International Journal of Engineering, Science and Mathematics
Vol. 12 Issue 10, Oct 2023,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, Email: editorijmie@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

An examination of fractional calculus operators and special functions

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DOI:ijesm.co.in.77303.34987

Abstract

Fractional differential equations and additional difficulties involving particular applications of modern physics, as well as their developments and oversimplifications in one or more variables, are often the result of mathematical modelling of real-world problems. Moreover, fractional order PDEs governs most physical phenomena in many other models, including those of fluid subtleties, quantum astronomy, power, ecological schemes, and many more. It is crucial to comprehend all of the recently developed and widely used techniques for solving small order PDEs, as well as the claims made by these techniques. This research generates exact formulas and grids of insufficient special functions using several categories of multiple fractional derivatives and integrals. Their application is also evaluated by the study.

Keywords: Fractional calculus, Functions, Integrals

1. Introduction

Recently, complex biological organisations with nonlinear performance and long-term memory have been described using the potent new method of fractional calculus. Fractional calculus was developed out of some simple questions about the derivation notion, like: if the first order derivative of a function defines its slope, what does its half order derivative tell us about it? Despite having a sophisticated mathematical background, fractional calculus was born out of these questions. By finding solutions to these puzzles, scientists were able to open up new avenues of inquiry between the mathematical and physical domains, which produced a plethora of fascinating new results and inquiries. For example, the fractional order plagiaristic of a constant function is not always zero, in contrast to the conventional derivative. The goal of this tutorial-based work is to provide extensive explanations of fractional calculus, its uses for model building, and the answers to the concerns raised above. In order to simulate a dynamic system and examine the mechanical behavior of a cell, a novel claim of this powerful implement is built after a thorough literature assessment of its concepts and uses. Riemann had arrived at the fractional integration expression as

$\frac{1}{\lceil 2}\int_0^v \frac{f(z)dz}{(v-z)^{1-m}}$, v>0

Maximum of the philosophy of fractional calculus is founded upon the accustomed differential machinist definite as,

$$D_y^w f(y) = \begin{cases} \frac{1}{\lceil w}\int_r^y (y-z)^{w-1} f(z)dz, Re(w) > 0 \\ \frac{d^r}{dy^r}\left[D_y^w f(y)\right] Re(w) < 0 \end{cases} \quad (1)$$

Where r is a progressive integer

Case(i)

If $r = o$, (i) decreases to traditional Riemann-Liouville fractional offshoots or integral of instruction w.

Case(ii)

If $r \rightarrow \infty$,equation (i) might be distinct by the description of the acquainted Wayl fractional operative of command w.Mishra (1988) consumes distinct the fractional imitative operative in the subsequent method.

$$D_y^\alpha(y^{v-1}) = \frac{\lceil v}{\lceil v-\alpha} y^{v-\alpha-1}, \alpha \neq v \ (2)$$

$$D_{K,av}(y^v) = \frac{\lceil v+1}{\lceil v-a+1} y^{v+k}, a \neq v+1 \ (3)$$

These operators stay oversimplification of Riemann-Liouville besides Weyl fractional integral operative.

The writerscontemporary in a combined manner a thorough account or slightly a transitoryreview of the Mittag-Leffler function (MLF), generalised MLF, MLFsort functions, besides their thought-provoking and valuable properties. The authors' motivation is primarily the achievement of the requests of the MLF in countless fields of knowledge and manufacturing. There are many examples of MLF being used in the physical and applied sciences. About every type of MLFsort function found in the poetry is given in this survey work.

To familiarise the reader with the current research trend in MLFsort functions and their requests, an attempt is made to give a practically full list of orientations pertaining to the MLF.Here, it is demonstrated that this equation may contain solutions that are not anywhere differentiable if one takes into account fractional derivative via fractional difference. About this issue, a number of equations are developed, and a number of unresolved issues are listed. The answer is derived using the Sumudu transform method. Here, fresh and compact representations of the results that are appropriate for numerical calculation are obtained in rapports of the generalised MLF. The limits of fractional integral machinists now comprise a generalised extended MLF kernel via various convexities.

In particular, the existing constraints for convex functions are investigated and further related with established findings. Moreover, similar findings were everyday to the parabolic function, springy recurrence families for MLF.Additionally, several fractional differential equations including MLF are created, and the Laplace transform technique is used to find their solutions.

