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"A generalized mathematical model for the study of Type 1technique of Reconstitution"

Nagarathna N
Associate professor
Department of Mathematics
Maharani's Science College for Women,
Bengaluru, Karnataka, India.

Abstract

The present investigation is carried out a general model for the technique of Reconstitution. In this investigation highly complicated system of nonlinear first order differential equations are considered. In Type -1, in order to elucidate the properties of reconstituted equations by applying the technique to a highly complicated system of nonlinear equations a general model consisting of coupled three nonlinear differential equations are considered. It is Illustrated that the Technique of Reconstitution increases the dimension of the manifold (i.e., the range of the parameters) and certainly produces an equation that is qualitatively correct in the extended solution space for a wider range of parametric values. The guiding principle of Reconstitution is discussed and the results are presented for a wide range of the parameters through the graph and the results are encouraging. The Reconstitution technique multiplies the solvability conditions by appropriate power of the small parameter & adds them together and finally gives the resultant equation essentially in terms of the original, un scaled and unexpanded variables. The advantage of the procedure of Reconstitution is that the physical process which could only slightly modify the leading - order solution is capable of interacting with the dominant dynamics of the leading -order evaluation equation. Finally, the principle of reconstitution can be stated as a reconstituted equations is one which upon substitution of a scaling and an expansion, gives a set of equations, which are exactly equivalent to the solvability conditions derived from the original equations through the same scaling and expansion.

Keywords: Reconstitution, solvability conditions, dominant dynamics, small parameter, leading – order evaluation, scaling and expansion, dimensions of manifold.

Introduction:

In recent years, the study of the phenomenon of the convective process in a horizontal fluid/porous layer has received remarkable attention owing to its very wide applications in Science, Engineering and industrial areas. Convection can be the dominant mode of heat and mass transport in many processes that involve the freezing and melting of the storage material. Natural convection is an omnipresent transport phenomenon in saturated porous geological structures. In fact, the study of convection is of most importance in geophysical, astrophysical and heat transfer problems. For example, the extraction of energy from geothermal sources is the most promising one among the other methods and it is believed that the fluid in these reservoirs is highly permeable and consists of multi-components rather than a single component. Therefore, buoyancy driven convection in a porous medium with water as the working fluid is an important mechanism of energy transport. In fact, the key feature of a major geothermal system while on the land or beneath the sea floor is a high intrinsic heat transport. In fact, the local thermal conditions and the physical properties of media are directly of great importance on the characteristics of the heat and mass transfer in such real configurations. Moreover, the nature of the fluid flow (2D, 3D, pattern and range) is drastically dependent on the complexity of geophysical sites, i.e., the geometry, heterogeneity and anisotropy of the domains. Fluid motions induced by free convection have tangible effects in geothermal areas, on the diffusion of pollutants or on the mineral diagenesis processes.

The technique of Reconstitution increases the dimension of the manifold (i.e., the range of the parameters) and certainly produces an equation that is qualitatively correct in the extended solution space for a wider range of the parametric values. The method is quite elegant and the desired accuracy can be achieved with much ease.

The study of convection in a fluid/porous layer in the absence of penetration has received considerable attention in the past few decades and Copious literature on this is now available (Chandrasekhar (1961); Nield (1968); Rudraiah and Srimani (1976); Turner (1979); Rudraiah and Srimani (1980): Srimani 1981; Rudraiah, Srimani and Friedrich (1982); Nield & Bejan (1991); Srimani (1990); Srimani (1991); Srimani and Anamika (1991); Srimani and Sudhakar (1992); Srimani and Sudhakar (1996); Chevalier et al (1999); Sezai and Mohamad (1999); Payne and Song (2000); Srimani and Nagarathna (2000) Aurnou and Olson (2001); Westerburg and Busse (2001).

It is observed that, in many physical problems the solution is dominated by a particular structure, and it is possible to derive a differential equation which is capable of describing the spatial and / or temporal evolution of this dominant structure. Such an evolution has its own limitations and is valid only for a restricted range of the parameters. But the technique of Reconstitution provides a rationale for systematically making corrections to this first approximation.

Roberts (1985) extended the method proposed by Spiegel (1981), for correcting such evolution equation by adding extra terms which bring new physics into the equation. He has illustrated the technique by considering a simple pair of coupled nonlinear differential equations and has shown that the advantage of the procedure of reconstitution is that the physical processes which previously could only slightly modify the leading-order solution can now interact with the dominant dynamics of the leading-order evolution equation.

1. Mathematical formulation and method of solution of the problem

In order to elucidate the properties of reconstituted equations by applying the technique to a highly complicated system of nonlinear equations, we consider the following set of nonlinear ordinary differential equations under the approximations and assumptions.

$$\frac{da}{dt} = Dra + Eab + Fbc \qquad (1.1) \qquad\qquad \frac{db}{dt} = Gb + Ha^2 + Iac \quad (1.2)$$

$$\frac{dc}{dt} = Jb^2 + Kac + Lab + Mc \qquad (1.3)$$

Where$D, E, F, G, H, I, J, K, L$ and $M$are known quantities and r is a parameter of the problem. In formulating the equation, we actually look at the equation that possesses a simple bifurcation.It is important to stress at this stage that near the bifurcation, one component of the solution will be of marginal stability and therefore, its evaluation will be automatically done over a time scale which is very much larger than that of other components. From (1.1)to (1.3) it is clear that, near the bifurcation (and for small amplitudes), b and $c$components evolve to zero on a fast time scale of order 1, whereas, a component evolves on a much lower time scale of order $1/r$.

The behavior of the exact solutions to (1.1) to (1.3) is investigated. Since the system under consideration is a third order autonomous system, the nature of the solution, stability etc., are best understood by looking at the fixed points. Clearly for $r \leq 0$, there exists exactlyone fixed point at (a, b, c) = (0,0,0) which is stable.However, for $r > 0$, there exists three fixed points:

(i) an unstable fixed point at$(a, b, c) = (0, 0,0)$ and

(ii) two stable fixed points at $(\pm\sqrt{r}, r, r)$.

It is of interest to study the nature of the solutions in detail near the fixed points. Therefore, small perturbations to (a, b, c), away from the fixed points are introduced. The perturbations have the time dependence of the form $e^{\lambda t}$ where the twovalues of λ are

(i) $\lambda = r$ and (ii) $\lambda = -1$

In the case of finite amplitude fixed points, the values ofλ are given by

$$\lambda = -\frac{1}{2}\left(1 \pm \sqrt{1 - 8r}\right)$$

2. The solvability conditions

In this section, we look for the solvability conditions along with their direct solutions. The procedure we adopt here is that, the unknown quantities are expanded in terms of a small perturbation parameter ε andthen appropriate slow space-time scales are introduced. These expansions are substituted into the full equations and then like powers of ε areequated. Thus, an approximate solution to thesystem (1.1) to (1.3) is determined and we then compare with the exact solution. By considering higher powers of ε, it is possible to find the dependence on some of the 'fast' variables by ignoring the dependence on the arbitrary 'slow' variables.

In this process, we find a solvability condition that gives an equation which actually governs the evolution of the previously determined structure over the slow scales. In the conventional procedure, the solutions to each solvability condition are multiplied by some appropriate powers of ε and then they are added up to give appropriate solution, while in the solvability conditions are multiplied by suitable powers of ε and then they are added up. The resulting equation, obviously will be in terms of the original unsealed, unexpanded variables.

Therefore, we are interested in forming only one equation which contains all the relevant information which was previously confirmed separately in the different solvability conditions.

