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SOME CENTRALIZING THEOREMS ON GENERALIZED$(\alpha, 1)$- REVERSE DERIVATIONS IN SEMI PRIME RINGS

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ABSTRACT

Let R be a semiprime ring, F:R→R be a generalized$(\alpha, 1)$- reverse derivation associated with $(\alpha, 1)$ - reverse derivation d and H: R→R be a right α -centralizer. If (i) $F(uv) \pm H(uv) = 0$; (ii) $F(uv) \pm H(vu) = 0$; (iii) $F(u)F(v) \pm H(uv) = 0$; (iv) $F(uv) \pm H(uv) \in C_{\alpha,1}$; (v) $F(uv) \pm H(vu) \in C_{\alpha,1}$; (vi) $F(u)F(v) \pm H(uv) \in C_{\alpha,1}$, for all $u, v \in R$.

KEY WORDS: Semiprime Ring; Right α-centralizer; $(\alpha, 1)$ - Reverse Derivation; Generalized $(\alpha, 1)$ - Reverse Derivation.

AMS Subject Classification: 16N60; 16S20; 16W25; 16U80.

1. INTRODUCTION

The concept of reverse derivation was first time introduced by Herstien [3]. Aboubakr et.al. [1] generalized the concept of reverse derivations to generalized reverse derivations and provided a study of relationship between generalized reverse derivations and generalized derivations. Inspired by this, Tiwari et.al. [11] gave the notion of multiplicative (generalized) reverse derivations. Yenigul and Argac [12] studied prime and semiprime rings with α- derivations. Ibraheem [4] and Asma Ali et.al. [2] studied generalized reverse derivation on semiprime or prime rings. Jaya Subba Reddy et.al. Proved some results on reverse derivations, generalized (σ, τ) derivations in semiprime rings, properties of left (α, 1) - derivations in prime rings and also proved some results on (α, 1) - reverse derivations on prime near-rings (See in [5-8]). Several authors have proved annihilator conditions of multiplicative (generalized) reverse derivations, some results of generalized reverse derivations for semiprime rings or prime rings ([10], [9]). In this paper, we proved some results on generalized (α, 1) - reverse derivations in semiprime rings.

2. PRELIMINARIES

Through out this paper R denote an associative ring with center Z. Recall that a ring R is semiprime if $aRa = \{0\}$ implies $a = 0$. For any $u, v \in R$, the symbol $[u, v]$ stands for the commutator $uv - vu$. The $(\alpha, 1)$ center of R denoted by $C_{\alpha,1}$ and defined by $C_{\alpha,1} = \{c \in R: c\alpha(r) = rc, \text{ for all } r \in R\}$. An additive mapping $d: R \to R$ is called a reverse derivation if $d(uv) = d(v)u + ud(v)$, for all $u, v \in R$. An additive mapping $d: R \to R$ is called a $(\alpha, 1)$ - reverse derivation if $d(uv) = d(v)\alpha(u) + vd(u)$, for all $u, v \in R$. An additive mapping $F: R \to R$ is called a generalized reverse derivation, if there exists a reverse derivation $d: R \to R$ such that $F(uv) = F(v)u + vd(u)$, for all $u, v \in R$. An additive mapping $F: R \to R$ is said to be a generalized $(\alpha, 1)$ - reverse derivation of R, if there exists a $(\alpha, 1)$ - reverse derivation $d: R \to R$ such that $F(uv) = F(v)\alpha(u) + vd(u)$, for all $u, v \in R$. An additive mapping $H: R \to R$ is called a right α-centralizer if $H(uv) = \alpha(u)H(v)$, for all $u, v \in R$, where α is an automorphism of R. Throughout this paper, we shall make use of the basic commentator identities:

$$[u, vw] = v[u, w] + [u, v]w;$$

$$[uv, w] = [u, w]v + u[v, w];$$

$$[uv, w]_{\alpha,1} = u[v, w]_{\alpha,1} + [u, w]v.$$

Lemma 2.1: Let R be a semiprime ring. If $F: R \to R$ is a generalized $(\alpha, 1)$ - reverse derivation associated with $(\alpha, 1)$- reverse derivation on d, then $d(uv) = d(v)\alpha(u) + vd(u)$, for all $u, v \in R$.