After that, Leibniz's communication with L'Hospital in 1695, wherein Leibniz outlined "paradoxes," made a prediction that "one day helpful conclusions will be deduced" since them, is when fractional calculus first appeared. Several of the "practical consequences" prophesied by Leibniz have been found, making the revision of non-integer instructions of diversity a vibrant extent of study not individual in arithmetic but too in other branches of knowledge like astronomy, biology, and manufacturing.

The field has developed so much, though, that experts are still divided on what a "fractional derivative" can be. Very recently have variable-order fractional operators been conceptualised and formally formalised. Subsequently it is conceivable to make evolutionary leading equations, these workers have remained successfully rummage-sale to characterizecomplex real-world matters in a diversity of fields, counting biology, mechanism, conveyanceprocedures, and switch theory.

Variable-order fractional calculus (VO-FC) is a relatively a smaller amountrecognised branch of calculus that suggestions remarkable chances to feign interdisciplinary developments. Identifying this unexploited potential, the logical community has stayed intensively discoveringsubmissions of VO-FC to the exhibiting of business and physical organizations.

This appraisal is envisioned to help as a starting opinionaimed at the reader attentive in impending this charming. The growth of VO-FC investigative and computational approaches with submission to the imitation of complicated physical organisations is summarised here in a clear and thorough manner. As far as we know, on September 30, 1695, the concept of fractional calculus was created as a result of a profound query asked in a letter from L'Hospital to Leibniz. Leibniz's prescient response to that challenging topic served as a major source of inspiration aimed at all succeeding cohorts of scientists and continues to inspire modern scientists.

Fractional calculus has been around for 325 years, and during that time it has maintained the interest of top mathematicians and developed into a self-same valuable implement for addressing the undercurrents of multifaceted systems from diverse disciplines of science and manufacturing. The current work also includes some of the introduced operator's properties. To examine the properties of fractional operators, the generalised Laplace transform is used. Here, the proposed derivative was used to simulate the FEL problem, and the mentioned Laplace transform was used to determine the solution. This study proposes a novel fractional derivative by a non-singular kernel connecting exponential then trigonometric purposes.The proposed fractional operator contains the Caputo-Fabrizio fractional unoriginal as a particular instance. This new idea is examined through theoretical and arithmeticaltrainings of fractional DE.

Then, a few RC-electrical circuit applications are given. The generalised k- Bessel function is subjected to the Saigo's k-fractional essential and derivative operators, which involve the k-hyper geometric purpose in the kernel. The consequences are articulated in terms of the k-Wright function, which is then second-hand to present the image formulations of integral converts, plus the beta transform. Moreover, Bessel functions and particular situations involving fractional calculus operators are taken into consideration.

The Multivariable H-Function

ThemultivariableWeylfractionalessentialworkerisdistinctastrails

$$\boldsymbol{W^{v_1 \dots v_k}\{g(y_1 \dots . y_k; v_1 \dots v_k); v_1 \dots v_k; z_1 \dots z_k\}} =$$

$$\int_{u_1}^{\infty} \dots \int_{u_k}^{\infty} \prod_{j=1}^{k} \left(\frac{(y_j-u_j)^{v_j-1}}{\lceil v_j}\right) H^{0,v:n_1m_1 \dots n_k m_k}_{q,p:q_1p_1 \dots p_k p_k}\begin{pmatrix} y_1(y_1-u_1)^{\sigma_1} \\ y_k(y_k-u_k)^{\sigma_k} \end{pmatrix} g(y_1 .. y_k v_1 \dots v_k) dy \dots dyk \quad \textbf{(4)}$$

Providing that the essential on right hand side of (4) congregates unquestionably,In (4) and to another place $[z_1 \dots\dots z_k]$ attitudeson behalf of the multivariable H-function presented by Srivastavathen Panda finisheda sequences of investigation identifications. This purpose is distinct and characterised in the subsequentcustom.

$$\mathrm{H}[z_1 \dots z_k] = \frac{1}{(2\pi w)^k} \int_{L_1} \dots \int_{L_r} \varphi(\varepsilon_1 \dots \varepsilon_k) \prod_{i=1}^{k} \{\varphi_1 \varepsilon_1 z_i^{\varepsilon_1} d\varepsilon_1\} \ (5)$$

