UNRAVELING THE MATHEMATICS OF SHAPES: A JOURNEY INTO GEOMETRIC ALGEBRA

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ABSTRACT

Much ink has been spilled these last 50 years over the notion (or whatever it is) of "geometric algebra" – sometimes in disputes so hot that one would believe it to be blood. However, nobody has seemed too interested in analyzing whether others have used the words in the same way as he has himself (he, indeed – as a feminist might declare, "all males, of course"). So, let us analyze what concepts or notions have been referred to by the two words in combination – if any The ideas that are presented here will be used in every subsequent maths course that you enrol in after this one. We'll get you started on some fun activities like plotting graphs and figuring out solutions to difficult equations. owing to the fact that we are now studying algebra and geometry in the classroom. The use of algebra in geometry, however, is an example of its application in the actual world. The social media platforms available these days have significantly advanced. Because we are unable to answer such figured riddles, we will need to resort to employing algebraic equations in order to do so.

keywords: *algebra, mathematics*

Introduction

The foundational area of mathematics known as algebra employs letters and symbols to represent numerical values and quantities, as well as the connections that exist between these things. It is an important component in a wide variety of mathematical subfields and has a wide range of applications in a variety of contexts. The following are some of the most important uses of algebra in mathematics: Finding solutions to equations Equations, which are mathematical assertions that involve unknown variables, may be solved by the use of algebra. It is useful in determining the values of these variables that are necessary to make the equation true. Equations may be found in a broad variety of academic disciplines, ranging from physics and engineering to economics and biology. Operations using polynomials Polynomials are the focus of the study of algebra. Polynomials are expressions that combine variables and constants via the operations of addition, subtraction, multiplication, and division. The ability to manipulate polynomials is necessary for solving issues involving calculus, number theory, and optimisation. The process of factoring includes expressing an algebraic expression or polynomial as a product of its component factors. Factoring may also be referred to as the factoring operation. Simplifying problems, solving equations, and working with different aspects of calculus and number theory all make use of it. Algebraic matrices and linear transformations: Numerous mathematical and scientific disciplines, such as engineering, physics, computer science, and economics, make extensive use of arrays of numbers known as matrices. The study of vector spaces as well as linear transformations is the focus of linear algebra, which also has applications in the resolution of systems of linear equations and the comprehension of geometric transformations. The study of graphs, which are mathematical representations of networks consisting of nodes and points that are linked to one another, is known as graph theory. The study of the characteristics of graphs via the use of matrices, polynomials, and other algebraic structures is known as algebraic graph theory. The study of algebraic structures such as groups, rings, and fields is the focus of abstract algebra. The fields of coding theory and cryptography, as well as many subfields of pure mathematics, may all make use of these structures. The study of numbers Techniques from the field of algebra are essential to the study of number theory, which examines the qualities of integers and the connections between them. Tools from algebraic theory are used to investigate mathematical ideas such as divisibility, prime factorization, and congruences. Algebra Plays a Significant Role in Calculus Calculus relies heavily on algebra, notably for the manipulation of functions, the solution of limit problems, and the operation of derivatives and integrals. Probability and statistics: In the fields of probability and statistics, algebra is absolutely necessary since equations are used to describe and analyse random processes, compute probabilities, and draw statistical conclusions. Algebraic procedures are used to solve issues in geometry and trigonometry that include unknown lengths, angles, and coordinates. These problems may be broken down into two categories: geometry and trigonometry.

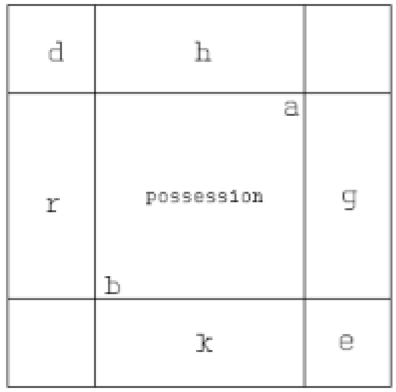
Before "geometric algebra"