Proof: We have $F(vu) = F(u)\alpha(v) + ud(v)$, for all $u, v \in R$.

Replacing v by wv in the above equation, we get

$F((wv)u) = F(u)\alpha(wv) + ud(wv)$, for all $u, v, w \in R$. (2.1)

On the other hand, we have

$F(w(vu)) = F(u)\alpha(vw) + ud(v)\alpha(w) + vud(w)$, for all $u, v, w \in R$. (2.2)

Equating equations (2.1) and equation (2.2), we get

$F(u)\alpha(wv) + ud(wv) = F(u)\alpha(vw) + ud(v)\alpha(w) + vud(w)$, for all $u, v, w \in R$.

$u(d(wv) - d(v)\alpha(w) - vd(w)) = 0$, for all $u, v, w \in R$. (2.3)

Left multiplying equation (2.3) by $d(wv) - d(v)\alpha(w) - vd(w)$, we get

$(d(wv) - d(v)\alpha(w) - vd(w))u(d(wv) - d(v)\alpha(w) - vd(w)) = 0$, for all $u, v, w \in R$.

$(d(wv) - d(v)\alpha(w) - vd(w))R(d(wv) - d(v)\alpha(w) - vd(w)) = 0$, for all $u, v, w \in R$.

Since R is semiprime ring, we get $d(wv) = d(v)\alpha(w) + vd(w)$, for all $v, w \in R$.

For this $d(uv) = d(v)\alpha(u) + vd(u)$, for all $u, v \in R$.

That is, d is a $(\alpha,1)$ - reverse derivation.

Lemma 2.2: Let R be a semiprime ring and $F: R \to R$ is a generalized $(\alpha, 1)$ - reverse derivation associated with $(\alpha, 1)$- reverse derivation d. If $F(uv) = 0$, for all $u, v \in R$, then $F = 0$ and $d = 0$.

Proof: We have $F(uv) = 0$, for all $u, v \in R$. (2.4)

Replacing u by wu in equation (2.4), we get $F(uv)\alpha(w) + uvd(w) = 0$.

Using (2.4) in the above equation, we get $uvd(w) = 0$, for all $u, v, w \in R$. (2.5)

Left multiplying equation (2.5) by $vd(w)$, we get $vd(w)uvd(w) = 0$, for all $u, v, w \in R$. $vd(w)Rvd(w) = 0$, for all $u, v, w \in R$.

Since R is semiprime ring, we get $vd(w) = 0$, for all $v, w \in R$. (2.6)

Left multiplying equation (2.6) by $d(w)$, we get $d(w)vd(w) = 0$, for all $v, w \in R$.

By the semiprimeness of R, we get $d(w) = 0$, for all $w \in R$. (2.7)

By the hypothesis $F(uv) = 0$, for all $u, v \in R$. $F(v)\alpha(u) + vd(u) = 0$, for all $u, v \in R$.

Using equation (2.7) in the above equation, we get $F(v)\alpha(u) = 0$, for all $u, v \in R$. (2.8)

Right multiplying equation (2.8) by $F(v)$, we get $F(v)\alpha(u)F(v) = 0$, for all $u, v \in R$.

Since α is an automorphism of R, we get $F(v)RF(v) = 0$, for all $v \in R$.

SinceR is semiprime ring, we get $F(v) = 0$, for all$v \in R$.

Hence $F = 0$and $d = 0$when $F(uv) = 0$, for all $u, v \in R$.

