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The study of customer behavior on non serial queues attached with multiple parallel serial queues

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ABSTRACT:

This paper considers the most appropriate & more general queuing model in respect of customers which are allowed to leave the system at any stage with or without getting service. The paper considers the steady-state behavior of the queuing processes when M service channels in series, are linked with N non-serial channels having balking & reneging phenomenon,wherein:

- Each of M service channels has identical multiple parallel channels.
- Poisson arrivals & exponential service times are followed.
- The service discipline follows SIRO rule (service in random order) instead of FIFO rule (first in first out).
- The customer becomes impatient in the queue after sometime and may leave the system without service.
- Customer behavior ,Balking and reneging is allowed only on N non serial servers.
- Waiting space is infinite.

Key Words: Poisson stream, Reneging, Balking, Traffic intensity.

INTRODUCTION:

The solutions of serial and non-serial queuing processes with reneging and balking phenomenon have been studied by [4]. The steady-state solution of serial and non-serial queuing processes with reneging and balking due to long queue and some urgent message and feedback phenomenon is obtained by [3]. In our present society, the impatient customers generate the most appropriate and modern models in the queuing theory. Incorporating this concept, we study the steady-state analysis of general queuing system in the sense that:

- M service channels in series are linked with N non serial channels having reneging and balking phenomenon where each of M service channels has identical multiple parallel channels.
- The input process is Poisson and the service time distribution is exponential.
- The service discipline follows SIRO-rule(service in random order) instead of FIFO-rule (first in first out)
- The customer becomes impatient in queue after sometime and may leave the system without getting service.
- The input process depends upon the queue size in non serial channels.
- Customer behavior ,Balking and reneging is allowedonly on N non serial servers.
- Waiting space is infinite.

FORMULATION OF MODEL:

The system consists of Q_i(i =1,2,….M) service phases where each service phase Q_i has c_i (i=1,2,….M) identical parallel service facilities andQ_{1j} channels (j=1,2,….N) with respective servers S_i(i=1,2,……M) and S_{1j}(j=1,2,……N).Customers demanding different types of service arrive from outside the system in Poisson distribution with parameters λ_i(i =1,2,….M) at Q_i service phase and λ_{1j}(j=1,2,…..N) at Q_{1j} service phase respectively . But the sight of long queue at Q_{1j}, may discourage the fresh customers from joining it and may decide not to enter the service channel Q_{1j}(j=1,2,….N)then the Poisson input rate λ_{1j}would be $\frac{\lambda_{1j}}{m_j+1}$ where $m_j$is the queue size of Q_{1j}. Further, the impatient customers joining any service channel Q_{1j} may leave the queue without getting service after a wait of certain time. The service time distribution for the server S_i(i=1,2……,M) and S_{1j} (j=1,2…..,N) are mutually independent negative exponential distribution with $\mu_i(i=1,2,.........,M)$ and $\mu_{1j}(j=1,2,....,N)$ respectively. After the completion of service at Q_i (i=1,2,……..M), the customers either leave the system with probability $p_i$or join the next phase with probability q_i such that p_i+q_i =1 (i= 1,2…..M-1). After completion of service at Q_M, the customers either leave the system with probability p_M orjoin any of the Q_{1j}(j=1,2,…….,N) with probability $\frac{q_{Mj}}{m_j+1}$ (j=1,2,….,N) such that $p_M+\sum_{j=1}^{N}\frac{q_{Mj}}{m_j+1}=1.$

If the customers are more than c_i in the Q_i service phase ,all the c_i servers will remain busy and each is putting out the service at mean rate μ_i and thus the mean service rate at Q_i is $c_i\mu_i$,on the other hand if the number of customers is less than c_i in the Q_i service phase ,only n_i out of the c_i servers will be busy and thus the mean service rate at Q_i is $n_i\mu_i$ (i=1,2,…….M).It is assumed that the service commences instantaneously when the customer arrives at an empty service channel.

FORMULATION OF EQUATIONS:

Define P(n_1 ,n_2 , ...n_M ; m_1 ,m_2,m_3, ...m_N ;t) as the probability that at time 't' , there are n_i customers (which may renege or after being serviced by the Q_i phase either leave the system or join the next service phase) waiting in the Q_i service phase (i=1,2,……..,M) , m_j customers (which may balk or renege or after being serviced leave the system)waiting before the servers S_{1j} (j=1,2,…….N) .

