International Journal of Engineering, Science and Mathematics
Vol. 11 Issue 02, February 2022,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in,
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

Mathematical Modelling on Conversion from Minor Stage to Major Stagein Diabetes Patient

Nirmala Mishra[1], Dr. Arun Kumar[2], Dr. Ashutosh Sharma[3]
[1]Research Scholar, Department of Mathematics, ShriJagdishprasadJhabarmalTibrewala University, Jhunjhunu, Rajasthan
[2]Assistant Professor, Department of Mathematics, ShriJagdishprasadJhabarmalTibrewala University, Jhunjhunu, Rajasthan
[3]Professor, Department of Mathematics, ShriJagdishprasadJhabarmalTibrewala University, Jhunjhunu, Rajasthan

Abstract: Diabetes mellitus is a group of diseases that affects the human body and creates various types of minor and major complications. The effect of diabetes has been analysed by the differential equations and solved by the mathematical methods. The outcomes are also compared with the RungeKutta Method and Taylor's series. These comparisons are represented graphically for the study of minor and major complications.

Keywords: Diabetes, Taylor's series, Differential Equations

Introduction: In present scenario, there is a risk in every health group to be confirmed as diabetic patient. In this situation most of them cure or control it at minor stage and some at major stage. The carbohydrates of the balanced food are converted immediately into monosaccharide glucose which is the main carbohydrate found in the blood. Some carbohydrates which are not converted to our fruit sugar mean fructose is used as cellular fuel as it is not converted to glucose and does not follow the insulin glucose metabolic regulator mechanism. The cellulose is not converted to glucose; the human body has no digestive pathway, capable of metabolizing cellulose.The most common cause of type-1 diabetes is autoimmune destruction, accompanied by antibodies that are directed against insulin and islet cell proteins. The principal treatment of Type-1 Diabetes during the early stages is to replace it with insulin. Without insulin, ketosis and diabetic ketoacidosis development will result in Coma or death.

In the early stages, the abnormality which reduces insulin sensitivity causes elevated levels of insulin in the blood. Hyperglycaemia can be reversed by medications that improve insulin sensitivity or reduce glucose production by the liver, with the complications of the disease, the impairment of insulin recreation is worse and the replacement of insulin often becomes necessary. The exact cause and mechanism for this resistance are due to its recreation of adipokines that impair glucose tolerance.

There is a rather stronger inheritance pattern for type-2 diabetes. Those having first-degree type-2 diabetes have a higher risk of developing type-2 diabetes. The monozygotic twins are close to 100% and about 25% of these have a family history of diabetes. It is also often connected to obesity, which is found in approximately 85% of patients diagnosed with this type. Sandhya et al. 2011, worked on glucose.

The model takes into account all plasma glucose concentrations, general insulin, and plasma insulin concentration. The numerical result provides the complex situation of diabetic patients. Diabetes must maintain a blood sugar level of 180 mg/dL after eating. In diabetics, the measurement is done two hours after a meal. Higher levels than 180 mg/dl show the person has taken too many carbohydrates or fatty or oily foods. Classical symptoms of diabetes mellitus such as frequent urination, excessive thirst, and fatigue accompany.

Taylor polynomials give a method for approximating a function near a particular point with a polynomial. We can use this approximation in order to replace a complicated function with a polynomial. One of the ways in which polynomials are simple is that it is easy to compute their derivatives and integrals. In this chapter, we see that it is possible to use Taylor polynomials and the Taylor series to obtain information about a function from information about its derivative.

The key observation is that if we know that

$$P(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)^2 + a_3(x - x_0)^3 + \cdots + a_n(x - x_0)^n$$

is the Taylor polynomial of degree n for f (x) cantered at x_o then

$$P^{'}(x) = a_1 + 2a_2(x - x_0)^2 + 3a_3(x - x_0)^3 + \cdots + na_n(x - x_0)^{n-1}$$

Is the Taylor polynomial of degree n-1 for $f^{'}$ centered at x_0.This is because the coefficient of x^j for the taylor polynomial of ‘f’ is

$$a_j = \frac{1}{j!}\frac{d^j f}{dx^j}$$

While the coefficient of x^j for the Taylor polynomial of $f^{'}$ is

$$ja_j = \frac{1}{(j-1)!}\frac{d^{j-1}(df/dx)}{dx^{j-1}}$$

Theorem: If f (x) is a continuously differentiable function and

$$f(x) = \sum_{n=0}^{\infty} a_n(\mathrm{x} - x_0)^n$$

with radius of convergence p then

$$f' = \sum_{n=1}^{\infty} na_n(\mathrm{x} - x_0)^{n-1}$$

and this series also has radius of convergence p.