Lemma 2.3: Let R be a semiprime ring and $F: R \to R$ is a generalized $(\alpha, 1)$- reverse derivation associated with $(\alpha, 1)$- reverse derivation d. If $F(uv) \in C_{\alpha,1}$, for all $u, v \in R$, then $[d(u), u]_{\alpha,1} = 0$, for all $u \in R$.

Proof: We have $F(uv) \in C_{\alpha,1}$, for all $u, v \in R$. (2.9)

Replacingu bywu in equation (2.9), we get $F(uv)\alpha(w) + uvd(w) \in C_{\alpha,1}$, for all $u, v, w \in R$. Using equation (2.9) in the above equation, we get $uvd(w) \in C_{\alpha,1}$, for all $u, v, w \in R$. $[uvd(w), w]_{\alpha,1} = 0$, for all $u, v, w \in R$.

$uv[d(w), w]_{\alpha,1} + [uv, w]d(w) = 0$, for all $u, v, w \in R$.

$uv[d(w), w]_{\alpha,1} + u[v, w]d(w) + [u, w]vd(w) = 0$, for all $u, v, w \in R$.

Replacing v by w in the above equation, we get

$uw[d(w), w]_{\alpha,1} + u[w, w]d(w) + [u, w]wd(w) = 0$, for all $u, w \in R$.

Again replacing w by u in the above equation, we get

$uu[d(u), u]_{\alpha,1} = 0$, for all $u \in R$. (2.10)

Left multiplying equation (2.10) by $u[d(u), u]_{\alpha,1}$, we get $u[d(u), u]_{\alpha,1}uu[d(u), u]_{\alpha,1} = 0$.

$u[d(u), u]_{\alpha,1}Ru[d(u), u]_{\alpha,1} = 0$, for all $u \in R$.

Since R is semiprime ring, we get $u[d(u), u]_{\alpha,1} = 0$, for all $u \in R$. (2.11)

Left multiplying equation (2.11) by $[d(u), u]_{\alpha,1} = 0$, we get

$[d(u), u]_{\alpha,1}u[d(u), u]_{\alpha,1} = 0$, for all $u \in R$.

By the semiprimeness of R, we conclude that $[d(u), u]_{\alpha,1} = 0$, for all $u \in R$.

Lemma 2.4: Let R be a semiprime ring, $F: R \to R$ is a generalized $(\alpha, 1)$- reverse derivation associated with $(\alpha, 1)$- reverse derivation d and $H: R \to R$ be a right α-centralizer. If the map $G: R \to R$ is defined as $G(u) = F(u) \pm H(u)$, for all $u \in R$, then G is a generalized $(\alpha, 1)$- reverse derivation associated with $(\alpha, 1)$- reverse derivation d.

Proof: We suppose that $G(u) = F(u) \pm H(u)$, for all $u \in R$. (2.12)

Replacingu byuv inequation (2.12), we get $G(uv) = F(uv) \pm H(uv)$, for allu, $v \in R$.

$G(uv) = F(v)\alpha(u) + vd(u) \pm \alpha(u)H(v)$, for all $u, v \in R$.

$G(uv) = (F(v) \pm H(v))\alpha(u) + vd(u)$, for all $u, v \in R$.

Using equation (2.12) in the above equation, we get $G(uv) = G(v)\alpha(u) + vd(u)$, for all $u, v \in R$. Then G is a generalized $(\alpha, 1)$- reverse derivation associated with $(\alpha, 1)$- reverse derivation d.

3. MAINRESULTS

Theorem 3.1: Let R be a semiprime ring, $F: R \to R$ is a generalized $(\alpha, 1)$- reverse derivation associated with $(\alpha, 1)$ - reverse derivation d and $H: R \to R$ be a right α -centralizer. If $F(uv) \pm H(uv) = 0$, for all $u, v \in R$, then $d = 0$. Moreover, $F(uv) = F(v)\alpha(u)$, for all $u, v \in R$ and $F = \pm H$.