Where w = $\sqrt{-1}$

$$\varphi(\varepsilon_1 \dots \varepsilon_k) = \frac{\prod_{j=1}^{m} \lceil 1-a_j + \sum_{j=1}^{k} a_j^i \varepsilon_1}{\prod_{j=1}^{m} \lceil 1-a_j + \sum_{j=1}^{k} a_j^i \varepsilon_1 \prod_{j=1}^{m} \lceil 1-b_j + \sum_{j=1}^{k} b_j^i \varepsilon_1} \ (6)$$

And

$$\varphi_i\varepsilon_i = \frac{\prod_{j=1}^{n_j}\left[\left(d_j^i-\delta_j^i\varepsilon_j\right)\prod_{j=1}^{m_j}\left[\left(1-e_j^i+\gamma_j^i\varepsilon_j\right)\right.\right.}{\prod_{j=n+1}^{D_j}\left[\left(1-d_j^i+\delta_j^i\varepsilon_j\right)\prod_{j=M+1}^{p_j}\left[\left(e_j^i-\gamma_j^i\varepsilon_j\right)\right.\right.}$$ i = 1,2,.k (7)

and an empty produce is taken as unity.

$|\arg(z_i)| < \frac{1}{2}\pi T_i \forall i \in \{1, \dots k\}$ (8)

Where

$\nabla_j = \sum_{j=1}^{p} a_j^i + \sum_{j=1}^{pj} x_j^i - \sum_{j=1}^{p} \beta_j^i + \sum_{j=1}^{p} \delta_j^i \leq 0$ (9)

Whenever here is no uncertainty or misperception, we will use a fine representation and write principal affiliate of equation (5) in next abbreviated procedure

$$H_{q,p:q_1p_1\dots p_kp_kg}^{0,v:n_1m_1\dots n_km_k}[Z_1 \dots Z_r]$$

Or

H$[Z_1 \dots Z_r]$ (10)

Supplementary we might memory the recognized asymptotic growth in the subsequent procedure

$[Z_1 \dots Z_r] = \begin{cases} 0(|Z_1|^{r_1} \dots |Z_k|^{r_k}), max\{|Z_1| \dots |Z_k|\} \to 0 \\ 0(|Z_1|^{p_1} \dots |Z_k|^{p_k}), m = 0, min\{|Z_1| \dots |Z_k|\} \to \infty \end{cases}$ (11)

Likewise, in lieu of the sake of brevity we custom the subsequent contracted symbolizations,

$$[(a)]_i = \prod_{j=1}^{p} |a_j|_i$$

$[(c^i)]_i = \sum_{j=1}^{p_j} [c_j^i]_j$ (12)

Everywhere a_m is the Pocchammer representation distinct by

$$[(a)]_i = \frac{\lceil a+m}{\lceil a}$$

Recently progress in philosophy of hyper geometric functions consumes gained ample interest outstanding to outline of certain novel generalized systems of hyper geometric functions. These purposes are Mac-Robert's E – function, Fox's H- function thenlately I – function. The I – functionconsumes been distinct by Saxena(2010) in the sequence of the explanation of double integral equation connecting H – function as kernels and remainedadditionalintentional by Verma(2014)The I – function familiarized by Saxena

(1982), is clear. JohnWallis,inhiseffortArithmeticalInfinitorumin1655,chiefsecondhandtheperiod‘hyper geometric’ (Greek word)tosignifyanyserieswhichremainedyonderthecommonplacegeometricseries$1+x+x2+\cdots$ Inexact,hecalculatedtheseries$1+a+a(a+1) +a(a+1)(a+2) +\cdots$

It is only logical to wonder if extra general functions may be created, such that the special meanings and uncomplicated functions stand essentially specialties of these universal functions, given the numerous relationships joining the unusual functions to bothsupplementarytoo to the elementary meanings. In reality, universal functions of this kind have been created, and they are discussed to as meanings of the hyper geometric type collectively. These functions come in a variety of forms, but the hyper geometric functions are the most prevalent.

Some significant findings relating to the hyper geometric function had already been made by Euler, but renowned German arithmetician C.F.The following infinite series, often known as the Gauss series or, supplementaryjust, the Gauss hyper geometric series, was studied by Gauss in 1812 and is a generalization of the basic geometric series. As a novel tool for modeling complicated systems, particularly viscoelastic materials, we intend to offer fractional calculus. The essentialnotions of fractional calculus are first briefly covered, along with the key steps of the fractionalization algorithm. Then, we explain how to solve fractional equations analytically and provide an interpretation of the fractional derivative. The demonstrating of viscoelastic organizations with the aid of this method is then briefly discussed.