The verification of this theorem has an easy part and a hard part. The easy part is the computation of the power series for f'. This just "differentiating termwise" and uses the usual rules for differentiating x^n over and over. This part is called the "formal computation". A differential equation is an equation that involves both a function and its derivative.

$$\frac{\mathrm{dy}}{\mathrm{dx}} = x^2$$

"Solving" this equation corresponds to finding a function y(x) such that the equation is true for all x. In this case, solving the differential equation is very easy. A function whose derivative is x^2 is

$$y(x) = \frac{x^3}{3} + c$$

The expression

$$\frac{\mathrm{dy}}{\mathrm{dx}} = x^2, y(0) = 5$$

The solution is

$$y(x) = \frac{x^3}{3} + 5$$

So, any anti-differentiation problem can be thought of as a differential equation. For example,

$$\frac{\mathrm{dy}}{\mathrm{dx}} = y$$

We cannot solve this equation, that is, find the unknown function y(x), by just anti-differentiating both sides because we would get

$$\int \frac{dy}{dx}dx = \int y(x)dx$$

While the integral on the left is easy, the one on the right is impossible because we do not know y(x) yet.As a last resort, you could say to yourself "Well, I don't know what y(x) is, but let's guess that whatever function it is, it is analytic." That is, you guess that the solution y(x) can be represented by its Taylor series. Starting with centre $x_0 = 0$, what you guess is

$$y(x) = \sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + \cdots$$

Now we move on to the test phase. We know that the derivative of this power series is given by

$$\frac{\mathrm{dy}}{\mathrm{dx}} = a_1 + 2a_2 x + 3a_3 x^2 + \cdots = \sum_{n=1}^{\infty} n a_n x^n$$

and if we plug this in to both sides of

$$\frac{\mathrm{dy}}{\mathrm{dx}} = y$$

We obtain

$$a_1 + 2a_2 x + 3a_3 x^2 + \cdots = a_0 + a_1 x + a_2 x^2 + \cdots$$

That is, the power series on the left is equal to the power series on the right. We are now faced with finding the coefficients a_0,a_1,a_2 . . . so that the power series on the left is equal to the power series on the right. This gives us the equations

$$a_1 = a_0$$

$$2a_2 = a_1$$

$$3a_3 = a_2$$

$$\vdots$$

It does not appear like we have made much progress. By guessing that the solution y(x) is a power series, we have replaced the differential equation

$$\frac{\mathrm{dy}}{\mathrm{dx}} = y$$

with infinitely many algebraic equations for the coefficients of the power series. Usually, we would rather be faced with one equation rather than infinitely many. However, looking at the equations above for the coefficients, we see that they have a very nice pattern. In particular

$$a_1 = a_0$$

so if we know a_0, then we know a_l. Next,

$$2a_2 = a_1$$

$$a_2 = \frac{a_1}{2} = \frac{a_0}{2}$$

and if we know a_0, then we know a_2. Next

$$3a_3 = a_2 = \frac{a_0}{2}$$

$$a_3 = \frac{a_0}{2 * 3}$$

and if we know a_0, then we know a_3. The pattern is already clear,

$$na_n = a_{n-1} = \frac{a_0}{(n-1)!}$$

$$a_n = \frac{a_0}{n!}$$

If we know a_0 then we know all the coefficients and

$$y(x) = a_0 + a_0 x + \frac{a_0}{2}x^2 + \frac{a_0}{2 * 3}x^3 \dots = \sum_{n=0}^{\infty} \frac{a_0}{n!}x^n$$

is the solution. Of course, we recognize this series as

$$y(x) = a_0 + \sum_{n=0}^{\infty} \frac{x^n}{n!} = a_0 e^x$$

The computation above is called finding a series solution to the differential equation.

