International Journal of Engineering, Science and Mathematics
Vol. 12 Issue 11, November 2023,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in , Email: ijesmj@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

Critical Buckling of Orthotropic Clamped Plate Resting on Elastic Foundation

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ABSTRACT

The main objective of this paper is to offer an analytic solution to study the critical buckling of orthotropic rectangular clamped plate resting on elastic foundation. An improved technique based on wide panel-transition matrix is presented here. Strip technique based on wide panels is implemented with exponential matrix to achieve the purposed analytical solutions. Reduction the number of strips of the decomposed domain of plate is successfully substituted by a little number of wide strips to save time and effort without decreasing the accuracy. The critical buckling of rectangular plate under in-plane compressive forces is studied under the effect of acting in-plane forces. Analytic results of buckling loads are obtained for orthotropic clamped plate. The study includes the effects of the aspect ratios and coefficients of elastic foundation on critical buckling of rectangular plates. Validity of the present method is examined by comparing the obtained results with those available in literature. Comparisons proved the good accuracy of the present technique.

Keywords:

Analytic;
Exponential Matrix;
Critical Buckling;
Orthotropic Plate;
Elastic Foundation.

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I. INTRODUCTION

Thin elastic plates subjected to in-plane compressive forces are often encountered in the aircraft, automotive industries and various engineering fields. Under none simply supported boundary conditions, analytical solution of plate subjected to buckling is not easy. Owing to the complicated mathematical structure of the general boundary conditions, closed form solution are generally difficult to be achieved. Approximate or numerical methods must, therefore, be resorted to the solutions. Pablo Moreno et al [1] offered a study on buckling and vibration of anisotropic plates and beams by Ritz method. L. N. Yshii et al [2] used the polynomial and the other trigonometric functions to check the accuracy of the linear buckling on anisotropic plates. Bank and Yin [3] studied the uniaxial buckling of orthotropic plates with free and rotationally restrained unloaded edges. A.V. Rodrigues et al [4] studied the laminated plates subjected to buckling force using differential quadrature method and Murakami's Zig-Zag theory. Ovesy and Fazilati [5] applied a Reddy type shear deformation theory to develop the semi- analytical and spline finite strip methods for prediction the behavior of thick plate. S.K. Lai and L.H. Zhang [6] studied the vibration and buckling on cracked orthotropic plates under the thermal effect. Hayder A. Rasheed et al [7] derived a closed form solution for anisotropic composite plates with simply supported edges under axial compressive force. Thermal buckling of laminated panels under the effects of locally distributed temperature and anisotropic properties has been analyzed by Jinqiang Li et al [8]. S.M. Ibrahim et al [9] used refined theory to analyze buckling of composite beams. Huu-Tai Thai and Dong-Ho Choi [10] applied an efficient refined theory to analyzed buckling of tapered plates. W eiZhang and Xinwei Wang [11] studied elastoplastic buckling of thick rectangular plates by using the differential quadrature method.

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A Rayleigh–Ritz numerical integration method is applied by Kaidas and Dickinson [12] to investigate the buckling and vibration for plates under combined in-plane forces. An analytical spectral stiffness method, depending on the advantages of superposition method was achieved by Xiang Liu et al [13] to studied buckling of plate resting on Winkler foundation. A Semi analytical technique based on finite strips and transition matrix, which was presented by Farag [14]-[17], has been modified her successfully for extended work of plate critical buckling. Farag [16] studied buckling and vibration of isotropic plate under in-plane force by a semi analytical solution based on transition matrix produced from Range Kutta fourth order. Sohan R. et al [18] offered a recent study in linear, nonlinear and post buckling of a stiffened plate with internal cutouts. In the present paper, limited numbers of equal wide strips have been used to express an analytic form of transition matrix. The present method is analytic technique to solve the critical buckling of clamped orthotropic plate CCCC under in plane forces. The effects of aspect ratios, un-axial and bi-axial in-plane forces and orthotropic properties on the critical buckling are investigated. The accuracy for the obtained results is examined. The obtained values are compared with those available in published papers with good agreement.