Now set

$\varepsilon^2 = r,\ \ \varepsilon, r \geq 0$ (2.1)

$$\left.\begin{aligned} a(t) &= \varepsilon A_0(s) + \varepsilon^3 A_1(s) + \varepsilon^5 A_2(s) + \ \ldots\ldots\ldots \\ b(t) &= \varepsilon^2 B_0(s) + \varepsilon^4 B_1(s) + \varepsilon^6 B_2(s) + \ \ldots\ldots\ldots \\ c(t) &= \varepsilon^2 C_0(s) + \varepsilon^4 C_1(s) + \varepsilon^6 C_2(s) + \ \ldots\ldots\ldots \end{aligned}\right\} \quad (2.2)$$

Where $s = \varepsilon^2 t$ (slow-time scale) (2.3)

We now expand the solution vector as

$$(a, b, c) = \varepsilon(a, b, c)_0 + \varepsilon^2 (a, b, c)_1 + \ldots\ldots\ldots\ldots \quad (2.4)$$

Substituting the expansions into (1.2) and equating the coefficients of ε^2, ε^4 and ε^6, the first three equations for B_n are obtained

$$\left.\begin{aligned} B_0 &= -\frac{H}{G} {A_0}^2 \\ B_1 &= -\frac{H}{G}\left[2A_0 A_1 + \frac{1}{G} 2A_0 {A_0}'\right] \\ B_2 &= -\frac{H}{G}\left[{A_1}^2 + 2A_0 A_2 + \frac{1}{G}\left\{(2A_0 {A_0}' + 2{A_0}' A_1) + \frac{1}{G}(2A_0 {A_0}'' + 2({A_0}')^2)\right\}\right] \end{aligned}\right\} \quad (2.5)$$

$$\left.\begin{aligned} {B_0}' &= -\frac{H}{G} 2A_0 {A_0}' \\ {B_1}' &= -\frac{H}{G}\left[2A_0 {A_1}' + 2{A_0}' A_1 + \frac{1}{G}(2A_0 {A_0}'' + 2({A_0}')^2)\right] \end{aligned}\right\} \quad (2.6)$$

Where $'$ denotes the differentiation with respect to the slow time. Substituting the expansions into (1.1) to (1.3) and utilizing the results from (2.5) and (2.6), we obtain

$$\left.\begin{aligned} C_0 &= 0 \\ C_1 &= -\frac{1}{M} J {B_0}^2 \\ C_2 &= -\frac{J}{M}\left[2B_0 B_1 + \frac{1}{M} 2B_0 {B_0}'\right] \\ C_3 &= -\frac{J}{M}\left[{B_1}^2 + 2B_0 B_2 + \frac{1}{M}\left\{(2B_0 {B_0}' + 2{B_0}' B_1) + \frac{1}{M}(2B_0 {B_0}'' + 2({B_0}')^2)\right\}\right] \end{aligned}\right\} \quad (2.7)$$

$${A_0}' = DA_0 - \frac{EH}{G} {A_0}^3 \quad (2.8)$$

$${A_1}' = DA_1 - \frac{EH}{G}\left[3{A_0}^2 A_1 + \frac{1}{G} 2{A_0}^2 {A_0}'\right] \quad (2.9)$$

$${A_2}' = DA_2 - \frac{EH}{G}\left[3{A_0}^2 A_1 + \frac{4}{G} A_0 A_1 {A_0}' + \frac{2}{G} {A_0}^2 {A_1}' + \frac{2}{G^2} A_0 ({A_0}')^2 + \frac{2}{G^2} {A_0}^2 {A_0}'' + 3A_0 {A_1}^2\right] \quad (2.10)$$

$$\left.\begin{aligned} C_1{}' &= 2JB_0B_1 + MC_2 \\ C_2{}' &= -\frac{J}{M}\left[2B_0B_0{}' + 2B_0{}'B_1 + \frac{1}{M}\left(2B_0B_0{}'' + 2(B_0{}')^2\right)\right] \end{aligned}\right\} \quad (2.11)$$

The set of equations (2.5) to (2.11)forms basis for the technique of Reconstitution.

3. Type I Reconstitution

By applying the principle of reconstitution accurate equations of the simplest form are derived as

$$\left.\begin{aligned} A_0{}' &= DA_0 - \frac{EH}{G}A_0{}^3 \\ A_1{}' &= DA_1 - \frac{EH}{G}\left[3A_0{}^2A_1 + \frac{D}{G}2A_0{}^3 - \frac{EH}{G^2}A_0{}^5\right] \\ A_2{}' &= DA_2 - \frac{EH}{G}\left[3A_0{}^2A_2 + 3A_0A_1{}^2 + \frac{6D}{G}A_0{}^2A_1 - \frac{10EH}{G^2}A_0{}^4A_1 + \frac{4D^2}{G^2}A_0{}^3 - 16\frac{DEH}{G^3}A_0{}^5 + 12\frac{E^2H^2}{G^4}A_0{}^7\right] \end{aligned}\right\}$$

(3.1)

It is evident that any set of direct solutions to the above equations is also a solution set of (2.8) to (2.10).

To derive more accurate evolution equations from (3.1)

put $a = \varepsilon A_0$ and $s = \varepsilon^2 t$ so that,$a = Dra - \frac{EH}{G}a^3$ (3.2)

Consider $\varepsilon^3(22) + \varepsilon^5(23) + O(\varepsilon^7)$ (3.3)

Set $a = \varepsilon A_0 + \varepsilon^3 A_1; s = \varepsilon^2 t$ (3.4)

$r = \varepsilon^2; A_0 = \frac{a}{\varepsilon}$ (3.5)

This procedure results in a more accurate evolution equation given by

$$\dot{a} = Dra - \frac{EH}{G}\left(1 + 2\frac{D}{G}r\right)a^3 + 2\frac{E^2H^2}{G^3}a^5 \quad (3.6)$$

The second reconstituted equation is derived in a similar manner by considering

$$\varepsilon^3(22) + \varepsilon^5(23) + \varepsilon^7(24) + O(\varepsilon^9) \quad (3.7)$$

yields

$$\dot{a} = Dra - \frac{EH}{G}\left[1 + 2\frac{D}{G}r + 4\frac{D^2}{G^2}r^2\right]a^3 + 2\frac{E^2H^2}{G^3}\left[1 + 8\frac{D}{G}r\right]a^5 - 12\frac{E^3H^3}{G^5}a^7 \quad (3.8)$$

Applying the same procedure, the reconstituted equations for b and c are derived as

$$\left.\begin{aligned} b &= -\frac{H}{G}a^2 \\ b &= -\frac{H}{G}\left[\left(1+2\frac{D}{G}r\right)a^2 - 2\frac{EH}{G^2}a^4\right] \\ b &= -\frac{H}{G}\left[\left(1+2\frac{D}{G}r+4\frac{D^2}{G^2}r^2\right)a^2 - 2\left(\frac{EH}{G^2}+6\frac{DEH}{G^3}r+2\frac{D}{G^2}\right)a^4 + \left(8\frac{E^2H^2}{G^4}+4\frac{EH}{G^3}\right)a^6\right] \end{aligned}\right\} \quad (3.9)$$

In a similar way,we get

$$\left.\begin{aligned} c &= -\frac{JH^2}{MG^2}a^4 \\ c &= -\frac{JH^2}{MG^2}\left[\left\{1+4\left(\frac{D}{G}+\frac{D}{M}\right)r\right\}a^4 - 4\frac{EH}{G}\left(\frac{1}{G}+\frac{1}{M}\right)a^6\right] \end{aligned}\right\} \quad (3.10)$$

$$c = -\frac{JH^2}{MG^2}\left[\left\{1+4\left(\frac{D}{G}+\frac{D}{M}\right)r + 4\left(\frac{3}{G^2}+\frac{4}{MG}+\frac{4}{M^2}\right)D^2r^2\right\}a^4 - \left\{4\frac{EH}{G}\left(\frac{1}{G}+\frac{1}{M}\right) + \left[\frac{8EH}{G}\left(\frac{4}{G^2}+\frac{6}{MG}+\right.\right.\right.\right.$$

5M2+8DG2ra6+E2H2G220G2+32MG+24M2+8EHG3a8 (3.11)

The solvability conditions (2.8) to (2.10) are transformed by substituting the second and higher order derivatives to get the final transformed equations as

$$\left.\begin{aligned} \dot{a} - Dra + \frac{EH}{G}a^3 &= 0 \\ \left(1+\frac{2}{G}a^2\right)\dot{a} - Dra + \frac{EH}{G}a^3 &= 0 \\ \frac{EH}{G^3}2\dot{a}a^2 + \left[1+2\left\{\frac{EH}{G^2}+\frac{EDH}{G^2}r\right\}a^2 - 6\frac{E^2H^2}{G^4}a^4\right]\dot{a} - Dra + \frac{EH}{G}a^3 &= 0 \end{aligned}\right\} \quad (3.12)$$

$$\left.\begin{aligned} b &= -\frac{H}{G}a^2 \\ b &= -\frac{H}{G}\left(a^2+\frac{2}{G}a\dot{a}\right) \\ b &= -\frac{H}{G}\left[a^2+\left\{2\left(\frac{1}{G}+\frac{D}{G^2}r\right)a - 6\frac{EH}{G^3}a^3\right\}\dot{a} + \frac{1}{G^2}2\dot{a}^2\right] \end{aligned}\right\}$$