Proof: By the hypothesis, weh ave $F(uv) - H(uv) = 0$, for all $u, v \in R$.

Using equation (2.12) in the above equation, we get $G(uv) = 0$, for all $u, v \in R$.

Using lemma 2.2 and lemma2.4, we get $G = 0$. So, we have $F = H$. (3.1)

By the hypothesis, we have $F(uv) - H(uv) = 0$, for all $u, v \in R$.

$F(v)\alpha(u) + vd(u) - \alpha(u)H(v) = 0$, for all $u, v \in R$.

Using equation (3.1) in the above equation, we get $vd(u) = 0$, for all $u, v \in R$. (3.2)

The equation (3.2) is same as equation (2.6) in lemma 2.2. Thus, by same argument of lemma 2.2, we can conclude the result $d(u) = 0$, for all $u \in R$. (3.3)

By the definition of F, we have $F(uv) = F(v)\alpha(u) + vd(u)$, for all $u, v \in R$.

Using equation (3.3) in the above equation, we get $F(uv) = F(v)\alpha(u)$, for all $u, v \in R$.

Similar proof shows that the same conclusion holds as$F(uv) + H(uv) = 0$, for all $u, v \in R$. In this case, we obtain $F = - H$. Hence the proof is completed.

Theorem 3.2: Let R be a semiprime ring, $F: R \to R$ is a generalized $(\alpha, 1)$- reverse derivation associated with $(\alpha, 1)$ - reverse derivation d and $H: R \to R$ be a right α -centralizer. If $F(uv) \pm H(vu) = 0$, for all $u, v \in R$, then $d = 0$. Moreover, $F(uv) = F(v)\alpha(u)$, for all $u, v \in R$ and $[F(u), \alpha(u)] = 0$, for all $u \in R$.

Proof: By the hypothesis, we have $F(uv) - H(vu) = 0$, for all $u, v \in R$. (3.4)

Replacing u by wv and v by u in equation (3.4), we get

$(F(vu) - H(uv))\alpha(w) + \alpha(w)H(uv) - \alpha(u)\alpha(w)H(v) + vud(w) = 0$, for all $u, v, w \in R$.

Using equation (3.4) in the above equation, we get

$\alpha(w)\alpha(u)H(v) - \alpha(u)\alpha(w)H(v) + vud(w) = 0$, for all $u, v, w \in R$.

$H(v)\alpha[w,u] + vud(w) = 0$, for all $u, v, w \in R$. (3.5)

Replacing w by u in equation (3.5), we get $vud(u) = 0$, for all $u, v, w \in R$. (3.6)

The equation (3.6) is same as equation (2.5) in lemma 2.2. Thus, by same argument of lemma 2.2, we can conclude the result $d(u) = 0$, for all $u \in R$. (3.7)

By the definition of F, we have $F(uv) = F(v)\alpha(u) + vd(u)$, for all $u, v \in R$.

Using (3.7) in the above equation, we get $F(uv) = F(v)\alpha(u)$, for all $u, v \in R$. (3.8)

Using equation (3.7) in equation (3.5), we get $H(v)\alpha[w,u] = 0$, for all $u, v, w \in R$. (3.9)

Replacing v by wv in equation (3.9), we get $H(wv)\alpha[w,u] = 0$, for allu, v, w $\in R$.

Using equation (3.4) in the above equation, we get $F(vw)\alpha[w,u] = 0$, for all $u, v, w \in R$.