II. GOVERNING DIFFERENTIAL EQUATION

The dimensionless partial differential equation governing buckling of orthotropic plate subjected to in-plane compression forces N_x and N_y is:

$$\chi_1 W_{\zeta\zeta\zeta\zeta} + 2\beta^2 \chi_2 W_{\zeta\zeta\eta\eta} + \beta^4 W_{\eta\eta\eta\eta} - \frac{N_x a^2}{D_y} W_{\zeta\zeta} - \beta^2 \frac{N_y a^2}{D_y} W_{\eta\eta} + \frac{K a^4}{D_y} W = 0 \qquad (1)$$

Where:

$\chi_1 = \dfrac{D_x}{D_y}$, $\chi_2 = \dfrac{H_{xy}}{D_y}$ and D_x, D_y, H_{xy} are rigidities of orthotropic plate [14],[15].

Value K is the modulus of elastic foundation while $\beta = \dfrac{a}{b}$ is the aspect ratio where a, b are the dimensions of plate in ζ, η directions respectively.

The displacement $W(\zeta, \eta, t)$ of plate is:

$$W(\zeta,\eta) = \sum_{m=1}^{M} \Phi_m(\zeta)\Psi_m(\eta) \qquad (2)$$

$\Psi_m(\eta)$ is unknown longitudinal function satisfying boundary conditions of the clamped edges of plate at $\eta = 0,1$. On the other hand, $\Phi_m(\zeta)$ is a basic function of plate in ζ direction, to be achieved according to the clamped conditions:

$$\Phi_m = \Phi_m' = 0 \quad \text{at} \quad \zeta = 0,1 \qquad (3)$$

Therefore:

$$\Phi_m = \sin \mu_{m\zeta} - \sinh \mu_m \zeta + \alpha_m (\cos \mu_m \zeta - \cosh \mu_m \zeta) \qquad (4)$$

Where:

$\alpha_m = \left(\dfrac{\sin \mu_m - \sinh \mu_m}{\cos \mu_m - \cosh \mu_m}\right)$; $\mu_1 = 4.73001$, $\mu_2 = 7.85398$, $\mu_3 = 10.9955$, in general, for a large number m one can find $\mu_m = (m + 0.5)\pi$.

Multiplying both sides of equation (1) by Φ_m and integrating them with respect to ζ from 0 to 1 , one can obtain:

$$\Psi_m'''' = \frac{1}{\beta^2}[-2\chi_2 \frac{c_m}{a_m} + \overline{N}_y]\Psi_m'' + \frac{1}{\beta^4}[-K_G + \frac{c_m}{a_m}\overline{N}_x - \chi_1 \frac{e_m}{a_m}]\Psi_m \qquad (5)$$

Where:

$$K_G = \frac{Ka^4}{D_y}, \bar{N}_x = \frac{N_x a^2}{D_y}, \bar{N}_y = \frac{N_y a^2}{D_y}$$

and

$$a_m = \int_0^1 \Phi_m \Phi_m d\zeta, \; c_m = \int_0^1 \Phi_m \Phi''_m d\zeta, \; e_m = \int_0^1 \Phi_m \Phi''''_m d\zeta$$

For more convenience, Eq. (5) becomes:

$$\Psi''''_m = \begin{bmatrix} \frac{-\chi_1 e_m + c_m \bar{N}_x - K_G a_m}{a_m \beta^4} & 0 & \frac{-2\chi_2 c_m + a_m \bar{N}_y}{a_m \beta^2} & 0 \end{bmatrix} \{\Psi_m\} \quad (6)$$

Where $\{\Psi_m\} = [\Psi_m \quad \Psi'_m \quad \Psi''_m \quad \Psi'''_m]^T$, T means transpose.

For orthotropic plate there is little number N of equal wide strips lieutenant with $N+1$ nodal lines $0,1,2,....j,......,N-1, N$. The general solution $\{\Psi_m\}_j$ of equation (6) due to the initial vector $\{\Psi_m\}_0$ is expressed at j^{th} nodal line as:

$$\{\Psi_m\}_j = [T_{k,l}]_j \{\Psi_m\}_{j-1} \quad ; \; k = 1,2,3,4 \;, l = 1,2,3,4 \quad (7)$$

Matrix $[T_{k,l}]_j$ is called the j^{th} transition matrix based on series expansion of the exponential matrix $e^{\frac{1}{N}[A_m]_j}$

Where $[A_m]_j$ is a fundamental matrix so that:

$$[A_m]_j = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ (\frac{-\chi_1 e_m + c_m \bar{N}_x - K_G a_m}{a_m \beta^4}) & 0 & (\frac{-2\chi_2 c_m + a_m \bar{N}_y}{a_m \beta^2}) & 0 \end{bmatrix} \quad (8)$$

Exponential matrix $e^{\frac{1}{N}[A_m]_j}$ is expanded analytically with an arbitrary truncation number Q of the power series expansion [19]. Calculations show that truncation number $Q = 12$ is enough for excellent accuracy.