Formulation of Mathematical Model:

The governing equation of the diabetes are given as below:

$$\frac{dX}{dt} = \Lambda - (\beta_1 + \beta_2)X - \mu X \quad (1)$$

$$\frac{dY_1}{dt} = \beta_1 X - \mu Y_1 - \eta_1 Y_1 - \gamma Y_1 \quad (2)$$

$$\frac{dY_2}{dt} = \beta_2 X - \mu Y_2 - \eta_2 Y_2 + \gamma Y_1 \quad (3)$$

Where,

Nomenclature:

X(t): Number of diabetic patients without complications

Y_1(t): Number of diabetic patients with minor complications

Y_2(t): Number of diabetic patients with major complications

N: Total populations (in Millions)

Λ: Incidence of diabetes

β_1: Growth rate of diabetic patients with minor complications

β_2: Growth rate of diabetic patients with major complications

γ:Conversion rate from minor complication to major complication

η_1: Death due to minor complications

η_2: Death due to major complications

t: time

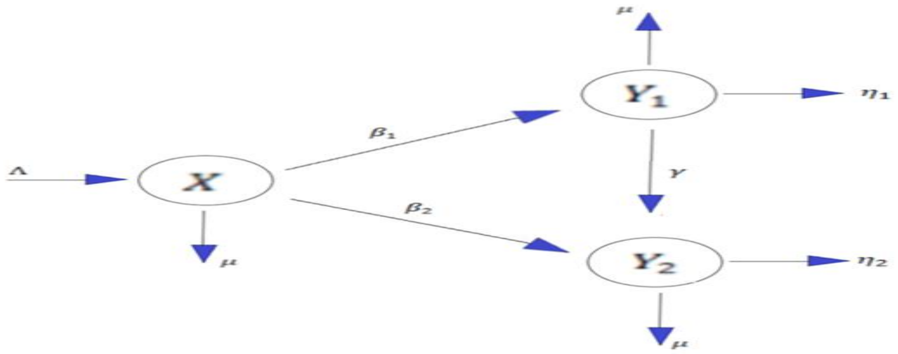

Figure 1: Structure of the Model XY_1Y_2

Analysis of Mathematical Model:

Method 1: RungeKutta method of order third.

In general, the approximation solution of the differential equation $\frac{dy}{dx} = f(x, y)$

with the condition $y(x_0) = y_0$

Compute, $x_1 = x_0 + h$

$$K_1 = f(x_0, y_0)$$

$$K_2 = f(x_0 + \frac{h}{2}, y_0 + \frac{h}{2} K_1)$$

$$K_3 = f(x_0 + h, y_0 - h\, K_1 + 2hK_2)$$

Therefore,

$$y_1 = y_0 + \frac{1}{6}\,(K_1 + 4K_2 + K_3)$$

Using the governing equation on RungeKutta method and developed the following system of eqautions:

$\frac{dX}{dt} = f_1 = \Lambda - (\beta_1 + \beta_2 + \mu)X$

$\frac{dY_1}{dt} = f_2 = \beta_1 X - (\eta_1 + \mu + \gamma)Y_1$

$\frac{dY_2}{dt} = f_3 = \beta_2 X - (\eta_2 + \mu)Y_2 + \gamma Y_1$

Using RungeKutta method of order third.

$X_{n+1} = X_n + \frac{1}{6}h\,[K_1 + 4K_2 + K_3].$

Here, $K_1 = f(x_n, y_n)$

$$K_2 = f(x_n + \frac{h}{2}, y_n + \frac{h}{2}K_1)$$

$$K_3 = f(x_n + h, y_n - hK_1 + 2hK_2)$$

So, the general equations for r ≥ 0 and h equals to time interval are given by,

$$X_{r+1} = X_r + \mathrm{K} = X_r + \frac{1}{6}[K_1 + 4K_2 + K_3] \quad (4)$$

$$(Y_1)_{r+1} = (Y_1)_r + \mathrm{L} = (Y_1)_r + \frac{1}{6}[L_1 + 4L_2 + L_3] \quad (5)$$

$$(Y_2)_{r+1} = (Y_2)_r + \mathrm{M} = (Y_2)_r + \frac{1}{6}[M_1 + 4M_2 + M_3] \quad (6)$$