Using equation(3.8) in the above equation, we get $F(w)\alpha(v)\alpha[w,u] = 0$, for all $u, v, w \in R$. Interchange u and w places in the above equation, we get

$F(u)\alpha(v)\alpha[u,w] = 0$, for all $u, v, w \in R$. (3.10)

Replacing v by vw in equation (3.10), we get

$F(u)\alpha(v)\alpha(w)\alpha[u,w] = 0$, for allu, v, w $\in R$. (3.11)

Left multiplying equation (3.10) by $\alpha(w)$, we get

$\alpha(w)F(w)\alpha(v)\alpha[u,w] = 0$, for all $u, v, w \in R$. (3.12)

We subtracting equation (3.12) from equation (3.11), we get

$[F(u),\alpha(w)]\alpha(v)[\alpha(u),\alpha(w)] = 0$, for all $u, v, w \in R$.

We replacing $\alpha(u)$ by $F(u)$ in the above equation, we get $[F(u),\alpha(w)]\alpha(v)[F(u),\alpha(w)] = 0$, for all $u, v, w \in R$. Again replacing w by u in the above equation, we get

$[F(w),\alpha(w)]\alpha(v)[F(w),\alpha(w)] = 0$, for all $v, w \in R$.

Since α is an automorphism of R, we get $[F(w),\alpha(w)]R[F(w),\alpha(w)] = 0$, for all $w \in R$. Since R is semiprime ring, we get $[F(u),\alpha(u)] = 0$, for all $u \in R$.

Similar proof shows that the same conclusion holds as $F(uv) + H(vu) = 0$, for all $u, v \in R$. Hence the proof is completed.

Theorem 3.3: Let R be a semiprime ring, $F: R \to R$ is a generalized $(\alpha, 1)$ - reverse derivation associated with $(\alpha, 1)$ - reverse derivation d and $H: R \to R$ be a right

α -centralizer. If $F(u)F(v) \pm H(uv) = 0$, for all $u, v \in R$, then $d = 0$. Moreover, $F(uv) = F(v)\alpha(u)$, for all $u, v \in R$ and $[F(u), \alpha(u)] = 0$, for all $u \in R$.

Proof: By the hypothesis, we have $F(u)F(v) - H(uv) = 0$, for all $u, v \in R$. (3.13)

Replacing u by uw in equation (3.13), we get

$(F(w)\alpha(u) + wd(u))F(v) - \alpha(u)H(wv) = 0$, for all $u, v, w \in R$.

$(F(w)F(v) - H(wv))\alpha(u) + wd(u)F(v) = 0$, for all $u, v, w \in R$.

Using (3.13) in the above equation, we get $wd(u)F(v) = 0$, for all $u, v, w \in R$. (3.14)

Replacing v by tv in equation (3.14), we get $wd(u)F(v)\alpha(t) + wd(u)vd(t) = 0$.

Using equation (3.14) in the above equation, we get $wd(u)vd(t) = 0$, for all $t, u, v, w \in R$. Replacing v by vw and t by u in the above equation, we get $wd(u)vwd(u) = 0$, for all $u, v, w \in R$.

By the semiprimeness of R, we conclude that $wd(u) = 0$, for all $u, w \in R$. (3.15)

The equation (3.15) is same as equation (2.6) in lemma 2.2. Thus, by same argument of lemma 2.2, we can conclude the result $d(u) = 0$, for all $u \in R$. (3.16)

By the definition of F, we get $F(uv) = F(v)\alpha(u) + vd(u)$, for all $u, v \in R$.

Using (3.16) in the above equation, we get $F(uv) = F(v)\alpha(u)$, for all $u, v \in R$. (3.17)

Replacing u by vu in equation (3.13), we get $F(vu)F(v) - H(vuv) = 0$, for all $u, v \in R$. $(F(u)F(v) - H(uv))\alpha(v) + ud(v)F(v) = 0$, for all $u, v \in R$.