4. Results and Discussion

The results of the present investigation are presented through graphs (figures 1 to 17) and are discussed. The profiles of b vs a and c vs a (equations (3.9) to (3.12)) are presented for different values of the parameters $D, E, G, H$and$r$. These results correspond to Type I, Reconstitution. These arethe approximate manifolds and are the successive approximations to the center manifolds for the reconstituted equations discussed in section 2. The graphs are plotted for different values and r viz., 0.0, 0.625,0.125 and 0.25 respectively. In fact, these equations described nothing more than the center manifold upon which the evolution takes place and are comparable with the results of the previous section. Based on the form of these approximations initial conditions can be chosen. The dynamics of the solution to the reconstituted equations (3.2), (3.6) and (3.10) can be easily understood by looking at the fixed points and their stability. Equations (3.6) has the fixed points $a = 0$for all values of$D$, E, G, H and for $r > 0$, $D = E = G = H = 1$, we have $a = \pm\varepsilon$. In other words, it is possible to choose the values of D, E, G, H in such a way that$a = \pm\varepsilon$. Accordingly, the fixed points$a = 0$, is stable if $r < 0$ and unstable if $r > 0$. It is observed that for a large a (3.6) yields $a \sim t^{-\frac{1}{4}}$ for$D = E = G = H = 1$and (3.8) gives$a \sim t^{-\frac{1}{6}}$, Thus, we observe that the above reconstituted equations clarify the center manifold on which the solutions are valid and certainly refines the accuracy of the evolution uniformly in time,

provided no singularities exist (Roberts 1985).

i) In figures 1 and2, the successive approximations to the center manifold are presented for r =0.0, 0.625,0.125,0.25 and $D = E = G = 1$ and$H = -1$.

ii) In figures3 and 4; the graphs are presented for$D = E = 1$, $G = 2$and$H = 1$, for the same values of r mentioned earlier. The behavior of the curves is totally different in these two cases.

iii) In drawing these graphs (i.e., b vs a curves) (i.e., figures1 to 12), the following sets of data are considered: $(D, E, G, H) = (1, 1, 1, -1)$; $(1, 1, 2, 1)$; $(0.5, -1, -1, 0.2)$; $(-1, 0.2, 0.4, 1)$; $(2, 2.5, 0.5, -1)$; $(0.4, 0.2, 0.5, 2)$. In drawing the graphs of c vs a (figures 13 to 18) the following sets of

data are considered:

$(D, E, G, H, J, M) = (1, 1, 1, -1, -1, 1)$;$(1, 1, 2, 1, 0.4, 0.2)$;$(-1, -0.2, 0.4, 1, 0.2, 0.5)$

where$r$ =0.0, 0.625, 0.125 and 0.25.

iv) Since, the graphs are presented for a wide range of the parameters the behavior pattern of

the profiles are clearly predicted. This enables us to apply the technique of Reconstitution under

different physical situations. Obviously, for$D = 1, E == 1, F = 0, G = -1, H = 1andI = J = K = L = M = 0$, the results of the present investigation coincide with those of Roberts (1985) and coincides with the exact solution. Since, the problem investigated here is of general type, the technique can be employed for different problems proper choice of the parameters.

5. Conclusions

1. The Reconstitution technique multiplies the solvability conditions by appropriate powers of the small parameter e, adds them together and finally gives the resultant equation essentially in terms of the original, unsealed and unexpanded variables. In other words, one equation which contains all the essential information that was originally contained separately in the different solvability conditions, is derived.

2. The advantage of the procedure of Reconstitution is that the physical process which could only slightly modify the leading-order solution is capable of interacting with the dominant dynamics of the leading-order evolution equation.

3. Finally, the principle of Reconstitution can be stated as "A Reconstituted equation is one which upon substitution of a scaling and an expansion, gives a set of equations, which are exactly equivalent to the solvability conditions derived from the original equations through the same scaling and expansion".

4. Although, some algebraic complexities are involved, the results are interesting and useful.

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Figures:

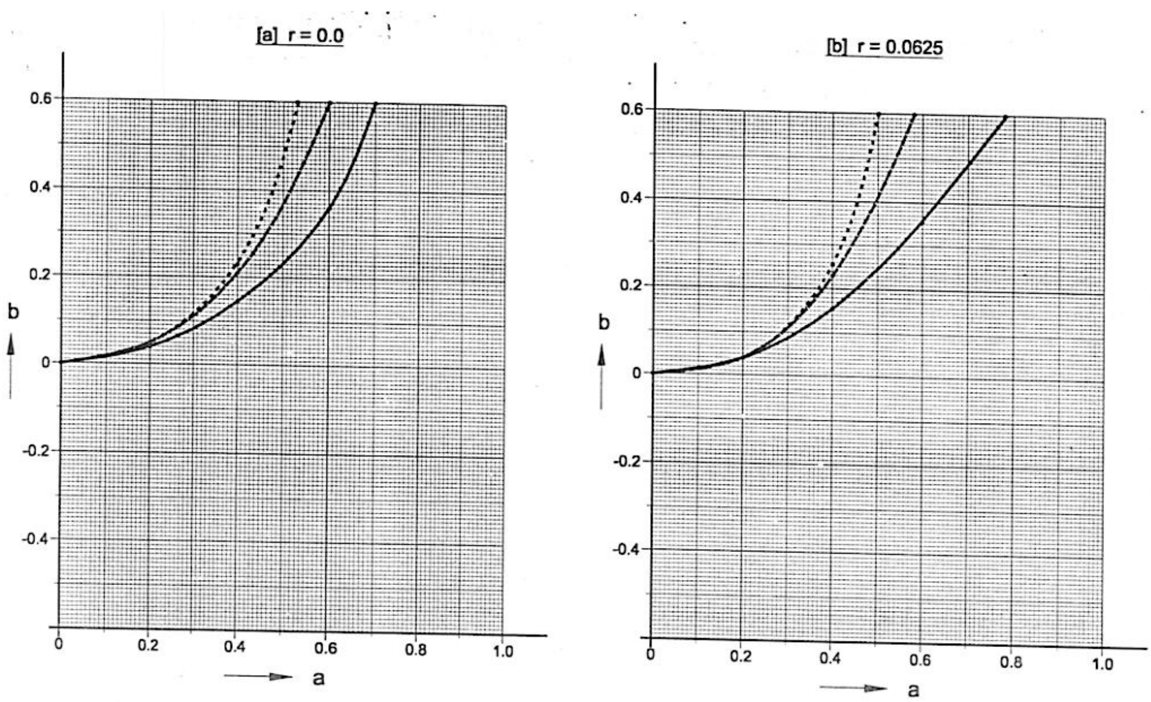

Figure 1: b vs a, $D = E = G = 1, H = -1$

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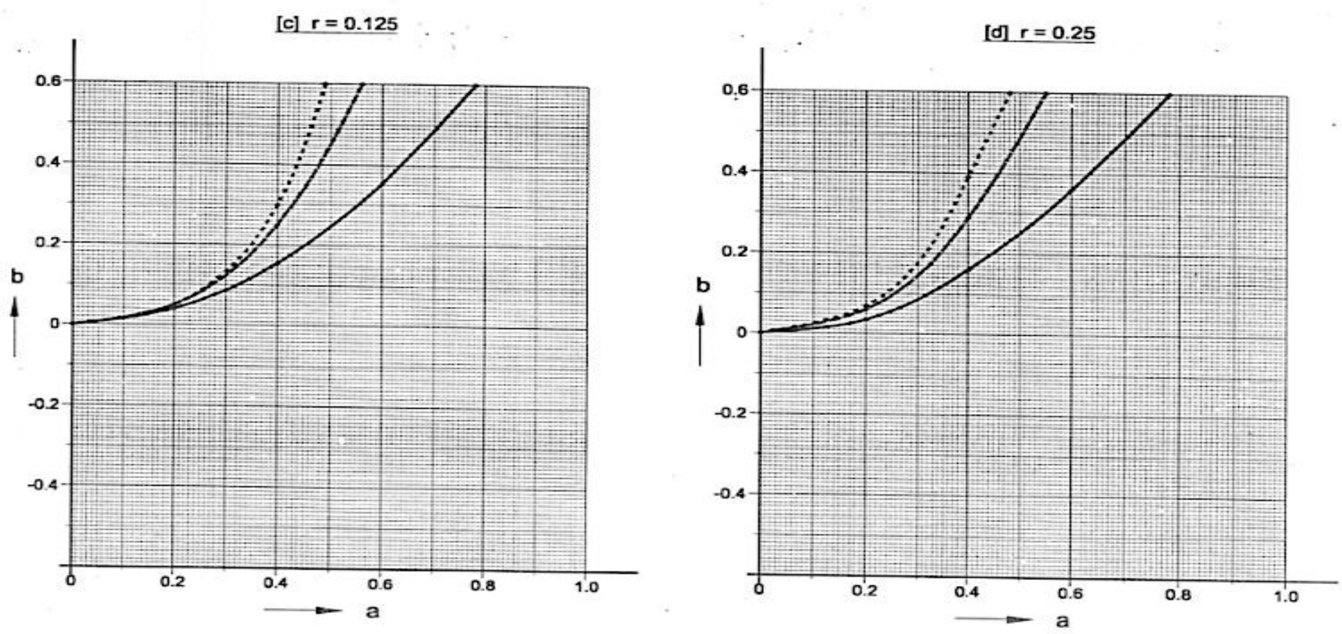