Equation (7) is applied for each orthotropic strip until the final end F of plate is covered. Then the final end vector $\{\Psi_m\}_F$ is:

$$\{\Psi_m\}_F = [T_{k,l}]^N \{\Psi_m\}_0 \quad (9)$$

III. BOUNDARY CONDITIONS at $\eta = 0,1$

The boundary conditions [20], [21] of clamped edges at $\eta = 0,1$ are:

$$\Psi_m = \Psi'_m = 0 \qquad \text{at } \eta = 0,1 \quad (10)$$

Initial vector $\{\Psi_m\}_0$ at $\eta = 0$ is:

$$\{\Psi_m\}_0 = [0 \quad 0 \quad \delta_1 \quad \delta_2]^T \quad (11)$$

Where δ_1 and δ_2 are two arbitrary constants. Boundary conditions for the final clamped edge of plate at $\eta = 1$, are applied in Eq. (9) to yield the Eigen values of critical buckling forces involved in:

$$\begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} \begin{Bmatrix} \delta_1 \\ \delta_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad (12)$$

IV. RESULTS AND DISCUSSIONS

The created method is used to study rectangle orthotropic clamped plates with different values of aspect ratio, orthotropic properties and in-plane forces. The achieved solutions are expressed analytically and represented graphically for the studied cases. The validity of the present technique is proved by comparing the achieved results with those available in literature. For orthotropic clamped CCCC plates under in-plane compressive forces N_x, N_y, the normalized critical biaxial buckling force $R = -\frac{\overline{N}_x}{\pi^2} = -\frac{\overline{N}_y}{\pi^2}$ and uniaxial buckling force $R = -\frac{\overline{N}_x}{\pi^2}$ or $-\frac{\overline{N}_y}{\pi^2}$ are applied when the analytical solutions are derived.

The normalized critical buckling force R are calculated for three different cases of orthotropic plates, as shown in table 1, for a range of aspect ratio $\beta = \frac{a}{b}$ varies from 0.25 to 2.00 and under unaxial load N_y while $N_x = 0$. The results are compared with those available in [16] showing a good accuracy for the present technique.

Table 1: Critical buckling forces R for rectangular orthotropic clamped plates CCCC under unaxial load N_y while $N_x = 0$.

β	$0.5D_x = 0.5D_y = H_{xy}$	$D_x = 0.5D_y = H_{xy}$	$D_x = D_y = H_{xy} = 1$	
	Present		Present	[16]
0.25	2.181540417	4.094836475	3.419013944	2.55739591
0.50	6.405446468	5.173544722	5.541231104	5.55123000
0.75	7.577612600	6.526286088	8.815086126	8.96983177
1.00	8.826204583	7.089331100	10.06367811	10.0697699
1.25	9.869557909	8.691031800	11.10703143	11.1073993
1.50	11.90967601	11.07755740	13.14714954	13.1472809
1.75	14.72141895	14.10604361	15.95889248	15.9588041
2.00	18.18422538	17.71162850	19.42169891	19.4215317

The analytical solutions for the normalized biaxial critical buckling force $R = -\frac{\overline{N}_x}{\pi^2} = -\frac{\overline{N}_y}{\pi^2}$ are derived for orthotropic clamped plates CCCC under in-plane compressive forces N_x, N_y as the first mode eigen value of R. The implicit formulae of critical bulking eigen values are expressed analytically and represented graphically for each case:

Case 1: A square orthotropic clamped plate with $D_x = 0.5D_y$, $H_{xy} = 0.5D_y$ under external in-plane compressive force $N_x = N_y$ where $K_G = 0$, is studied and the closed form solution of the critical buckling force R is expressed by:

$$f(R) = -.1840750062\,R + 0.2370521(10)^{-5}R^6 - 0.0076157961\,R^3 + 0.00073002507\,R^4 - 0.4853420(10)^{-4}R^5 + 1.52041325(10)^{-24}R^{20} - 4.27183125(10)^{-26}R^{21} + 8.84493140(10)^{-28}R^{22} - 5.8319^{-19}R^{15} + 1.160088(10)^{-19}R^{16} + 0.0505206391\,R^2 + .2665745391 - 1.209438(10)^{-20}R^{17} - 2.234(10)^{-19}R^{14} - 8.23(10)^{-17}R^{12} - 6.993(10)^{-15}R^{11} - 8.79623(10)^{-8}R^7 + 2.48772(10)^{-9}R^8 - 5.2324(10)^{-11}R^9 + 7.782(10)^{-13}R^{10} + 1.940(10)^{-17}R^{13} + 1.077737142(10)^{-31}R^{24} - 1.275893999(10)^{-29}R^{23} + 8.293607(10)^{-22}R^{18} - 4.0940043(10)^{-23}R^{19} \quad (13)$$

The critical buckling force R is evaluated in the first mode, (See Fig. 1), where $f(R) = 0$ and $R = 3.687913836$.

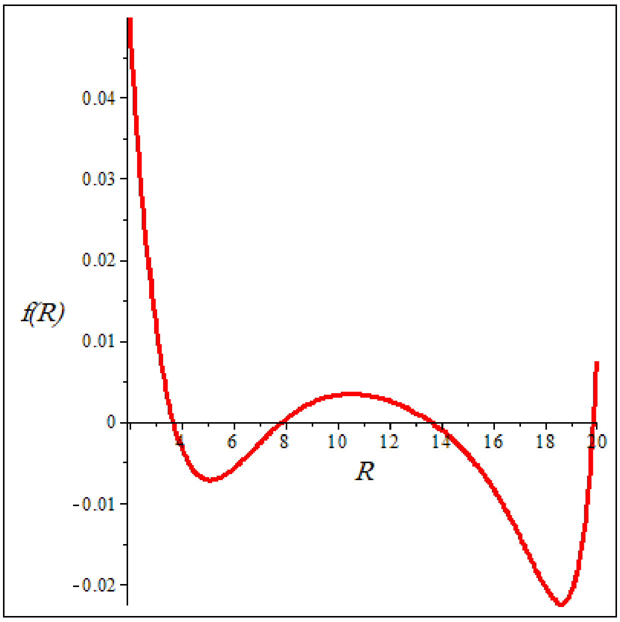

Fig.1 Critical buckling force R for orthotropic square clamped plate under in-plane force where $D_x = 0.5D_y$, $H_{xy} = 0.5D_y$, $N_x = N_y$, , $K_G = 0$

Case 2: A square orthotropic clamped plate with $D_x = 0.5D_y$, $H_{xy} = 0.5D_y$ under external in-plane compressive force $N_x = N_y$ and resting on homogenous subgrade base with $K_G = 100$, is studied and the closed form solution of the critical buckling force R is expressed by:

$$f(R) = -.2021766646\,R - 0.0080060049R^3 + 0.00075878208R^4 - 0.50054229(10)^{-4}R^3 + 0.2431114(10)^{-5}R^6 + 0.0540203977R^2 - 4.0088273(10)^{-23}R^{19} + 1.50196076(10)^{-24}R^{20} - 4.24405099(10)^{-26}R^{21} + 1.077737142(10)^{-31}R^{24} + 2.823(10)^{-17}R^{13} - 2.0340(10)^{-18}R^{14} - 3.8543(10)^{-19}R^{15} + 1.017446(10)^{-19}R^{16} + .3061441681 + 8.82059056(10)^{-28}R^{22} - 1.275894001(10)^{-29}R^{23} - 8.98039(10)^{-8}R^7 + 2.52938(10)^{-9}R^8 - 1.135343(10)^{-20}R^{17} + 8.004473(10)^{-22}R^{18} - 5.2995(10)^{-11}R^9 + 7.863(10)^{-13}R^{10} - 7.245(10)^{-15}R^{11} - 8.17(10)^{-17}R^{12} \quad (14)$$

The critical buckling force R is evaluated in the first mode, (See Fig. 2), where $f(R) = 0$ and $R = 4.0930128817$