Using (3.13) in the above equation, we get $ud(v)F(v) = 0$, for all $u, v \in R$. (3.18)

Right multiplying equation (3.13) by $\alpha(v)$, we get

$F(u)F(v)\alpha(v) - H(uv)\alpha(v) = 0$, for all $u, v \in R$. (3.19)

Using equation (3.18) in the above equation, we get

$F(u)F(v)\alpha(v) - \alpha(v)H(uv) = 0$, for all $u, v \in R$. (3.20)

Subtracting equation (3.20) from equation (3.19), we get

$F(u)[F(v), \alpha(v)] = 0$, for all $u, v \in R$. (3.21)

Replacing u by ru in equation (3.21), we get

$F(u)\alpha(r)[F(v), \alpha(v)] + ud(r)[F(v), \alpha(v)] = 0$, for all $r, u, v \in R$.

Using equation (3.15) in the above equation, we get

$F(u)\alpha(r)[F(v), \alpha(v)] = 0$, for all $r, u, v \in R$. (3.22)

Replacing r by tr in equation (3.22), we get

$F(u)\alpha(t)\alpha(r)[F(v), \alpha(v)] = 0$, for all $u, v, r, t \in R$. (3.23)

Left multiplying equation (3.22) by $\alpha(t)$, we get

$\alpha(t)F(u)\alpha(r)[F(v), \alpha(v)] = 0$, for all $u, v, r, t \in R$. (3.24)

Subtracting equation (3.24) from equation (3.23), we get

$[F(u), \alpha(t)]\alpha(r)[F(v), \alpha(v)] = 0$, for all $u, v, r, t \in R$.

Replacing t by u and v by u in the above equation, we get

$[F(u), \alpha(u)]\alpha(r)[F(u), \alpha(u)] = 0$, for all $u, r \in R$.

Since α is an automorphism of R, we get $[F(u), \alpha(u)]R[F(u), \alpha(u)] = 0$, for all $u \in R$.

Since R is semiprime ring, we get $[F(u), \alpha(u)] = 0$, for all $u \in R$.

Similar proof shows that the same conclusion holds as $F(u)F(v) + H(uv) = 0$, for all $u, v \in R$. Hence the proof is completed.

Theorem 3.4: Let R be a semiprime ring, $F: R \rightarrow R$ is a generalized $(\alpha, 1)$ - reverse derivation associated with $(\alpha, 1)$ - reverse derivation d and $H: R \rightarrow R$ be a right α-centralizer. If $F(uv) \pm H(uv) \in C_{\alpha,1}$, for all $u, v \in R$, then $[d(u), u]_{\alpha,1} = 0$, for all $u \in R$.

Proof: By the hypothesis, we have $F(uv) \pm H(uv) \in C_{\alpha,1}$, for all $u, v \in R$.

Using equation (2.12) in the above equation, we get $G(uv) \in C_{\alpha,1}$, for all $u, v \in R$.

Using lemma 2.3 and lemma2.4, we get$[d(u), u]_{\alpha,1} = 0$, for all $u \in R$.

Hence the proof is completed.

Theorem 3.5: Let R be a semiprime ring, $F: R \rightarrow R$ is a generalized $(\alpha, 1)$ - reverse derivation associated with $(\alpha, 1)$ - reverse derivation d and $H: R \rightarrow R$ be a right α-centralizer. If $F(uv) \pm H(vu) \in C_{\alpha,1}$, for all $u, v \in R$, then $[d(u), u]_{\alpha,1} = 0$, for all $u \in R$.

Proof: By the hypothesis, we have $F(uv) - H(vu) \in C_{\alpha,1}$, for all $u, v \in R$. (3.25)

Replacing u by wv and v by u in equation (3.25), we get

$F(vu)\alpha(w) + vud(w) - \alpha(u)\alpha(w)H(v) \in C_{\alpha,1}$, for all $u, v, w \in R$.

$(F(vu) - H(uv))\alpha(w) + H(uv)\alpha(w) - \alpha(w)\alpha(u)H(v) + vud(w) \in C_{\alpha,1}$, for all $u, v, w \in R$.

Using equation (3.25) in the above equation, we get

$\alpha(u)\alpha(w)H(v) - \alpha(w)\alpha(u)H(v) + vud(w) \in C_{\alpha,1}$, for all $u, v, w \in R$.