Figure 2: b vs a, $D = E = G = 1, H = -1$

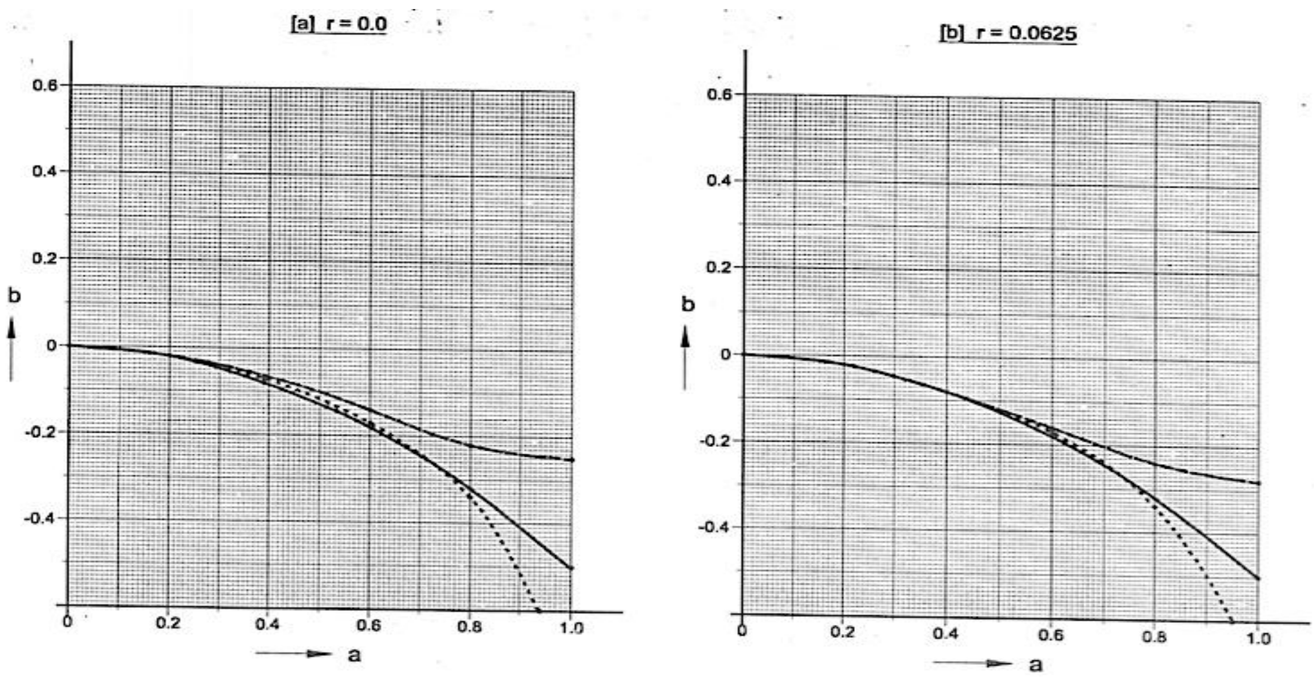

Figure 3: b vs a, $D = E = 1, G = 2, H = 1$

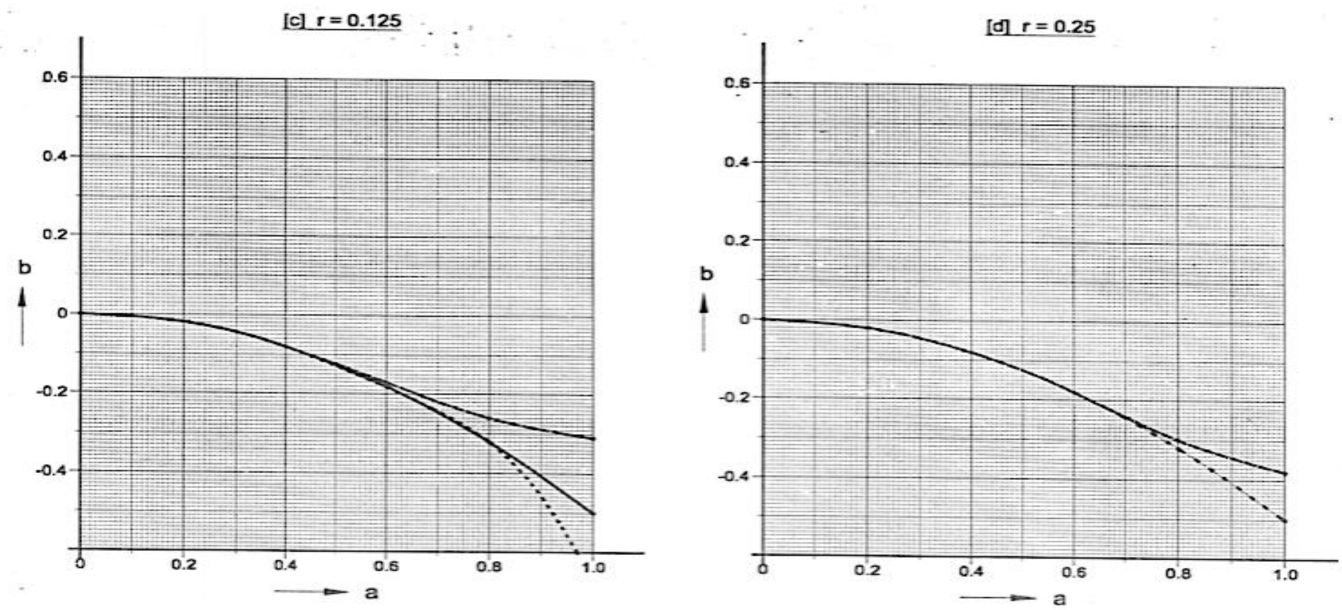

Figure 4: b vs a, $D = E = 1, G = 2, H = 1$

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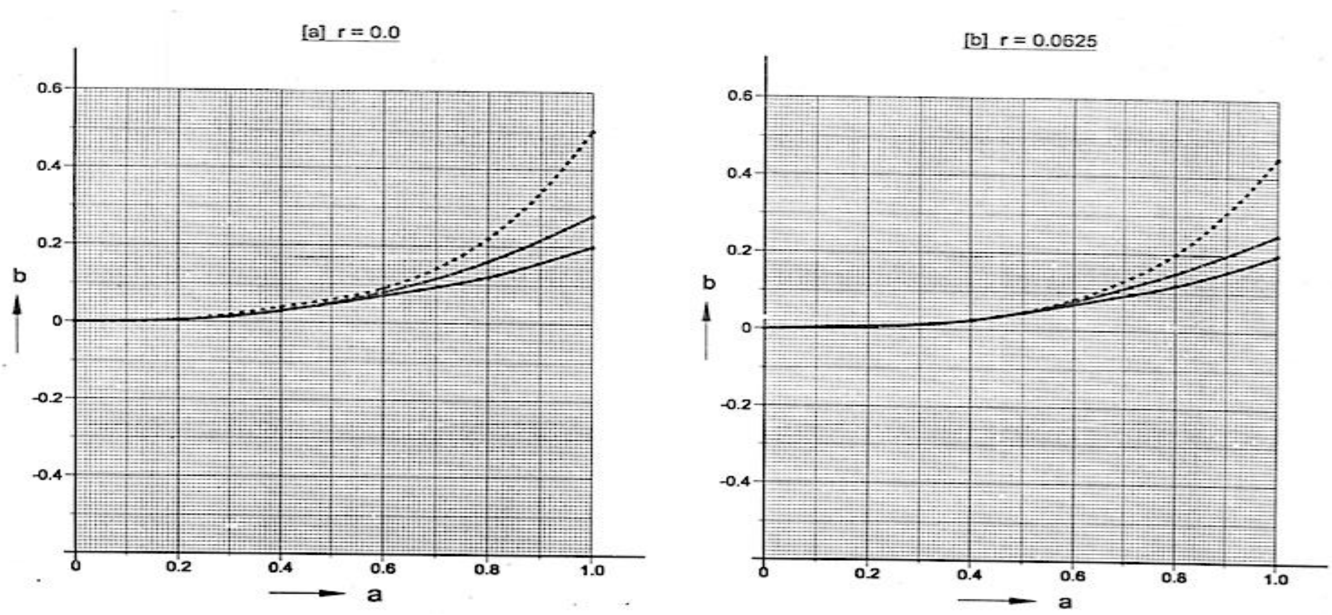