Here K, L, and M are defined as

$$K_1 = \Lambda h - (\beta_1 + \beta_2 + \mu)hX_0$$

$$K_2 = \Lambda h - (\beta_1 + \beta_2 + \mu)h\,(X_0 + \frac{1}{2}h\,K_1)$$

$$K_3 = \Lambda h - (\beta_1 + \beta_2 + \mu)h\,(X_0 + h\,K_2)$$

Then, $K = \frac{1}{6}[K_1 + 4K_2 + K_3] = 1240868.1$

$$L_1 = h\beta_1 X_0 - h(\eta_1 + \mu + \gamma)(Y_1)_0$$

$$L_2 = h\beta_1\left(X_0 + \frac{h}{2}L_1\right) - h(\eta_1 + \mu + \gamma)\{(Y_1)_0 + hL_1\}$$

$$L_3 = h\beta_1(X_0 + hL_2) - h(\eta_1 + \mu + \gamma)\{(Y_1)_0 + hL_2\}$$

Then, $L = \frac{1}{6}[L_1 + 4L_2 + L_3] = 2389.04.$

$$M_1 = h\beta_2 X_0 - h(\eta_2 + \mu)(Y_2)_0 + \gamma(Y_1)_0$$

$$M_2 = h\beta_2\left(X_0 + \frac{h}{2}M_1\right) - h(\eta_2 + \mu)\left\{(Y_2)_0 + \frac{1}{2}hM_1\right\} + \gamma\{(Y_1)_0 + \frac{1}{2}hM_1\}$$

$$M_3 = h\beta_2(X_0 + hM_2) - h(\eta_2 + \mu)\{(Y_2)_0 + hM_2\} + \gamma\{(Y_1)_0 + hM_2\}$$

Then, $M = \frac{1}{6}[M_1 + 4M_2 + M_3] = 5194.03$

In the report, provided by the ICMR that the prevalence of diabetes in Chandigarh was 13%, 20% of prediabetics and 50% don't know that they have diabetes unless there is complication with one or the other organ.

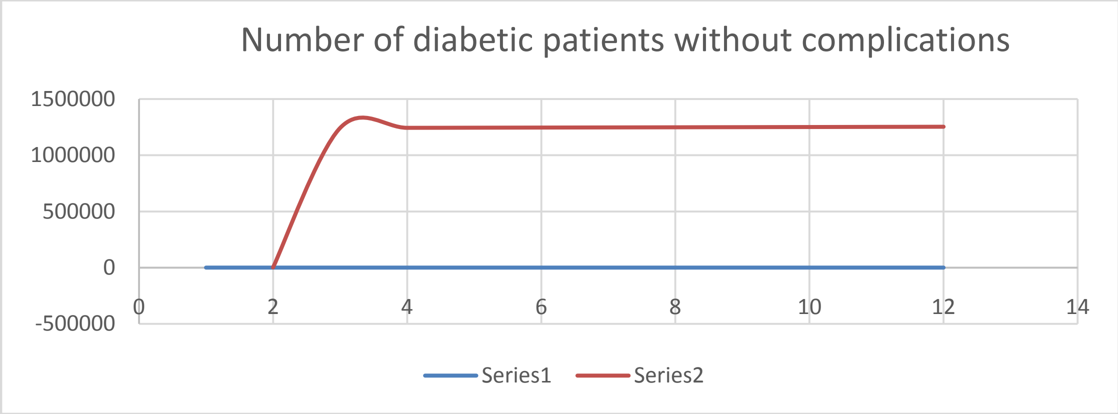

Figure 2: In the above graph, the number of diabetes patients without complications has highest conversion in yellow line with the range 2 – 4 and after that it likely to be constant and provides a straight line.

