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Development of some fixed point Theorem in Parametric B-Metric Space

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Abstract:-
In this paper we develop some parametric B-Metric space and explore the existence of fixed point results for transforming contacts in the literature .Some went fixed point in triangular order generalized fuzzy B- Metric spaces the addition, some examples and their application are provided.

KEYWORDS:- Fixed point ,parametric metric space on parametric B- metric

1. Introduction

The development of fixed point theorem a parametric B- metric Spaces has been extended in many directions . It has developed by Czerarik (1993) Hussain (2009)Rao (2014), krishnakur (2016), Daheriya (2016), Boricreanu (2009), Alghamdi (2013)
In the recent years the different kinds of extension are developed by different types of mapping. There types of results are improve in existing literature.

2. Preliminaries

In this section we recall some important definition and their concept of parametric space for use of our develop results.
Def. (2.1) let x be a non -empty set ,$\emptyset \geq 1$be a real number and
$\emptyset : X * X * (0,+\infty) \rightarrow [0,+\infty]$be a function. We Ray $\emptyset$ is called parametric B- metric on X ^{0}of

$$< A1 > \emptyset(x,y,t) = 0 \; \forall t > 0 \; iffx = y$$

$< A2 > \emptyset(x,y,t) = \emptyset$ (x, y ,t) $\forall t > 0$

$$< A3 > \emptyset(x,y,t) \leq [\emptyset(x,y,t) + \emptyset(z,y,t)] \forall x,y,t \in X, t > 0 \; andp \geq 1$$

and the pair (X,$\emptyset$) is a parametric space with paramet s$\geq 1 \; andifs = 1 \; thenitiscalled$parametric B- Metric space is reduces form.
Def. (2.2) Let the sequence $< x >$ is a sequence in parametric metric space (x,$\emptyset, p$) if

$$< A1 >< x > convergenttox \in Xif \lim_{n\to\infty} \emptyset(x,y,t) \quad = 0,$$

define as $\lim_{n\to\infty} x_n = x \forall t > 0$.
$< A2 >< x >$is said to cauchy sepauce x∈X if lim $\emptyset(x,y,t) = 0 \forall t > 0$
$< A3 >$(x,$\emptyset, p$) is called complete if sequence is convergent se ...... convergent ....

Example: Suppose X = $[0,+\infty]$and $\emptyset : X * X * (0,+\infty) \rightarrow (0,+\infty) writtenas$

This $\emptyset$is a parametric B- metric with constant p=2^s

Definition (2.3) Suppose $(x,\emptyset,p)$ be a parametric b-metric space and mapping T: X $\rightarrow X$is continuous type mapping at x is X, if

$\lim_{n\rightarrow\infty} x^n$ =x => $\lim_{n\rightarrow\infty} Tx^n$ =T_x

3. lemmas

lemma (3.1) Suppose (X ,$\emptyset,p$)is a b-metric space with p=1 and $< xn >_{n=1}^{\infty}$is convergent to x and above$< xn >_{n=1}^{\infty}$is converge to you then x=y, means $< xn >_{n=1}^{\infty}$ is unique.

Lemma (3.2) Suppose (X ,$\emptyset,p$) is a b- metric with p=1 and the sepace$< xn >_{n=1}^{\infty}$ is convergent to x,

then

1/p $\emptyset$ (x, y, t) $\leq \lim_{n\rightarrow\infty} \emptyset (x_n, y, t) \leq p\emptyset(x, y, t) \forall$ y$\in X$ and $\forall$t>0.

lemma (3.3) Suppose (X,$\emptyset,p$) is b-metric space with corfficient p=1 and$< xn >_{n=1}^{\infty}$ CX.

Then

$\emptyset(x_n,x_0,t) \leq p\, \emptyset (x_0,x_1,t)+ p^2\emptyset(x_2,x_3,t) +.............+ p^{n-2}\emptyset(x_{n-2},x_{n-1},t) + p^{n-1}\emptyset(x_{n-1},x_n,t)$.

Lemma (3.4) Suppose (x,$\emptyset$,p) is a b-metric space with coefficient p=1.

Let the sequence$< xn >_{n=1}^{\infty}$ be the point of x s.t.