Figure 5: b vs a, $D = 0.5$,$E = -1$, $G = -1$, $H = 0.2$

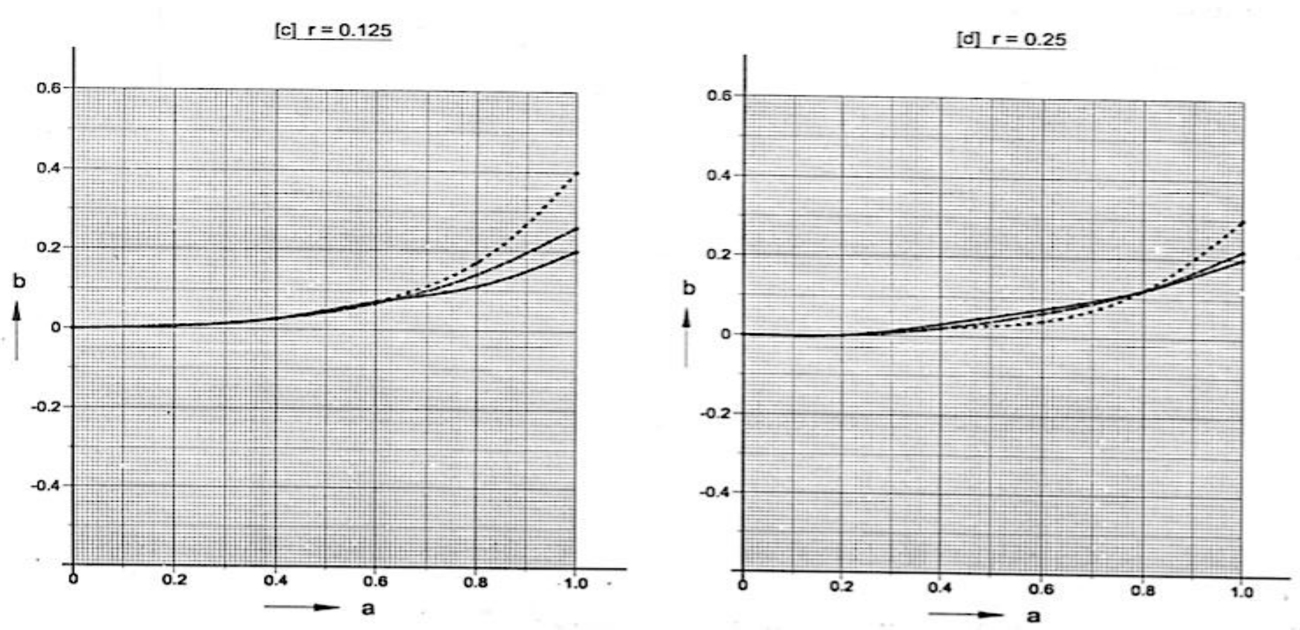

Figure 6: b vs a, $D = 0.5, E = -1$, $G = -1, H = 0.2$

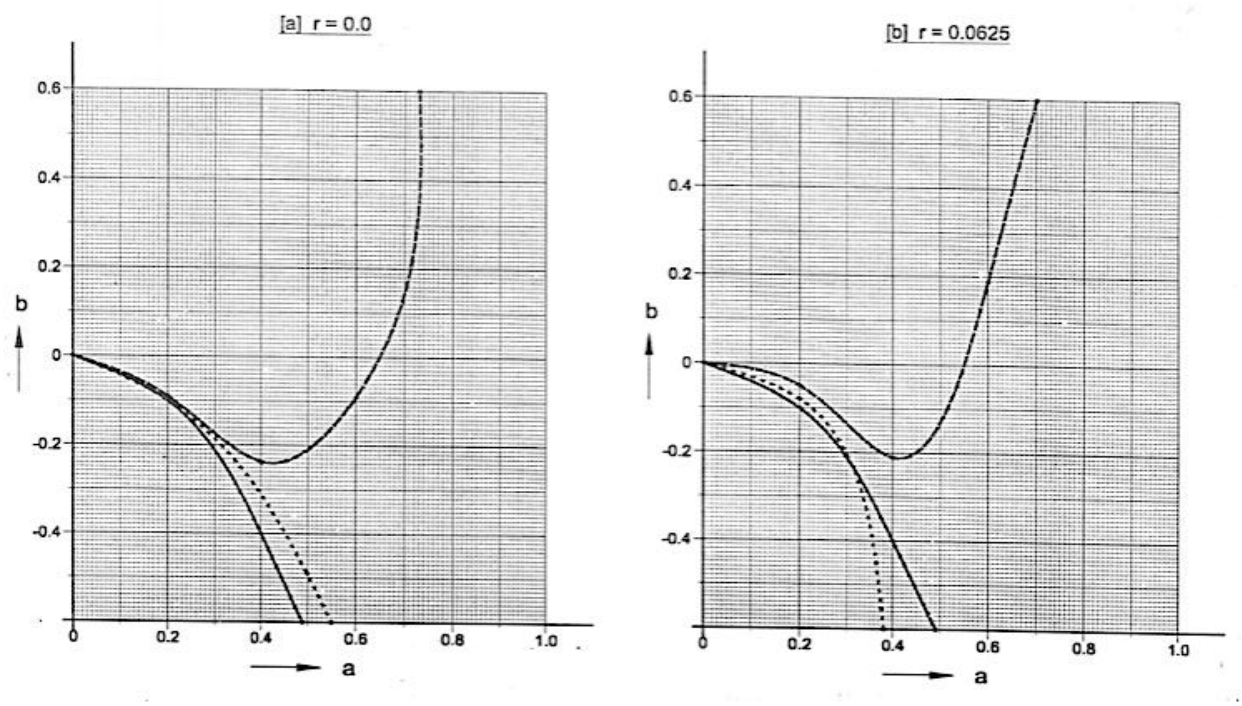

Figure 7: b vs a, $D = -1, E = 0.2$, $G = 0.4, H = 1$

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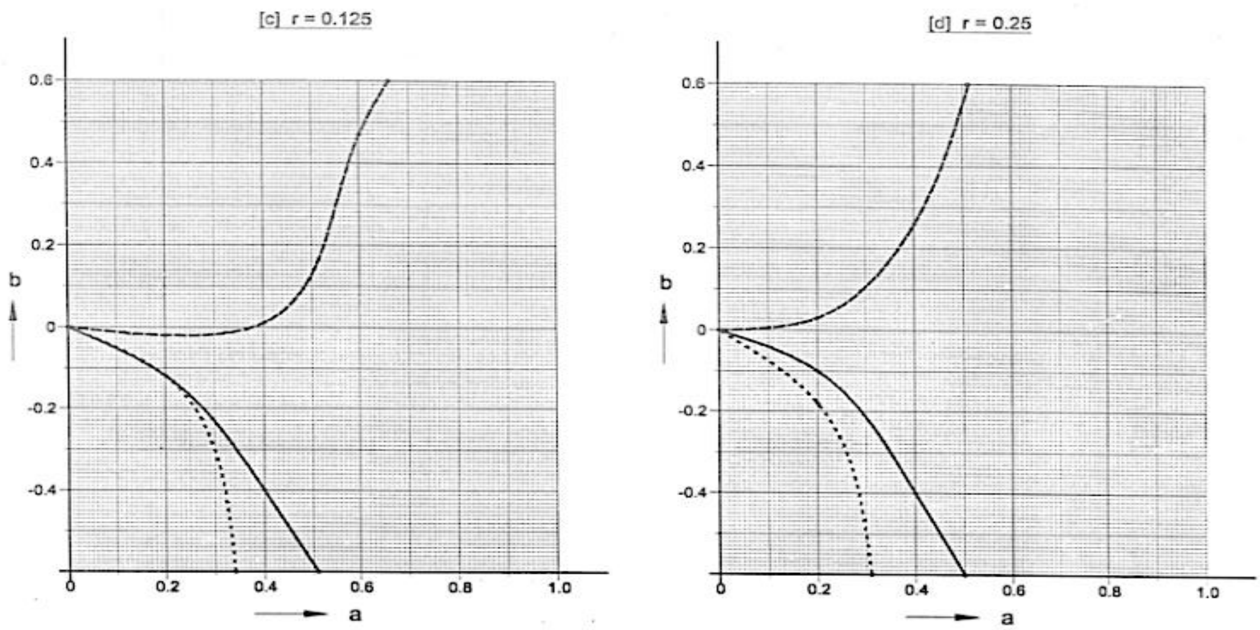