Since the possibly earliest written treatment of algebra carrying that name, algebra and the geometry of rectangles and line segments have been linked. When al-Khwārizmī was asked by the caliph al-Ma'mūn to write a brief presentation of the art of al-jabr wa'l-muqābalah, he decided not only to let it contain "what was most subtle in this calculation and what is most noble, and what people need" in various commercial and mensurational practices.2 Knowing from his familiarity with those who were engaged in translation of Greek mathematics (so we may reasonably surmise) that mathematics ought to be based on argument, he also provided geometric proofs for the algorithmic prescriptions for the solution of the mixed second-degree equation types – not by appealing to the propositions of Elements II, which his target group was perhaps not likely to know, but by borrowing from the surveyors' geometric riddle tradition.3 This is most obvious in the case of the algorithm for the equation type "possession and square roots equals number", normally translated

$x^2+\alpha x=\beta,$[4] whose algorithm corresponds to the formula

$$x=\sqrt{\beta+\left(\frac{\alpha}{2}\right)^2}-\frac{\alpha}{2}$$

The diagram shows the justification as it appears in Gherardo's version [ed. Hughes 2013] (the Arabic version only differs by using Arabic letters). It is adapted to the equation "a possession and 10 roots equal 39 dirham". The central square represents the possession, and the 10 roots are represented by four rectangles, each of breadth 2½ and length equal to the side of the square, that is, the square root of the possession. The four corners, with area 4×2½×2½ = 25, are filled out, and the resulting larger square thus has an area equal to 39+25 = 64; etc. This is fairly different in style from what we find in Euclid, not strictly deductive but an appeal to what can be "seen immediately".



According to the rules of grammar, "geometric algebra", whatever it means, must refer to some kind of algebra, only modified or restricted by the adjective. Accordingly, this is not geometric algebra but merely a justification of a certain algebraic procedure by means of a borrowing from a different field. Half a century later, Thābit ibn Qurrah offered new proofs He does not mention al-Khwārizmī at all but only refers to the procedures of the ahl al-jabr, the "al-jabr people"– presumably those reckoners whose technique al-Ma'mūn had asked al-Khwārizmī to write about. Most likely, Thābit did not regard al-Khwārizmī 's justifications as proofs proper; his, indeed, are in strict Euclidean style, with explicit reduction to Elements II.5–6.

Abū Kāmil does refer to al-Khwārizmī in his algebra, and he writes a full treatise on the topic; but his proofs are equally and explicitly Euclidean [ed., trans. Rashed 2013: 354 and passim]. Roshdi Rashed (p. 37) speaks of Thābit 's and Abū Kāmil's proofs as "geometric algebra" (Rashed's quotes).6 Later Arabic algebrists are no different, and there is no reason to discuss them separately. The same can be said about Fibonacci's Liber abbaci, and about the use of geometric justifications from Pacioli to Cardano and his contemporaries. However, what we see is once again not ("geometric") algebra but merely a justification by means of a borrowing – this time from rigorous Euclidean and not from intuitively obvious geometry.

Slightly different is the case of Jordanus de Nemore's De numeris datis. In his attempt to create a theoretically coherent stand-in for Arabic algebra based on axiomatic arithmetic,7 When did it start? Tannery or Zeuthen? Jordanus created the arithmetical (and quasi-algebraic) analogues of a number of theorems from Elements II (and much more). If the phrase had not already been occupied by a different signification, it would not be totally misleading to speak of this reversely as "algebraic geometry", that is, geometry translated into something like algebra. In any case, it is no more "geometric algebra" than what we have already discussed. Nor is, of course, Nuñez' or Descartes' use of algebra as a tool for solving geometric problems (on very different levels, to be sure).

DEFINITION OF ALGEBRA

Mathematically speaking, algebra is a subfield of mathematics that focuses on relations, operations, and the constructs of those three. It is a fundamental concept in mathematics and has a wide range of uses in our everyday lives. It is also one of the building blocks of mathematics. Algebra, in addition to its importance as a foundational topic in mathematics, is of great assistance to students of all ages in establishing a comprehensive grasp of other advanced subfields of mathematics, such as calculus, geometry, arithmetic, and so on. A field of mathematics that involves the generalisation of arithmetic operations and connections via the use of alphabetic symbols to represent unknown numbers or members of defined sets of numbers. The area of mathematics concerned with the study of more abstract formal structures, such as sets, groups, and other such things.