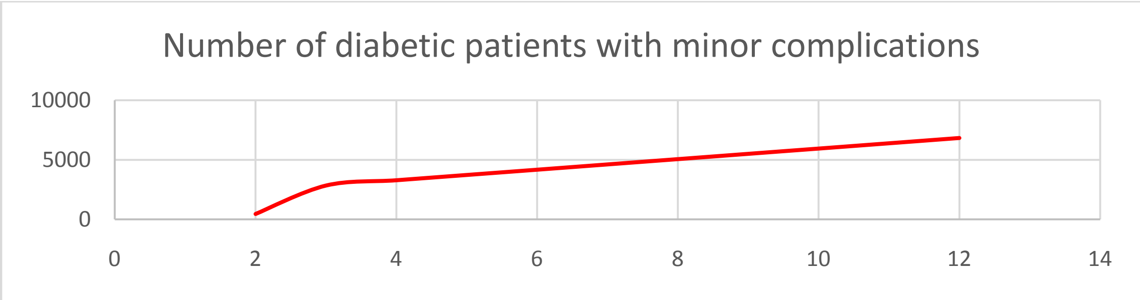

Figure 3: In the above graph, the number of diabetes patients with minor complications has highest conversion in red line with the range 2 – 4 and after that it again increases with respect to time.

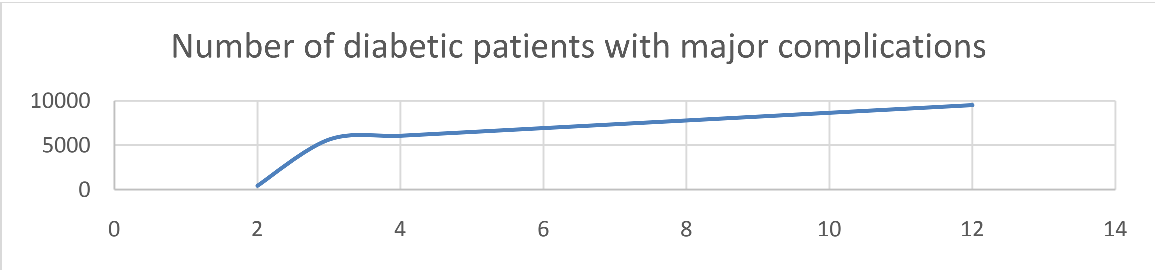

Figure 4: In the above graph, the number of diabetes patients with major complications has highest conversion in blue line with the range 2 – 4 and after that it again increases with respect to time.

Method 2: Taylor Method

Consider the given system as below

$$f_1 = \wedge - (\beta_1 + \beta_2 + \mu)X$$

$$f_2 = \beta_1 X - (\eta_1 + \mu + \gamma)Y_1$$

$$f_3 = \beta_2 X - (\eta_2 + \mu)Y_2 + \gamma Y_1$$

Since, Taylor series are polynomials that measure function for the given system of three variables, Taylors's series depends on first, second and so on.

Partial derivatives at some initial points $(x_0, y_0, z_0) \equiv (X_0, Y_{1(0)}, Y_{2(0)})$.

$\therefore\ f_1(X) = -(\beta_1 + \beta_2 + \mu), f_1(XX) = 0, f_2(X) = \beta_1, f_2(XX) = 0$,

$f_2(Y_1) = -(\eta_1 + \mu + \gamma), f_2(Y_1Y_1) = 0, f_3(X) = \beta_1, f_3(XX) = 0,$

$f_3(Y_1) = \gamma, f_3(Y_1Y_1) = 0, f_3(Y_2) = -(\eta_2 + \mu), f_3(Y_2Y_2) = 0,$

$f_1(XY_1) = 0, f_1(XY_2) = 0$and, so on.