Figure 8:b vs a, $D = -1$,$E = 0.2$, $G = 0.4$, $H = 1$

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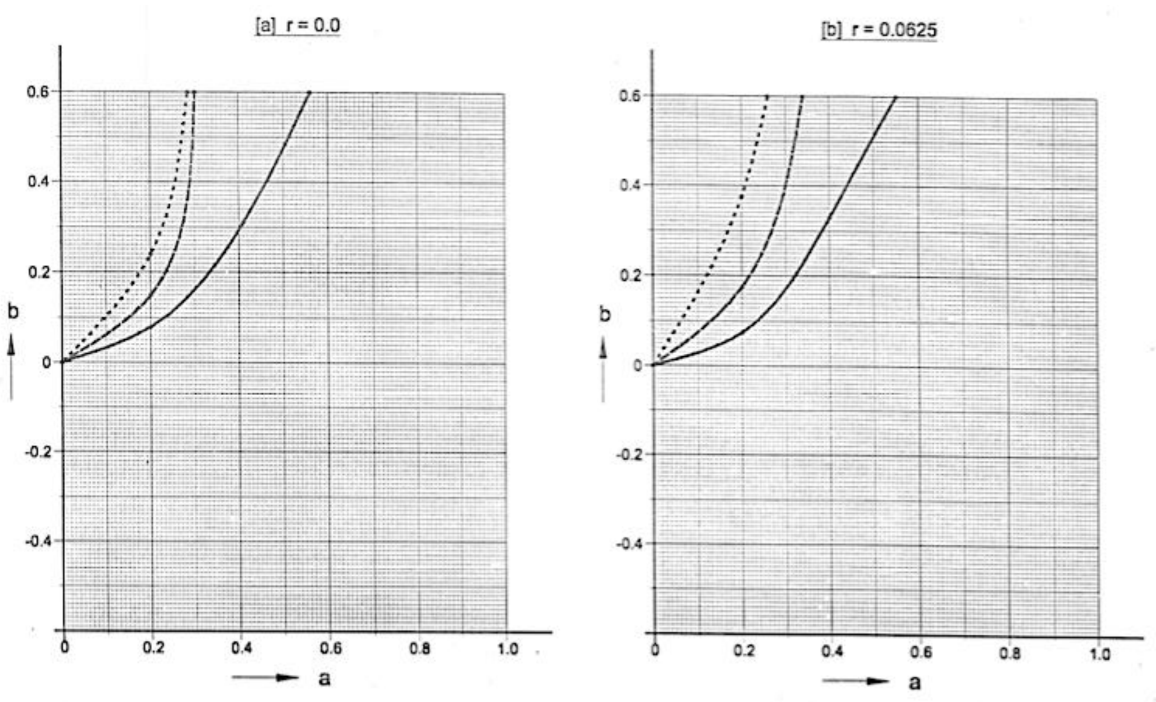

Figure 9: b vs a, $D = 2$,$E = 2.5$, $G = 0.5$, $H = -1$

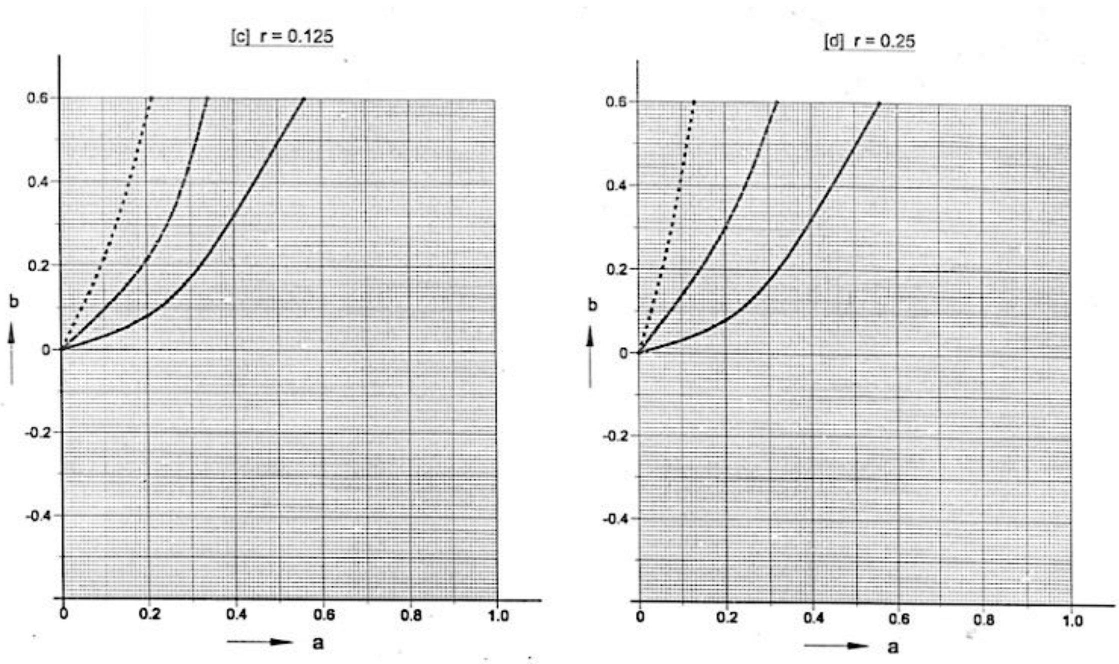

Figure 10: b vs a, $D = 2, E = 2.5, G = 0.5, H = -1$

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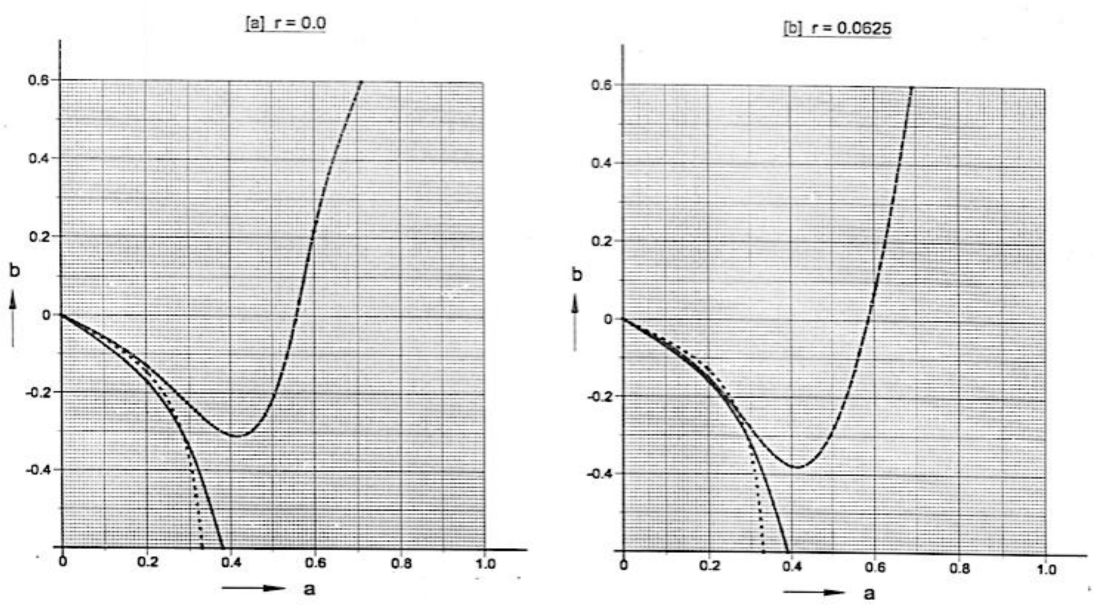