POLYNOMIAL

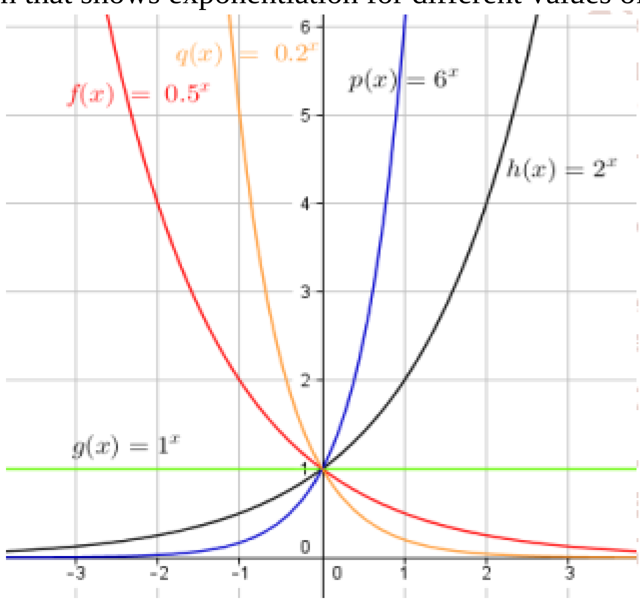
A polynomial is constructed with a limited number of monomials that are connected to one another using the arithmetic operations of addition and subtraction. It is possible to define the order of a polynomial by referring to the order of the monomial with the greatest degree that is included in the mathematical statement. Polynomial of order 3 in a single variable, the expression 2x3 + 4x2 + 3x – 7 is as follows. Polynomials also occur in many variables. A polynomial with the variables x and y is represented by the expression x3 + 4x2y + xy5 + y 2 – 2.

EXPONENT

An action in mathematics known as exponentiation is represented by the symbol an, where a represents the base and n, also known as the power, index, or exponent, is a positive integer. One way to think about the operation of exponentiation is as the act of continually multiplying a number by itself, with the exponent serving as a representation of the total number of times the number is multiplied.

- In a3, a is multiplied with itself 3 times i.e. a x a x a.
- a 5 translates to a x a x a x a x a (a multiplied with itself 5 times).

Shown below is a graph that shows exponentiation for different values of bases a.



After examining the graph, we have reached the conclusion that as the exponent value increases, the values that are less than one become closer and closer to zero. On the other hand, when the exponentiation index is increased by a bigger amount for integers that are greater than one, the exponentiation values tend to approach infinity. Every equation that we have seen up to this point has been of the linear kind. The following is the most typical distinction that can be made between the two kinds of equations:

TYPES OF EQUATION

LINEAR EQUATIONS

- A simple linear equation is of the form: y = mx + c
- A linear equation looks like a straight line when graphed.
- It has a constant slope value.
- The degree of a linear equation is always 1.
- Superposition principle is applicable to a system characterized by a linear equation.
- The output of a linear system is directly proportional to its input.

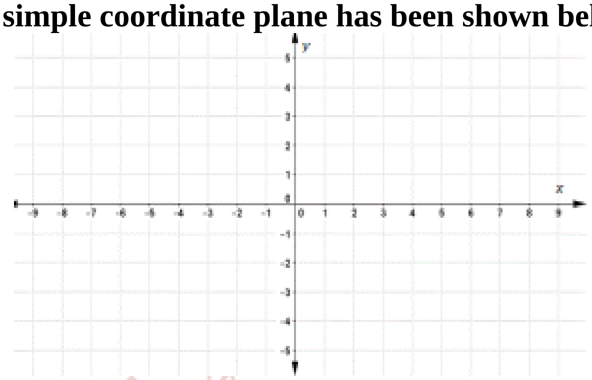
NON-LINEAR EQUATIONS

- A simple non-linear equation is of the form: ax2 + by2 = c
- A non-linear equation looks like a curve when graphed.
- It has a variable slope value.
- The degree of a non-linear equation is at least 2 or other higher integer values. With the increase in the degree of the equation, the curvature of the graph increases.
- Superposition principle does not apply to the systems characterized by non-linear equations.
- The input and output of a non-linear system is not directly related.

GRAPHING A LINEAR EQUATION IN ONE VARIABLE

A co-ordinate plane is required if an equation is to be graphed. It is made up of two lines that are perfectly straight, one going in a horizontal direction and the other going in a vertical direction. The term "x-axis" is used to refer to the horizontal line, while "y-axis" is used to refer to the vertical line. The term "origin" refers to the place in space where the two lines meet.