The expansion of the governing equations in a Taylor's series around the initial points are as given below:

$$\begin{aligned} f(x,y,z) = f(x_0,y_0,z_0) + (x-x_0)f_x(x_0,y_0,z_0) + (y-y_0)f_y(x_0,y_0,z_0) \\ + (z-z_0)f_z(x_0,y_0,z_0) \\ + \frac{1}{2}\big[(x-x_0)^2 f_{xx}(x_0,y_0,z_0) + (x-x_0)(y-y_0)f_{xy}(x_0,y_0,z_0) \\ + (x-x_0)(z-z_0)f_{xz}(x_0,y_0,z_0) + (x-x_0)(y-y_0)f_{yx}(x_0,y_0,z_0) \\ + (y-y_0)(z-z_0)f_{yz}(x_0,y_0,z_0) + (y-y_0)^2 f_{yy}(x_0,y_0,z_0) \\ + (z-z_0)(x-x_0)f_{zx}(x_0,y_0,z_0) + (z-z_0)(y-y_0)f_{zy}(x_0,y_0,z_0) \\ + (z-z_0)^2 f_{zz}(x_0,y_0,z_0)\big] + R_3(x,y,z) \end{aligned}$$

After using initial values, the expansion provides the results,

$$X^* = f_1\big(X_0, Y_{1(0)}, Y_{2(0)}\big) + (X - X_0)(-\beta_1 - \beta_2 - \mu)$$

$$Y_1{}^* = f_2\big(X_0, Y_{1(0)}, Y_{2(0)}\big) + (X - X_0)\beta_1 + \big(Y_1 - Y_{1(0)}\big)(-\eta_1 - \mu - \gamma)$$

$$Y_2{}^* = f_3\big(X_0, Y_{1(0)}, Y_{2(0)}\big) + (X - X_0)\beta_2 + \big(Y_2 - Y_{2(0)}\big) - (\eta_2 - \mu) + \big(Y_2 - Y_{1(0)}\big)\gamma$$

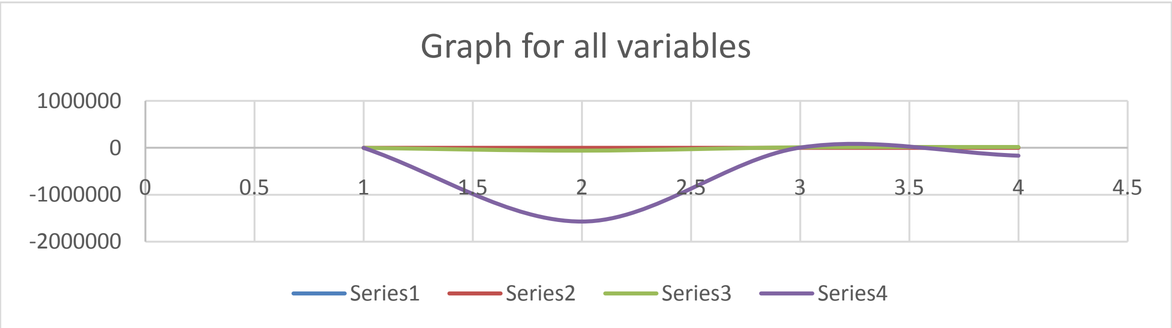

Figure 5: Comparison of all data

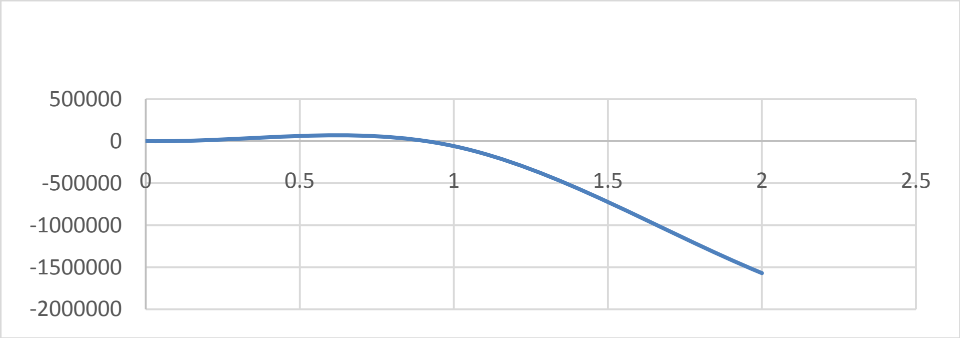

Figure 6: Graph for X, the curve is increased with respect to time but after medication it reduces the number of susceptibilities for X due to diabetes.