$\emptyset(x_n,x_{n+1},t) \leq p\, \emptyset (x_{n-1},x_n,t)$

where$p \in [0,1/n]$, n=1,2,3,..........

then$< xn >_{n=1}^{\infty}$is cauchysepace.λ

L_1. Main Results

Theorem (a.1) suppose $(\propto,\emptyset)$ be a complete parametric B – Metric space and Y is a continuous mapping define the condition

$$T_{x,}T_{y,}T_{z,}t+\lambda_1\left\{\frac{\emptyset(x,T_{y,}T_{z,}t)}{1+\emptyset(T_{x,}y,t)\emptyset(y,T_{y,}T_{z,}t)}\right\}$$
$$+\lambda_2\left\{\frac{\emptyset(y,T_{y,}T_{z,}t)}{1+\emptyset(T_{x,}y,z,t)\emptyset(y,T_{y,}T_{z,}t)}\right\}$$
$$+\lambda_3\left\{\frac{\emptyset(z,T_{z,}t)}{1+\emptyset(T_{x,}y,z,t)\emptyset(z,T_{z,}t)}\right\}$$
$$\leq \lambda_4(x,y,z,t)+\lambda_5 max[\emptyset(x,T_{y,}T_{z,}t),\emptyset(y,T_{y,}T_{z,}t),\emptyset(z,T_{z,}t)]$$
$$\lambda_4(x,y,z,t)$$
$$\vee x,y,z \in X, x\neq y\neq z and \lambda_1,\lambda_2,\lambda_3,\lambda_4 \geq 0\ and (\lambda_4+\lambda_5-2\lambda_1-\lambda_2-\lambda_3)<1$$

Then T Ram a fixed point is x.

Pr

$T_{xn} = X_{n+1}, T_{yn} = Y_{n+1} and T_{zn} = Z_{n+1} \nless n = 1,2,3,4$ ...oof: As we know by d above definition

Now

$$\emptyset\ (T_{xn}, T_{xn+1}, t) + \lambda_1 \left\{\frac{\emptyset\ (T_{xn}, T_{xn+1}, t)}{1 + \emptyset\ (T_{xn}, T_{xn+1}, t)\emptyset\ (X_{n+1}, T_{xn+1}, t)}\right\} + \lambda_2 \left\{\frac{\emptyset\ (X_{n+1}, T_{xn+1}, t}{1 + \emptyset\ (T_{xn}, X_{n+1}, t)\emptyset\ (X_{n+1}, T_{xn+1}, t)}\right\} + \lambda_3 \emptyset(X_{n,} X_{n+1}, t) + \lambda_4 max\left[\emptyset(X_{n,} T_{xn,} t)\emptyset\ (X_{n+1}, T_{xn+1}, t\right]$$

Similarly

$$\emptyset\ (X_{n+1}, X_{n+2}, t) + \lambda_1 \left\{\frac{\emptyset\ (X_n, X_{n+2}, t)}{1 + \emptyset\ (X_{n+1}, X_{n+1}, t)\emptyset\ (X_{n+1}, X_{n+2}, t)}\right\}$$

$$+\lambda_2 \left\{\frac{\emptyset\ (X_{n+1}, X_{n+2}, t)}{1 + \emptyset\ (X_{n+1}, X_{n+1}, t)\emptyset\ (X_{n+1}, X_{n+2}, t)}\right\} \leq \lambda_2 \emptyset\ (X_n, X_{n+1}, t) \leq \lambda_4 \max[\emptyset\ (X_n, X_{n+1}, t), \emptyset\ (X_{n+1}, X_{n+2}, t)]$$

Therefore

$$P\ (X_{n+1}, X_{n+2}, t) + \lambda_1 \emptyset\ (X_n, X_{n+2}, t) + \lambda_2\ (X_{n+1}, X_{n+2}, t)$$

Where

$$\leq \lambda_3 \emptyset\ (X_n, X_{n+1}, t) \leq \lambda_4 \max[\emptyset\ (X_n, X_{n+1}, t), \emptyset\ (X_{n+1}, X_{n+2}, t)]$$

By the same way, it is define as

$$P\ (Y_{n+1}, Y_{n+2}, t) + \lambda_1 \emptyset\ (Y_n, Y_{n+2}, t) + \lambda_3\ (Y_{n+1}, Y_{n+2}, t)$$

Where

$$\leq \lambda_3 \emptyset(Y_n, Y_{n+1}, t) \leq \lambda_4 \max[\emptyset\ (Y_n, Y_{n+1}, t), \emptyset\ (Y_{n+1}, Y_{n+2}, t)]$$

And

$$P\ (Z_{n+1}, Z_{n+2}, t) + \lambda_1 \emptyset\ (Z_n, Z_{n+2}, t) + \lambda_3\ (Z_{n+1}, Z_{n+2}, t)$$

