,0.2] \rangle$ the membership, non membership and hesitation degree for x=5 respectively be [0.5, 0.7], [0.1, 0.2] and [0.4, 0.1].

If $[\mu_{\tilde{P}}^{L}, \mu_{\tilde{P}}^{U}] = [1,1]$ and $[\upsilon_{\tilde{P}}^{L}, \upsilon_{\tilde{P}}^{U}] = [0,0]$ then the IVTIFN $\tilde{P}$ degenerates to $\langle [p,q,r,s]; 1, 0 \rangle$, a trapezoidal fuzzy number (TFN). Thus, the concept of IVTIFN is a generalization of TFN. In addition, if $p = q$, $\mu_{\tilde{P}}^{L} = \mu_{\tilde{P}}^{U}$ and $\upsilon_{\tilde{P}}^{L} = \upsilon_{\tilde{P}}^{U}$ then $\tilde{P}$ reduces to a TIFN.

Operational Laws of IVTIFNs: (Wan 2011)

Let $\tilde{P}_1 = \langle [p_1, q_1, r_1, s_1]; [\mu^L_{\tilde{P}_1}, \mu^U_{\tilde{P}_1}]; [\upsilon^L_{\tilde{P}_1}, \upsilon^U_{\tilde{P}_1}] \rangle$ and $\tilde{P}_2 = \langle [p_2, q_2, r_2, s_2]; [\mu^L_{\tilde{P}_2}, \mu^U_{\tilde{P}_2}]; [\upsilon^L_{\tilde{P}_2}, \upsilon^U_{\tilde{P}_2}] \rangle$ be two IVTIFNs, then

$$\tilde{P}_1 \oplus \tilde{P}_2 = ([\, p_1 + p_2, q_1 + q_2, r_1 + r_2, s_1 + s_2 \,]; [\, \mu^L_{\tilde{P}_1} + \mu^L_{\tilde{P}_2} - \mu^L_{\tilde{P}_1}\mu^L_{\tilde{P}_2}, \mu^U_{\tilde{P}_1} + \mu^U_{\tilde{P}_2} - \mu^U_{\tilde{P}_1}\mu^U_{\tilde{P}_2}]; [\upsilon^L_{\tilde{P}_1}\upsilon^L_{\tilde{P}_2}, \upsilon^U_{\tilde{P}_1}\upsilon^U_{\tilde{P}_2}\,]) \quad (1.1)$$

$$\tilde{P}_1 \otimes \tilde{P}_2 = ([\, p_1p_2, q_1q_2, r_1r_2, s_1s_2]; [\mu^L_{\tilde{P}_1}\mu^L_{\tilde{P}_2}, \mu^U_{\tilde{P}_1}\mu^U_{\tilde{P}_2}]; [\upsilon^L_{\tilde{P}_1} + \upsilon^L_{\tilde{P}_2} - \upsilon^L_{\tilde{P}_1}\upsilon^L_{\tilde{P}_2}, \upsilon^U_{\tilde{P}_1} + \upsilon^U_{\tilde{P}_2} - \upsilon^U_{\tilde{P}_1}\upsilon^U_{\tilde{P}_2}]) \quad (1.2)$$

$$\lambda \tilde{P}_1 = ([\lambda p_1, \lambda q_1, \lambda r_1, \lambda s_1]; [1-\left(1-\mu^L_{\tilde{P}_1}\right)^{\lambda}, 1-\left(1-\mu^U_{\tilde{P}_1}\right)^{\lambda}]; [\left(\upsilon^L_{\tilde{P}_1}\right)^{\lambda}, \left(\upsilon^U_{\tilde{P}_1}\right)^{\lambda}]) \quad (1.3)$$

$$\left(\tilde{P}_1\right)^{\lambda} = ([(p_1)^{\lambda}, (q_1)^{\lambda}, (r_1)^{\lambda}, (s_1)^{\lambda}]; [\left(\mu^L_{\tilde{P}_1}\right)^{\lambda}, \left(\mu^U_{\tilde{P}_1}\right)^{\lambda}]; [1-\left(1-\upsilon^L_{\tilde{P}_1}\right)^{\lambda}, 1-\left(1-\upsilon^U_{\tilde{P}_1}\right)^{\lambda}]) \quad (1.4)$$

AGGREGATION OPERATORS OF IVTIFNS

Accumulation of data is a critical issue in decision-making (Li, (2007a, 2007b, 2009 and 2010), Yager (1988, 2004a &2004b)). The old style weighted accumulation is the basic added substance weighted averaging technique (Li, 2003 &2010, Hwang et al., 1981). Yager (1988) presented one more significant accumulation administrator known as Ordered Weighted Averaging (OWA). Further, in 2004, Yager (2004a) presented the Generalized Ordered Weighted Averaging (GOWA) Operator. In this segment, collection administrators of IVTIFNs are reviewed.