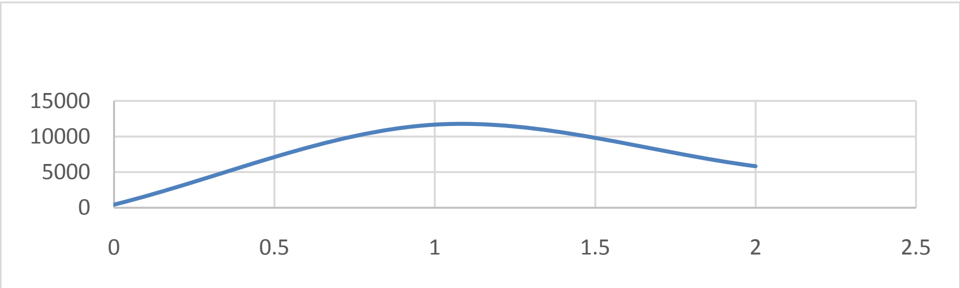

Figure 7: Graph for Y1, the curve is increased with respect to time but after medication it reduces the number of susceptibilities for Minor Cases Y1 due to diabetes.

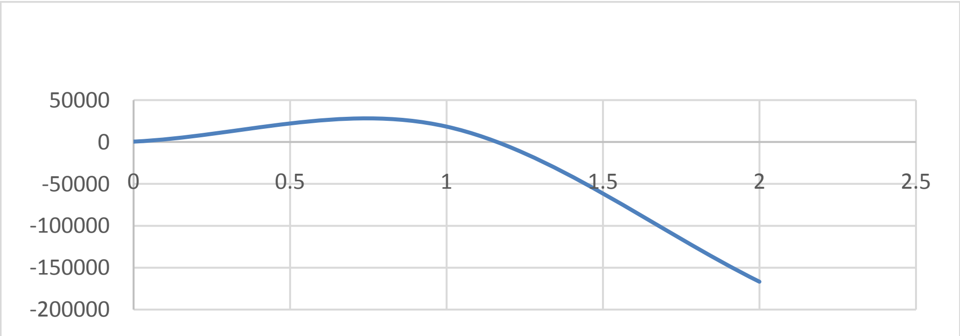

Figure 8: Graph for Y2, the curve is increased with respect to time but after medication it reduces the number of susceptibilities for major cases Y2 due to diabetes.

Conclusion: Diabetes model is helpful for understanding the conversion of diabetic patient form the minor to major complications. Diabetes mellitus is a group of diseases that affects the human body and creates various types of minor and major complications. The effect of diabetes has been analysed by the differential equations and solved by the mathematical methods. The outcomes are also compared with the RungeKutta Method and Taylor's series. These comparisons are represented graphically for the minor and major complications.From the Figure 1, 2, 3: the number of diabetes patients get instant growth after a short period of time interval 2 to 4 with initially minor stage to major stage.

References:

- Modu, G.U., Hadejia, Y.A., Ahmed, I., Kumam, W. and Thounthong, P., 2021. Analysis of Linear and Nonlinear Mathematical Models for Monitoring Diabetic Population with Minor and Major Complications. Thai Journal of Mathematics, 19(3), pp.1004-1027.
- Daud, A.A.M., Toh, C.Q. and Saidun, S., 2020. Development and analysis of a mathematical model for the population dynamics of Diabetes Mellitus during pregnancy. Mathematical Models and Computer Simulations, 12(4), pp.620-630.
- Kompas, S., Aldila, D. and Handari, B.D., 2020, November. Modelling the spread of diabetes transmission through social contact. In AIP Conference Proceedings (Vol. 2296, No. 1, p. 020098). AIP Publishing LLC.
- Anusha, S. and Athithan, S., 2021. Mathematical Modelling Co-existence of Diabetes and COVID-19: Deterministic and Stochastic Approach.
- Boutayeb, A., Twizell, E.H., Achouayb, K. and Chetouani, A., 2004. A mathematical model for the burden of diabetes and its complications. *Biomedical engineering online*, *3*(1), pp.1-8.