A simple coordinate plane has been shown below



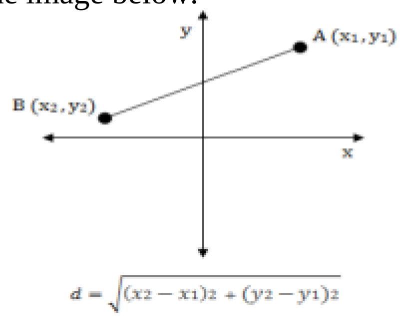
On the plane that represents coordinates, there is an endless number of points. A single point may be stated by using the two coordinate values x and y, and this is done by representing the point as an ordered pair using the notation (x,y). In this context, the values of x and y may be anything at all. Utilising a coordinate plane allows us to plot a linear equation with one variable as the only variable. Let's illustrate that with a real-world scenario, shall we?

DISTANCE FORMULA

The name "Distance Formula" gives away the purpose of this tool, which is to determine the shortest possible distance (in a straight line) between two sites.

PYTHAGOREAN THEOREM

When this well-known theory is used, one may easily derive a straightforward derivation of the formula. This theorem states that the hypotenuse of a right-angled triangle may be found by substituting the values x2 and y2 into the equation for h2. When it comes to the formula for calculating the distance, we may get the value of x by taking the difference between x1 and x2 and subtracting it. In a similar manner, the value of y may be calculated by subtracting y1 from y2 as illustrated in the image below.



Eventually, the straight line distance d between the two points (x1,y1) and (x2,y2) is given by

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

MIDPOINT FORMULA

A point (its coordinate values) that is positioned precisely between two other points in a plane may be found with the use of a formula known as the midway formula. The formula has a tonne of uses, but its most important one is in geometry. The following equation may be used to get the coordinates of the point (x, y) that is precisely in the middle of the two points (x1, y1) and (x2, y2):

$$x = x_1 + x_2,\ y = y_1 + y_2$$

In a similar vein, if we wish to identify the midpoint of a segment in the space that is three-dimensional, we may get the midpoint by utilising the following methods:

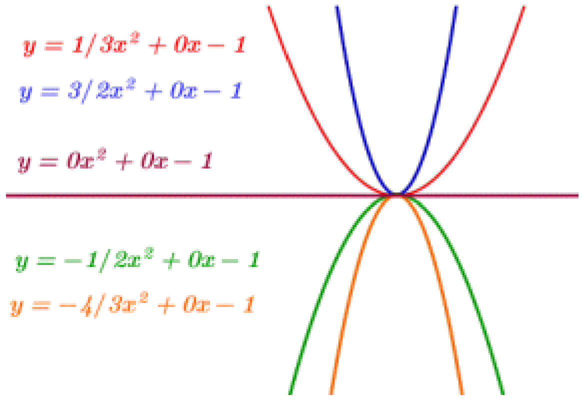
$$x = x_1 + x_2, y = y_1 + y_2, z = z_1 + z_2$$

The figure shown below gives an illustration of the midpoint formula. A quadratic equation is a polynomial of second degree in a single variable. It is expressed as

$$ax^2+bx+c=0$$

In the above equation, a,b,c are constants where a!= 0.

The figure below shows the plot of a quadratic equation $y=ax^2+bx+c$. The values of all three coefficients are changed one at a time, one at a time; that is, while an is changed, b and c continue to have their original values, and so on. We may deduce from the illustration that the graph of a quadratic equation is a parabola, and that changing the values of the three coefficients causes a movement of the parabola along the coordinate axis.



QUADRATIC FORMULA

For a general quadratic equation of the form

$ax^2+bx+c=0$

where a,b,c are constants (can be -ve) and where a !=0, the quadratic formula is given by

x= (-b±√(b^2-4*a*c)) /2*a

This answer suggests that the quadratic equation has two different points where it might be solved. And they are offered in exchange for:

x= (-b+√(b^2-4*a*c)) /2*a

x= (-b-√(b^2-4*a*c)) /2*a

i.e. one with the positive sign, while the other has a negative sign.

OPERATIONS WITH POLYNOMIALS

In mathematics, an expression is referred to as a polynomial if it is built using just the three arithmetic operations of addition, subtraction, and multiplication and if it contains one or more variables and constants. While the exponents of the variables are positive integers, the constants of the polynomials are represented by real values.

Example: x^2+5x-3 is a polynomial in a single variable x.