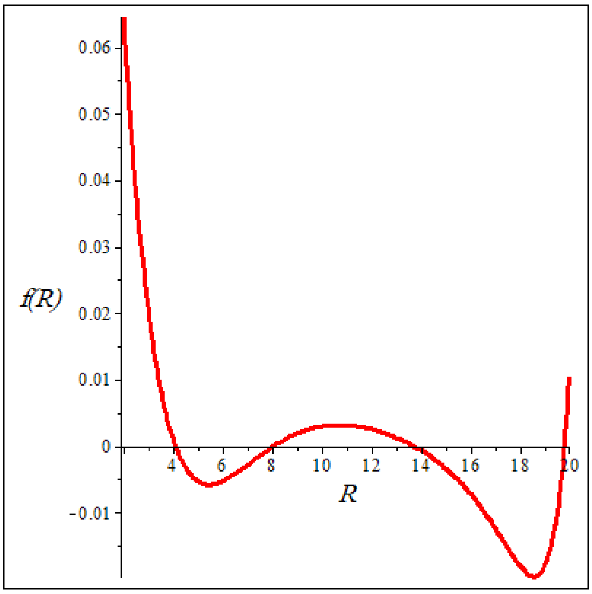

Fig. 2 Critical buckling force R for orthotropic square clamped plate under in-plane force where $D_x = 0.5D_y$, $H_{xy} = 0.5D_y$, $N_x = N_y$, , $K_G = 100$

Case 3: A square orthotropic clamped plate with $D_x = 0.5D_y$, $H_{xy} = 0.5D_y$ under external in-plane compressive force $N_x = N_y$ and resting on homogenous subgrade base with $K_G = 500$, is studied and the closed form solution of the critical buckling force R is expressed by:

$$\begin{aligned} f(R) = &-.2819700669R - 9.73892(10)^{-8}R^7 + 2.69987(10)^{-9}R^8 - 5.5700(10)^{-11}R^9 + 8.163(10)^{-13}R^{10} \\ &- 9.049(10)^{-15}R^{11} + 1.263(10)^{-16}R^{12} + 3.847(10)^{-17}R^{13} - 7.3030(10)^{-18}R^{14} - 0.00966189332R^3 \\ &+ 0.00087950245R^4 - 0.56388516(10)^{-4}R^5 + 0.26821587(10)^{-5}R^6 + 2.9524(10)^{-19}R^{15} \\ &+ 4.92894(10)^{-20}R^{16} - 8.53374(10)^{-21}R^{17} + 6.880962(10)^{-22}R^{18} - 3.6733595(10)^{-23}R^{19} \\ &+ 1.42863019(10)^{-24}R^{20} - 4.13292992(10)^{-26}R^{21} + 8.72322710(10)^{-28}R^{22} - 1.275894001(10)^{-29}R^{23} \\ &+ 1.077737142(10)^{-31}R^{24} + 0.06910523736R^2 + .4865472790 \end{aligned} \quad (15)$$

The critical buckling force R is evaluated in the first mode, (See Fig. 3), where $f(R) = 0$ and $R = 5.696344196$

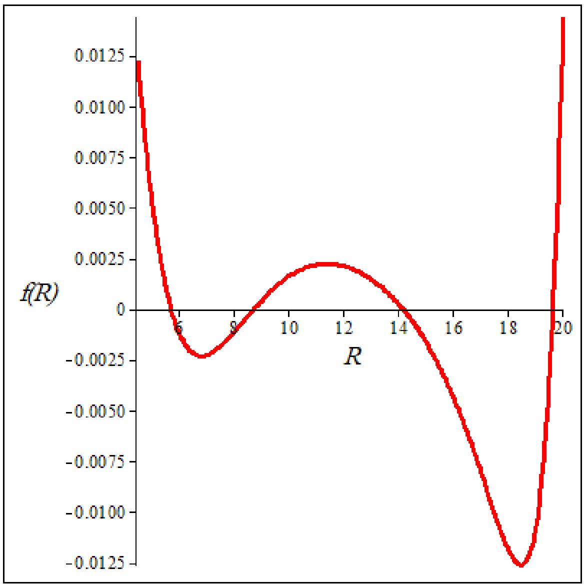

Fig. 3 Critical buckling force R for orthotropic square clamped plate under in-plane force where $D_x = 0.5D_y$, $H_{xy} = 0.5D_y$, $N_x = N_y$, , $K_G = 500$