Where

$$\leq \lambda_3 \emptyset(Z_n, Z_{n+1}, t) \leq \lambda_4 \max[\emptyset\ (Z_n, Z_{n+1}, t), \emptyset\ (Z_{n+1}, Z_{n+2}, t)]$$

Corollary (i)

$$\max[\emptyset\ (X_n, X_{n+1}, t), \emptyset\ (X_{n+1}, X_{n+2}, t)] = \emptyset\ (X_n, X_{n+1}, t)$$

We get

$$\emptyset\ (X_{n+1}, X_{n+2}, t) + \lambda_1\{\emptyset\ (X_n, X_{n+1}, t) + \emptyset\ (X_{n+1}, X_{n+2}, t)\} + \lambda_2 \emptyset(X_{n+1}, X_{n+2}, t) \leq \lambda_3 \emptyset\ (X_n, X_{n+1}, t), \lambda_4 \emptyset\ (X_n, X_{n+1}, t)$$

$$\Rightarrow (1 + \lambda_1 + \lambda_2)\emptyset\ (X_{n+1}, X_{n+2}, t) \leq (\lambda_3 + \lambda_4 - \lambda_1)\emptyset\ (X_n, X_{n+1}, t)$$

$$\Rightarrow \emptyset\ (X_{n+1}, X_{n+2}, t) \leq \frac{(\lambda_3 + \lambda_4 - \lambda_1)}{(1 + \lambda_1 + \lambda_2)} \emptyset(X_n, X_{n+1}, t)$$

$$\Rightarrow \emptyset\ (X_{n+1}, X_{n+2}, t) \leq \propto \emptyset(X_n, X_{n+1}, t)$$

Where $\propto = \frac{(\lambda_3+\lambda_4-\lambda_1)}{(1+\lambda_1+\lambda_2)} < 1$

Similarly we get

$$\emptyset\ (Y_{n+1}, Y_{n+2}, t) \leq \propto \emptyset\ (Y_n, Y_{n+1}, t)$$

Where $\propto = \frac{(\lambda_3+\lambda_4-\lambda_1)}{(1+\lambda_1+\lambda_2)} < 1$

And

$$\emptyset\ (Z_{n+1}, Z_{n+2}, t) \leq \emptyset\ (Z_n, Z_{n+1}, t)$$

Where $\propto= \frac{(\lambda_3+\lambda_4-\lambda_1)}{(1+\lambda_1+\lambda_2)} < 1$

By the same way

$$\Longrightarrow \emptyset\,(X_{n+1}, X_{n+2}, t) \leq \propto^2\, \emptyset(X_{n+}, X_n, t)$$
$$\dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots$$
$$\Longrightarrow \emptyset\,(X_{n+1}, X_{n+2}, t) \leq \propto^n\, \emptyset(X_0, X_1, t)$$
$$\emptyset\,(Y_{n+1}, Y_{n+2}, t) \leq \beta^2 \emptyset\,(Y_n, Y_{n+1}, t)$$
$$\dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots$$
$$\emptyset\,(Y_{n+1}, Y_{n+2}, t) \leq \beta^n \emptyset\,(Y_0, Y_1, t)$$

And

$$\emptyset\,(Z_{n+1}, Z_{n+2}, t) \leq \gamma^2\,(Z_{n-1}, Z_n, t)$$
$$\dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots$$
$$\emptyset\,(Z_{n+1}, Z_{n+2}, t) \leq \gamma^n\,(Z_0, Z_1, t)$$

Using lemma $\{X_n\}_{n\in N}, \{Y_n\}_{n\in N}$ and $\{Z\}_{n\in N}$ is a Cauchy space is X. But X be a complete parametric space and converges if taleip limit $x_{n,}y_{n,}z_n \longrightarrow \propto andn \to \infty$.

We get

$$T_\propto = T(\lim_{n\to\infty} x_n) = \lim_{n\to\infty} T x_n = \lim_{n\to\infty} x_{n+1=\propto}$$
$$T_\beta = T(\lim_{n\to\infty} y_n) = \lim_{n\to\infty} T y_n = \lim_{n\to\infty} y_{n+1=\propto}$$

And

$$T_\gamma = T(\lim_{n\to\infty} z_n) = \lim_{n\to\infty} T z_n = \lim_{n\to\infty} z_{n+1=\propto}$$

Therefore, T has a fixed point of X.