$H(v)\alpha[u, w] + vud(w) \in C_{\alpha,1}$, for all $u, v, w \in R$.

$[H(v)\alpha[u, w] + vud(w), w] = 0$, for all $u, v, w \in R$.

$[H(v)\alpha[u, w], w]_{\alpha,1} + [vud(w), w]_{\alpha,1} = 0$, for all $u, v, w \in R$.

$[H(v)\alpha[u, w], w]_{\alpha,1} + vu[d(w), w]_{\alpha,1} + v[u, w]d(w) + [v, w]ud(w) = 0$, for all $u, v, w \in R$.

Replacing w by v in the above equation, we get

$[H(v)\alpha[u, v], v]_{\alpha,1} + vu[d(v), v]_{\alpha,1} + v[u, v]d(v) = 0$, for all $u, v \in R$.

Replacing u by v in the above equation, we get $vu[d(v), v]_{\alpha,1} = 0$, for all $v \in R$. (3.26)

The equation (3.26) is same as equation (2.10) in lemma 2.3. Thus, by same argument of lemma 2.3, we can conclude the result $[d(u), u]_{\alpha,1} = 0$, for all $u \in R$.

Similar proof shows that the same conclusion holds as $F(uv) + H(vu) \in C_{\alpha,1}$, for all $u, v \in R$. Hence the proof is completed.

Theorem 3.6: Let R be a semiprime ring, $F: R \to R$ is a generalized $(\alpha, 1)$ - reverse derivation associated with $(\alpha, 1)$ - reverse derivation d and $H: R \to R$ be a right α-centralizer. If $F(u)F(v) \pm H(uv) \in C_{\alpha,1}$, for all $u, v \in R$, then $[d(u), u]_{\alpha,1} = 0$, for all $u \in R$.

Proof: By the hypothesis, we have $F(u)F(v) - H(uv) \in C_{\alpha,1}$, for all $u, v \in R$. (3.27)

Replacing u by wu in equation (3.27), we get

$(F(u)\alpha(w) + ud(w))F(v) - \alpha(w)H(uv) \in C_{\alpha,1}$, for all $u, v, w \in R$.

$(F(u)F(v) - H(uv))\alpha(w) + ud(w)F(v) \in C_{\alpha,1}$, for all $u, v, w \in R$.

Using equation (3.27) in the above equation, we get

$ud(w)F(v) \in C_{\alpha,1}$, for all $u, v, w \in R$. (3.28)

Replacing v by vt in equation (3.28), we get $ud(w)F(t)\alpha(v) + ud(w)td(v) \in C_{\alpha,1}$, for all $t, u, v, w \in R$.

Using equation (3.28) in the above equation, we get $ud(w)td(v) \in C_{\alpha,1}$, for all $t, u, v, w \in R$.

Replacing $td(v)$ by v in the above equation, we get $ud(w)v \in C_{\alpha,1}$, for all $u, v, w \in R$.

$[ud(w)v, w] = 0$, for all u, v, w ∈ R. (3.29)

$uv[d(w), w]_{\alpha,1} + [u, w]v + u[v, w] = 0$, for all u, v, w ∈ R.

Replacing w byu in the above equation, we get $uv[d(u), u]_{\alpha,1} + u[v, u] = 0$, for all u, v ∈ R.

Again replacing v b yu in the above equation, we get

$uu[d(u), u]_{\alpha,1} = 0$, for all u ∈ R. (3.30)

The equation (3.30) is same as equation (2.10) in lemma 2.3. Thus, by same argument of lemma 2.3, we can conclude the result $[d(u), u]_{\alpha,1} = 0$, for all u ∈ R.

Similar proof shows that the same conclusion holds as $F(u)F(v) + H(uv) \in C_{\alpha,1}$, for all u, v ∈ R. Hence the proof is completed.

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