Figure 11: b vs a, $D = 0.4$,$E = 0.2$, $G = 0.5$, $H = 2$

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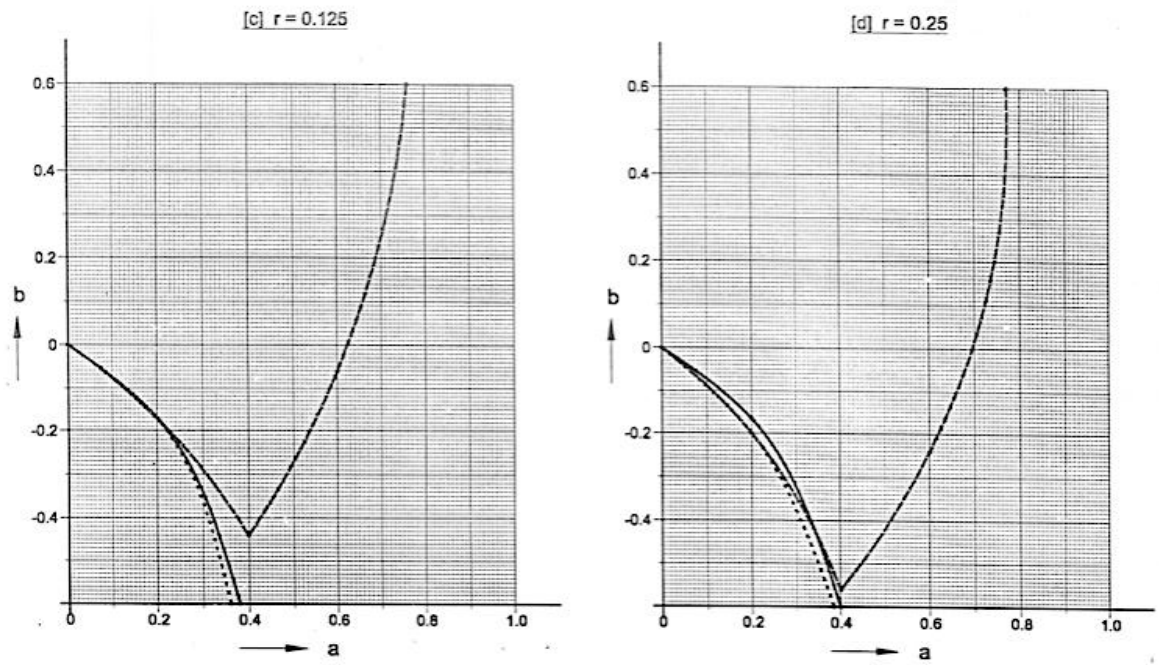

Figure12:b vs a, $D = 0.4$,$E = 0.2$, $G = 0.5, H = 2$

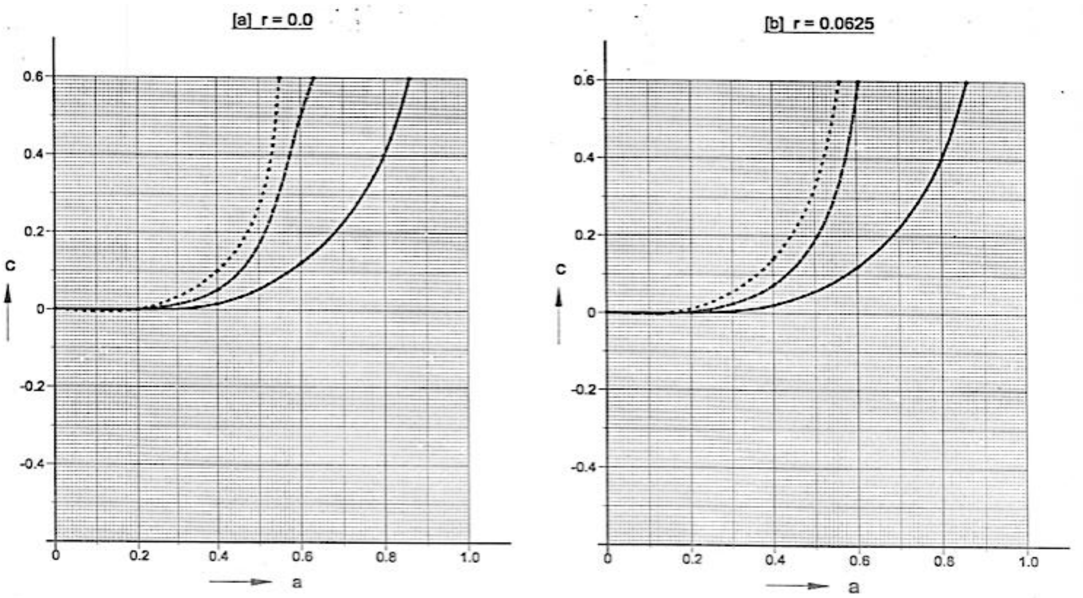

Figure 13: c vs a, $D = 1$,$E = 1$, $G = 1$, $H = -1$, $J = -1$, $M = 1$

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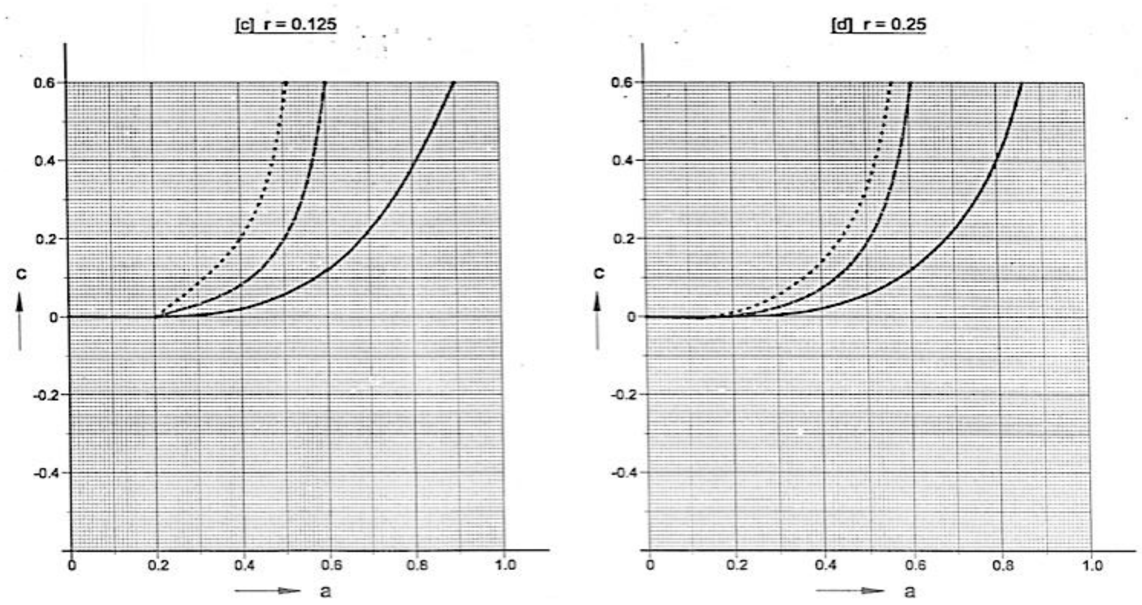