Corollary (ii) .if

$$\max[\emptyset\,(X_{n+1}, X_n, t), \emptyset\,(X_{n+2}, X_{n+1}, t), \emptyset\,(X_{n+2}, X_n, t)] = \emptyset\,(X_{n+2}, X_n, t)$$

There $\emptyset\,(X_n, X_{n+1}, t), \lambda_1 \emptyset\,(X_{n+2}, X_{n+1}, t) \geq \lambda_2 \emptyset\,(X_{n+1}, X_{n+2}, t) + (\lambda_3 + \lambda_4 + \lambda_5)\emptyset\,(X_{n+1}, X_n, t)$

$$[1 - (\lambda_3 + \lambda_4 + \lambda_5)]\emptyset\,(X_n, X_{n+1}, t) \geq (\lambda_2 - \lambda_1)\emptyset\,(X_{n+1}, X_{n+2}, t)$$
$$(a_2 - a_1)\emptyset\,(X_{n+1}, X_{n+2}, t) \leq 1 - (\lambda_3 + \lambda_4 + \lambda_5)]\emptyset\,(X_n, X_{n+1}, t)$$

$$(a_2 - a_1)\emptyset\,(X_{n+1}, X_{n+2}, t) \leq 1 - (\lambda_3 + \lambda_4 + \lambda_5)]\emptyset\,(X_n, X_{n+1}, t)$$
$$\Longrightarrow (1 - \lambda_1 - \lambda_3 - \lambda_4 - \lambda_5)]\emptyset\,(X_{n+1}, X_{n+2}, t) \geq (\lambda_2 - \lambda_1)\emptyset\,(X_{n+1}, X_{n+2}, t)$$
$$\emptyset\,(X_{n+1}, X_{n+2}, t) \leq \frac{(1 - \lambda_3 - \lambda_4 - \lambda_5)}{(\lambda_2 - \lambda_1)}\emptyset\,(X_{n+1}, X_{n+2}, t)$$

Proof$\frac{(1-\lambda_3-\lambda_4-\lambda_5)}{(\lambda_2-\lambda_1)}$

$$\Longrightarrow \emptyset\,(X_{n+1}, X_{n+2}, t) \leq P\emptyset\,(X_{n+1}, X_n, t)$$

By mathematical induction method,

We get

$$\emptyset\,(X_{n+1}, X_{n+2}, t) \leq P^{n+1}\emptyset\,(X_0, X_n, t)$$

similarly

$$\emptyset\,(Y_{n+1}, Y_{n+2}, t) \leq Q^{n+1}\emptyset\,(Y_0, Y_n, t)$$

and

$$\emptyset\,(Z_{n+1}, Z_{n+2}, t) \leq R^{n+1}\emptyset\,(Z_0, Z_n, t)$$

since X is complex parametric b-metric space and hence $\{X_n\}_{n\in N}, \{Y_n\}_{n\in N}$ and $\{Z\}_{n\in N}$ are converges a limit z, by lamma of Cauchy sequence. The $a \in X, x_{n,}y_{n,}Z_n \to aash \to \infty$

there we get

$$T(\lim_{n\to\infty y_n, z_n,} x_n) = \lim\{X_{n+1}, Y_{n+1}, Z_{n+1}\}$$
$$i, e., T_a = a$$

There T hence a fixed point is sequence X.

Uniqueness:

Let x^*, y^*, z^* be and then fixed point of T is X; then

$$Tz^* = z^*, Ty^* = y^* and Tx^* = x^*.$$

Now

$$\emptyset(Tx^*, Ty^*, Tz^*, t) \geq \lambda \frac{\emptyset(x^*, Tx^*, t)\emptyset(y^*, Ty^*, t)\emptyset(z^*, Tz^*, t)}{\emptyset(x^*, y^*, z^*, t) + \emptyset(x^*, Tx^*, t) + \emptyset(y^*, Ty^*, t) + \emptyset(z^*, Tz^*, t)}$$

$$+ \propto \emptyset(x^*, y^*, z^*, t) - \beta\emptyset(y^*, Ty^*, t) - \gamma\emptyset(z^*, Tz^*, t)$$

$$\Rightarrow \emptyset(x^*, y^*, z^*, t) \geq \lambda\emptyset(x^*, y^*, t) - \beta\emptyset(x^*, Ty^*, t) - \gamma\emptyset(x^*, y^*, z^*, t)$$

$$\geq (\lambda - \beta - \gamma)\emptyset(x^*, y^*, z^*, t)$$

$$\Rightarrow \emptyset(x^*, y^*, z^*, t) \leq \frac{1}{(\lambda - \beta - \gamma)}\emptyset(x^*, y^*, z^*, t)$$

The above result is hence for

$$\emptyset(x^*, y^*, z^*, t) = 0$$

So that

$$x^* = y^* = z^*$$

Hence T has a unique fixed point in X.

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