Case 4: A square orthotropic clamped plate with $D_x = 0.5D_y$, $H_{xy} = 0.5D_y$ under external in-plane compressive force $N_x = N_y$ and resting on homogenous subgrade base with $K_G = 1000$, is studied and the closed form solution of the critical buckling force R is expressed by:

$$\begin{aligned} f(R) = & -0.3994035258R + 0.30160597(10)^{-5}R^6 - 0.01195464353R^3 + 0.001043688532R^4 \\ & -0.64897797(10)^{-4}R^5 + 4.74(10)^{-18}R^{13} + 1.077737142(10)^{-31}R^{24} - 1.275893999(10)^{-29}R^{23} \\ & + 5.550278(10)^{-22}R^{18} - 3.2658126(10)^{-23}R^{19} + 2.92215(10)^{-9}R^8 - 5.9084(10)^{-11}R^9 \\ & + 8.3683(10)^{-13}R^{10} + 0.09052966351R^2 + .7659312444 - 1.1254(10)^{-14}R^{11} - 1.0737986(10)^{-7}R^7 \\ & - 5.32701(10)^{-21}R^{17} - 9.9021(10)^{-18}R^{14} + 6.868(10)^{-16}R^{12} + 1.33804546(10)^{-24}R^{20} \\ & - 3.99402859(10)^{-26}R^{21} + 8.60152274(10)^{-28}R^{22} + 9.1311(10)^{-19}R^{15} - 6.2991(10)^{-21}R^{16} \end{aligned} \quad (16)$$

The critical buckling force R is evaluated in the first mode, (See Fig. 4), where $f(R) = 0$ and $R = 7.651058294$

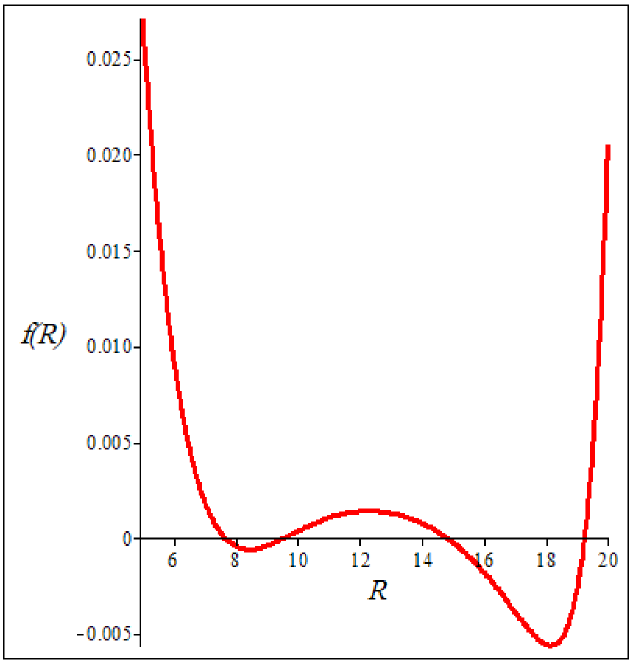

Fig. 4 Critical buckling force R for orthotropic square clamped plate under in-plane force where $D_x = 0.5D_y$, $H_{xy} = 0.5D_y, N_x = N_y$, , $K_G = 1000$

V. CONCLUSION

A combination between the series expansion and strip method is presented here to derive a closed form solutions for eigen values on buckling of plates subjected to in-plane forces. The plate under study is clamped orthotropic resting on elastic homogenous sub-grade base. On the present method the plate domain is divided into a limited number of wide strips (panels) to be solved by the power series expansion. A limited number of strips can be applied with increasing the number of terms of the expanded series to preserve a high accuracy in the achieved solution. The present method is illustrated and the accuracy is verified via several numerical examples examining buckling and vibration of orthotropic plate under the un-axial and biaxial in-plane forces and elastic coefficients of subgrade. The study shows a good agreement in comparisons which prove the validity and applicability of the present technique.

REFERENCES

[1] Pablo Moreno-Garcia1, José V. Araujo dos Santos and Hernani Lopes. A Review and Study on Ritz method admissible functions with emphasis on buckling and free vibration of isotropic and anisotropic beams and plates. *Arch Computat Methods Eng (2018) 25:785–815*. Springer.