Figure 14: c vs a, $D = 1$,$E = 1$, $G = 1$,$H = -1$, $J = -1$, $M = 1$

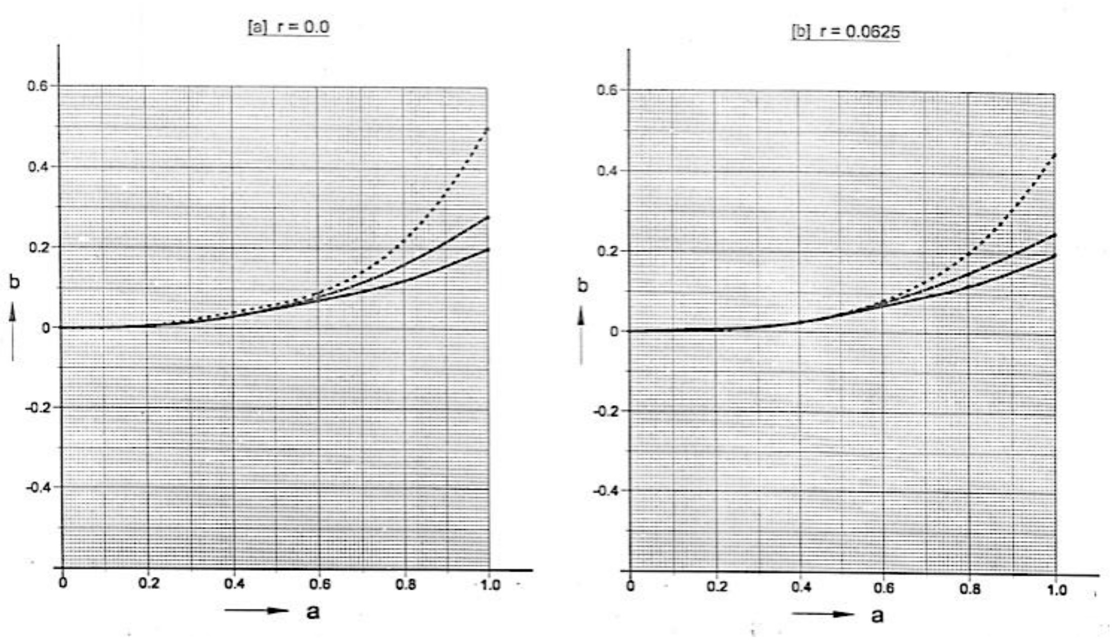

Figure 15: b vs a, $D = 0.5, E = -1$, $G = -1, H = 0.2$

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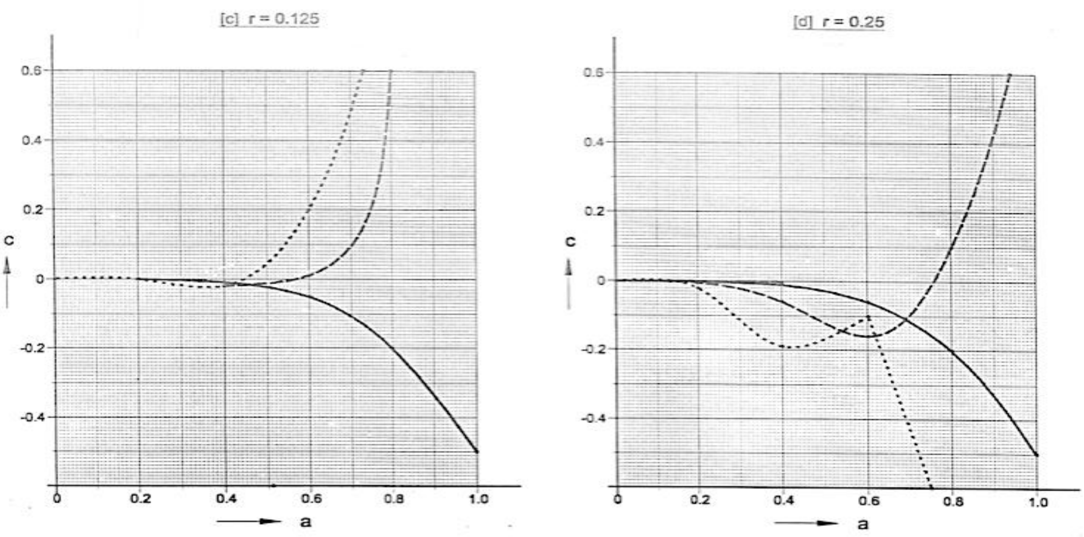


Figure 16:c vs a, $D = 1$,$E = 1$, $G = 2$,$H = 1$, $J = 0.4$, $M = 0.2$

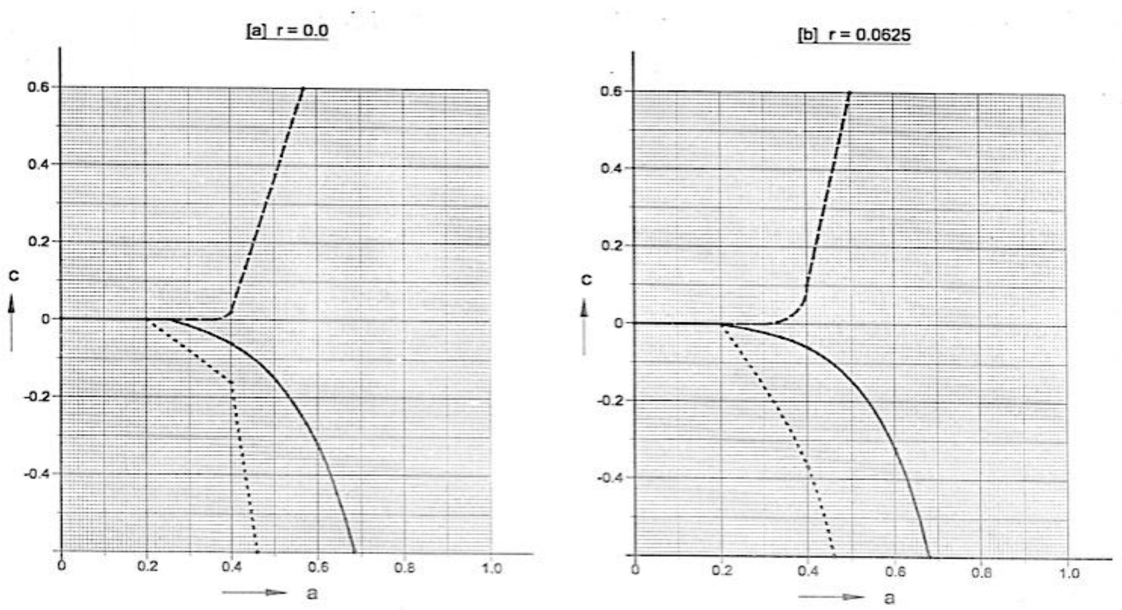
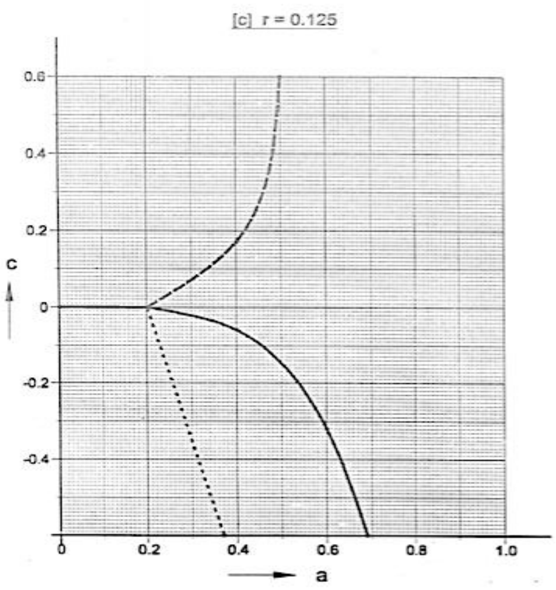


Figure 17: c vs a, $D = 1$,$E = 0.2$, $G = 0.4$,$H = 1$, $J = 0.2$, $M = 0.5$



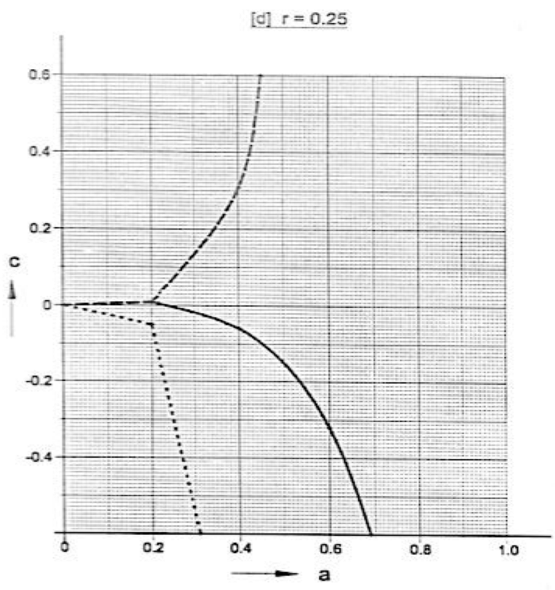


Figure 18: c vs a, $D = -1$,$E = -0.2$, $G = 0.4$, $H = 1$, $J = 0.2$, $M = 0.5$