The Degree, the Variables, and the Coefficients of a Polynomial The value of the variable that is raised to the greatest power in a polynomial is referred to as the degree of the polynomial. The degree of the polynomial in the case that was just shown is 2.

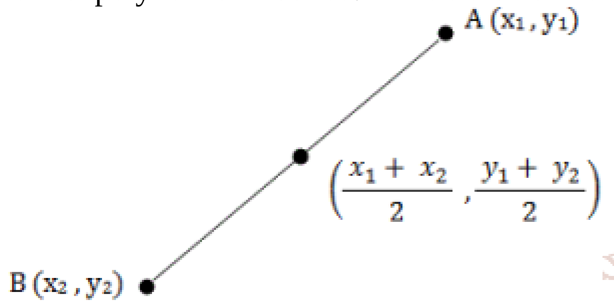
Coefficients and coefficient values are terms that refer to the constant values that are included inside a polynomial. The constants 1, 5, and -3 were utilised in the previous polynomial expression. In a polynomial, the letters a through z, as well as other letters like as x, y, and z, are referred to as variables. They are referred to as variables due to the fact that they may be assigned any value within a specified range (hence the name "variables"). The variable in the above illustration is denoted by the letter x. Polynomials may also exist that make use of more than one variable at a time.

Example: +5xy−3 is a polynomial of degree 2 in two variables x and y.

Note: Only the definition of a polynomial has been covered up to this point in the discussion. Nevertheless, it is important to be explicit about what does not constitute a polynomial for a variety of reasons. If you do not provide this specification, there is a good chance that you will convert a non-polynomial into a polynomial.

SUBTRACTING POLYNOMIALS CALCULATOR

The process of subtracting polynomials from a number is quite similar to the process of adding them. Simply put, we may say that "the process of subtracting one polynomial from another polynomial is equivalent to adding the second polynomial into the first polynomial with all of the signs of the first polynomial inverted." This statement is accurate.



EXPLANATION

If we suppose that point A has the coordinates x1 and y1 and that point B has the coordinates x2 and y2, then we can calculate the midpoint between the two sets of coordinates for a line segment. Using the technique for finding the midpoint that was just presented, you can determine where Point A and Point B meet by taking the average of their x and y coordinates. Using this method, it is feasible to locate the midpoint of a line segment in any orientation, including vertically, horizontally, and even diagonally.

PARENTHESES RULES DEFINITION

Expressions in algebra and mathematics often make use of brackets for the primary purpose of altering the conventional sequence of operations. As a result, when there are brackets included in an expression, the words that are contained inside the brackets () are evaluated first.

- a+(−b)=a−b
- a−(−b)=a+b
- a·(−b)=−ab
- (−a)(−b)=ab

The following are several instances involving Parentheses Rules that will assist you in comprehending the importance of these rules as well as the manner in which they are used.

MULTIPLYING POLYNOMIALS CALCULATOR

In algebra and mathematics more generally, the multiplication of polynomials is an operation that comes up rather often. When working with the multiplication of polynomials, we make heavy use of the following three qualities throughout the whole process. Take note that all that is left are the rules governing exponents. First, we will describe these principles, and then we will go on to discussing the multiplication of polynomials.

RULES OF EXPONENT

Let a be a real number, and let m,n be any positive integers then am·an=am+n

If a is any real number, and m,n are two positive integers then (am)n=amn

If we assume that a and b are two real numbers and that n is a positive integer, we may deduce that (ab)n=an.bn These qualities may be ascertained with little effort because to their straightforward nature. Before moving on to the multiplication of polynomials, you have to have a firm grasp on these concepts and commit them to memory.

CONCLUSION

The work that the Australian Bureau of Statistics does including problem-solving, inquiry, testing, design, and analysis relies heavily on mathematics. It enables the development of a complete data base of information in a manner that is efficient with regard to costs. It helps us to get value from the data by exploring the patterns that are contained within it and estimating the level of confidence that can be fairly placed in the conclusions that are formed from the data. One of the most interesting facets of the use of mathematics in business is the fact that, in the real world, no one solution ever appears to immediately apply. Rather, the concepts behind previous results need to be understood as a foundation for the development of subsequent solutions, which will lead to the enhancement and development of new theories to match the actual world. Researchers and graduate practitioners who have a very deep comprehension of the principles underlying the known theory and the foundation on which it has been established are required to be able to expand and correctly apply that theory in the context of practise. This understanding is necessary in order to fulfil this need.

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