After briefly reviewing a few recent everything, we finally present anbid of the tactic for simulating the biomechanical characteristics of a cell. We show that the projected model accurately envisages cell behavior compared to earlier spring-dashpot models, and that the model yields are consistent with untried results. To summarize, we will provide just enough information to allow readers to "get their feet wet" and begin creating fractional calculus models for complex systems. The issue of meaning extension is the source of fractional calculus.For instance, real numbers can be extended to complex numbers, natural number factorials can be converted to generalized factorials, gamma functions can be used, and many other examples.

Can the denotation of the derivative of number order dnydxn be expanded to include aimportthe minute n is a fraction? This was the initial query that gave rise to the term of fractional calculus. Subsequently, the query changed to: Can n be any number, whether it is complex, irrational, or fractional? Because the answer to this question was in the positive, the term "fractional calculus" has come to be misunderstood; integration and differentiation to an arbitrary order would be a more appropriate name.Similar to how fractional champions are anoffice off of fansper integer value, fractional calculus is a branch off of the usual characterization of calculus fundamental and imitative operators.

Nevertheless, this is not the case for fractional-order integration besidesdifference, which represents a rapidly expanding subject in terms of both concept and applicability to real-world issues. There hasn't been a suitable geometric or physical reading of these operations for supplementary than 300 years, ever since the concept of variation and mixing of arbitrary order first emerged. In Podlubny (2016), it was revealed that the physical explanation of fractional integration's geometric meaning, "gloom of the precedent," is "gloom on the fortifications."Function has been used in countlessmethodical and technical fields recently, with fluid flow, rheology, diffusive transport, electrical networks, electromagnetic theory, and probability, by way of well as in investigations of visco-elastic supplies. Several definitions of fractional offshoots and integrals will be taken into consideration in the current chapter. For a number of simple functions, it is possible to find the explicit formula for the fractional derivative and integral. Electrical and control theory dynamical systems use fractional calculus.

2. Conclusion:

Extended a number of important results from the literature by creating certain widely used fractional essential operators with the multivariable H-Function, I-Function, and generic class of polynomials. Certain theorems pertinent to N fractional calculus of creation employing I-function and H-function must continue to be created in order to give unification and extension of numerous (known and novel) results already found in the works. A number of new integrals from Wright's GeneralisedHypergeometric Function and I-Function have persisted; these integrals are believed to be useful in both pure and applied mathematics.

References:

- Agarwal, R.P., A proposd'une note de M.PierreHumbert, C.R. Séances Acad. Sci., 236 (21) (1953):2031-2032.
- Erdelyi A. (ed.), Tables of Integral Transforms, McGraw-Hill, York,1 (1954).
- Fahad, Fernandez, Rehman, &Siddiqi, 2020, Tempered and Hadamard-Type Fractional Calculus with Respect to Functions.
- Baleanu, Agarwal, 2021, Fractional Calculus in the Sky.
- Baleanu, Fernandez, 2019, Fractional Operators and Their Classifications.
- G. Jumarie, "On the solution of the stochastic differential equation of exponential growth driven by fractional Brownian motion," Applied Mathematics Letters, vol. 18, no. 7, pp. 817–826, 2005.
- G. Jumarie, "An approach to differential geometry of fractional order via modified Riemann-Liouville derivative," ActaMathematicaSinica, vol. 28, no. 9, pp. 1741–1768, 2012.
- Alshabanat, Jleli, Kumar, Samet, 2021, Generalization of Caputo-Fabrizio Fractional Derivative and Applications to Electrical Circuits.
- Assaleh K. and Ahmad W.M., Modeling speech signals using fractional calculus, 9th international symposium on signal processing and its applications, ISSPA,12 – 15 Feb., (2007): 1-4.
- Chen, Farid, Rehman, &Latif, 2020, Estimations of Fractional Integral Operators for Convex Functions and Related Results.
- H. M. Srivastava, M. Saigo, and S. Owa, A class of distortion theorems involving certain operators of fractional calculus, J. Math. Anal. Appl., 131(1988), 412-420.