[2] L. N. Yshii; E. Lucena Neto; F. A. C. Monteiro; and R. C. Santana. Accuracy of the Buckling Predictions of Anisotropic Plates. *J. Eng. Mech., ASCE, 2018, 144(8).*
[3] Bank, L.C. and Yin, J., "Buckling of orthotropic plates with free and rotationally restrained unloaded edges", *Thin-Walled Structures*, 24(1), 83-96, 1996.
[4] J.D. Rodrigues, C.M.C. Roque, A.J.M. Ferreira, M. Cinefra and E. Carrera. Radial basis functions-differential quadrature collocation and a unified formulation for bending, vibration and buckling analysis of laminated plates, according to Murakami's Zig-Zag theory. Computers and Structures Journal 90–91 (2012) pp107–115
[5] H.R. Ovesy, J. Fazilati. Buckling and free vibration finite strip analysis of composite plates with cutout based on two different modeling approaches, Composite Structures 94 (2012) 1250–1258
[6] S.K. Lai, L.H. Zhang. Thermal effect on vibration and buckling analysis of thin isotropic and orthotropic rectangular plates with crack defects. *Engineering Structures Volume 177,(2018), Pages 444-458*
[7] Hayder A. Rasheed, Rund Al-Masri and Bacim Alali. Closed form stability solution of simply supported anisotropic laminated composite plates under axial compression compared with experiments. *Engineering Structures 151 (2017) 327–336.*
[8] Jinqiang Li, Yoshihiro Narita and Zhihua Wang. The effects of non-uniform temperature distribution and locally distributed anisotropic properties on thermal buckling of laminated panels. *Composite Structures 119 (2015) 610–619*
[9] S.M. Ibrahim, E. Carrera, M. Petrolo, E. Zappino, Buckling of composite thin-walled beams by refined theory, journal of Composite Structures 94 (2012) 563–570
[10] Huu-Tai Thai, Dong-Ho Choi, A n efficient and simple refined theory for buckling analysis of functionally graded plates. Journal of Applied Mathematical Modelling 36 (2012) 1008–1022
[11] W eiZhang and Xinwei Wang. Elasto plastic buckling analysis of thick rectangular plates by using the Differential quadratur emethod, Computers and Mathematics with Applications Journal 61(2011) 44–61
[12] M. M. Kaidas and S. M. Dickinson, Vibration and buckling calculations for rectangular plates subject to complicated in-plane stress distributions by using numerical integration in a Rayleigh–Ritz analysis, *Journal of Sound and Vibration*, 75, 151–162, 1981.
[13] Xiang Liu, Xiao Liu and Wei Zhou. An analytical spectral stiffness method for buckling of rectangular plates on Winkler foundation subject to general boundary conditions. *Applied Mathematical Modeling 86 (2020) 36–53.*
[14] A. M. Farag, Mathematical Analysis of Free and Forced Vibration of Rectangular Plate, Ph.D. Thesis, Faculty of Engineering, Alexandria University, Alexandria, (1994).
[15] A. Farag, A. Ashour. Free Vibration of orthotropic skew slates, *Journal of Vibration and Acoustics*, Vol. 122, pp. 313, July 2000.
[16] A. M. Farag, Buckling and Flexural Vibration of Plates Subjected to In-plane Forces, *International Review of Mechanical Engineering*, Vol. 3, N.2, pp. 239-246, March 2009.
[17] A. Ashour, A. Farag. A Combination between Laplace Transform, Strip Method and Transition Matrix for Determination of Dynamic Response and Damping Effect of Plates, *International Journal of Acoustics and Vibration*, Vol. 5, No. 4, pp. 191-195, 2000.
[18] R. Sohan, S. Likith, Jeevan Sai1, Shashank Vadlamani1, Linear, nonlinear and post buckling analysis of a stiffened panel with cutouts, *International Conference on Materials Science and Engineering* ICMSE, 2022.
[19] Ahmed M. Farag El sheikh. Analytic Formulation on Vibration of Orthotropic Plate Under In-Plan Forces, *International Journal of Engineering, Science and Mathematics,* Vol. 9 Issue 10, October 2020.
[20] Ahmed M. Farag El sheikh. Exact Series Solutionfor Vibrationof Restrained Composite Plate, *International Journal of Scientific Research & Engineering Trends*, Vol. 6 Issue 4, August 2020.
[21] Ahmed M. Farag El sheikh. Analytical Solution of Higher Order Partial Differential Equations, *International Journal of Mathematics and Physical Science Research,* Vol. 8 Issue 9, September 2020.