Interval Valued Trapezoidal Intuitionistic Fuzzy Weighted Arithmetic Aggregation (IVTIFWAA) Operator: (Wan, 2011)

Let ω be set of IVTIFNs and $\tilde{P}_k \in \omega,\ for\ k = 1, 2, 3, \ldots m$. An IVTIFWAA: $\omega^m \to \omega$ is defined as

$$IVTIFWAA_w(\tilde{P}_1, \tilde{P}_2, \ldots, \tilde{P}_m) = w_1(\tilde{P}_1) \oplus w_2(\tilde{P}_2) \oplus \ldots \oplus w_m(\tilde{P}_m) \quad (1.5)$$

Where $w=(w_1,w_2,...w_m)^T$ be the weight vector of $\tilde{P}_k,\ k=1,2,3,...m,$ such that $w_k \geq 0, \sum_{k=1}^{m} w_k = 1$

Interval Valued Trapezoidal Intuitionistic Fuzzy Ordered Weighted Arithmetic Aggregation (IVTIFOWAA) Operator: (Wan, 2011)

For, $\tilde{P}_k \in \omega,\ k=1,2,3,...,m,$ an IVTIFOWAA: $\omega^m \to \omega$ is defined as IVTIFOWAA $_w(\tilde{P}_1,\tilde{P}_2,...,\tilde{P}_m)=w_1(\tilde{P}_{\sigma(1)})\oplus w_2(\tilde{P}_{\sigma(2)})\oplus...\oplus w_m(\tilde{P}_{\sigma(m)})$ where $w=(w_1,w_2,...w_m)^T$ be the weight vector of $\tilde{P}_k,\ k=1,2,3,...,m,$ such that $w_k \geq 0, \sum_{k=1}^{m} w_k = 1$ and $\{\sigma(1),\sigma(2),\cdots,\sigma(m)\}$ is permutation of (1,2,…,m) such that $\tilde{P}_{\sigma(k-1)} \geq \tilde{P}_{\sigma(k)}$ for all k.

Interval Valued Trapezoidal Intuitionistic Fuzzy Weighted Geometric Aggregation (IVTFIWGA) Operator: (Wu and Liu, 2013)

For, $\tilde{P}_k \in \omega,\ k=1,2,3,...m,$ an IVTIFWGA: $\omega^m \to \omega$ is defined as

IVTIFWGA $_w(\tilde{P}_1,\tilde{P}_2,...,\tilde{P}_m)=(\tilde{P}_1)^{w_1}\oplus(\tilde{P}_2)^{w_2}\oplus...\oplus(\tilde{P}_m)^{w_m}$ where $w=(w_1,w_2,...w_m)^T$ be the fuzzy weight vector of $\tilde{P}_k,\ k=1,2,3,...,m,$ such that $w_k \geq 0, \sum_{k=1}^{m} w_k = 1$

Interval Valued Trapezoidal Intutitionistic Fuzzy Ordered Weighted Geometric Aggregation (IVTIFOWGA) operator: (Wu and Liu, 2013)

For, $\tilde{P}_k \in \omega,\ k=1,2,3,...,m,$ an IVTIFOWGA: $\omega^m \to \omega$ with weighting vector $w=(w_1,w_2,...w_m)^T$ such, that $w_k \geq 0, \sum_{k=1}^{m} w_k = 1,$ is defined as

$$IVTIFOWGA_w(\tilde{P}_1,\tilde{P}_2,...,\tilde{P}_m)=(\tilde{P}_{\sigma(1)})^{w_1}\oplus(\tilde{P}_{\sigma(2)})^{w_2}\oplus...\oplus(\tilde{P}_{\sigma(m)})^{w_m}$$

(1.7) where $\{\sigma(1),\sigma(2),\cdots,\sigma(m)\}$ is permutation of (1,2,…,m) such that $\tilde{P}_{\sigma(k-1)} \geq \tilde{P}_{\sigma(k)}$ for all k.

MULTI CRITERIA DECISION MAKING (MCDM)

Multi Criteria Decision Making (MCDM) is most perceived part of functional exploration, manages decision problems under the presence of number of decision measures. MCDM is separated into multi target decision-making (MODM) and multi characteristic decision-making (MADM) (Climaco, 1997). There are a few strategies in both MODM and MADM in particular, "need based", "out positioning", "distance based and blended" techniques. Every technique has its own qualities. These strategies are ordered into single or collective choice making techniques dependent on number of decision producers.

Objective of the study

1. To study on Interval Valued Trapezoidal Intutitionistic Fuzzy Ordered
2. To study on Interval Valued Intuitionistic Fuzzy Set Concept

Conclusion

The current theory examines diverse dynamic issues with measures and without models under the dubious/unsure climate. For taking care of such dynamic issues, a few strategies/approaches have been thought about. As dynamic is the vital piece of postulation, the questionable climate under which it is settled, is likewise on prime mode due to its reality wherever for all intents and purposes. To adapt to such circumstances we use the intuitionistic fluffy set (IFS) hypothesis and its augmentation stretch esteemed intuitionistic fluffy set (IVIFS).

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