We define the operators $T_{i.}$ And $T_{.i}$to act upon the vector $\tilde{n}=(n_1,n_2,.............,n_M)$ and $T_{j.}$ and $T_{.j}$ and $T_{.j,j+1.}$ to act upon the vector $\tilde{m}=(m_1,m_2,.............,m_N)$ as follows:

$T_{i.}(\tilde{n})$ = (n_1,n_2,………n_i-1…n_M)

$T_{.i}(\tilde{n})$ = (n_1,n_2,………n_i+1…n_M)

$T_{.j,j+1.}(\tilde{m})$ = (m_1 ,m_2 ,……,m_j +1,m_{j+1} -1….m_N)

$T_{j.}(\tilde{m})$ = (m_1,m_2,………m_j-1…m_N)

$T_{.j}(\tilde{m})$ = (m_1,m_2,………m_j+1…m_N)

Following the procedure given by [4], we write difference differential equations as :

$$\frac{dP(\tilde{n},\tilde{m};t)}{dt} = -\left[\begin{array}{l}\sum_{i=1}^{M}\lambda_i + \sum_{j=1}^{N}\frac{\lambda_{1j}}{m_j+1} + \sum_{i=1}^{M}\delta(n_i)(\mu_{in_i} + \delta_{n_i-c_i} r_{in_i}) \\ + \sum_{j=1}^{N}\delta(m_j)\{(\mu_{1j}) + R_{jm_j}\}\end{array}\right] P(\tilde{n},\tilde{m};t)$$

$$+ \sum_{i=1}^{M}\lambda_i P(T_{i.}(\tilde{n}),\tilde{m};t)) + \sum_{j=1}^{N}\frac{\lambda_{1j}}{m_j}P(\tilde{n};T_{j.}(\tilde{m});t)$$

$$+ \sum_{i=1}^{M}\delta_{n_i-c_i}(r_{in_{i+1}})P(T_{.i}(\tilde{n}),\tilde{m};t) + \sum_{i=1}^{M-1}q_i\mu_{in_{i+1}}P(T_{.i\,i+1.}(\tilde{n}),\tilde{m};t) +$$

$$\sum_{i=1}^{M}p_i\mu_{in_{i+1}}P(T_{.i}(\tilde{n}),\tilde{m};t) + \sum_{j=1}^{N}\mu_{Mn_M+1}\frac{q_{Mj}}{m_j}P(n_1,n_2,.....n_M+1,T_{j.}(\tilde{m});t)$$

$$+ \sum_{j=1}^{N}(\mu_{1j} + R_{jm_j+1})P(\tilde{n};T_{.j}(\tilde{m});t) \qquad ....(1)$$

for $n_i \geq 0$, $mj \geq 0$, (i = 1,2,….M) , (j= 1,2….N) .

Where

$$\delta(x) = \begin{bmatrix} 1 & when & x \neq 0 \\ 0 & when & x = 0 \end{bmatrix}$$

$$\delta(n_i) = \begin{bmatrix} 0 & when & n_i = 0 \\ 1 & when & n_i \neq 0 \end{bmatrix}$$

$$\delta_{(n-c)} = \begin{bmatrix} 0 & when & n < c \\ 1 & when & n \geq c \end{bmatrix}$$

$$\mu_{in_i} = \begin{bmatrix} n_i\mu_i & when & 1 \leq n_i < c_i \\ c_i\mu_i & when & n_i \geq c_i \end{bmatrix}$$

$$r_{in_i} = \mu_i e^{\frac{-\mu_i T_{0i}}{n_i}} \Big/ (1 - e^{\mu_i e^{\frac{-\mu_i T_{0i}}{n_i}}}) \qquad ; i = 1, 2, ...M$$

$$R_{jm_j} = \mu_{1j} e^{\frac{-\mu_{1j} T_{0j}}{m_j}} \Big/ 1 - e^{\frac{\mu_{1j} T_{0j}}{m_j}}$$

Where r_{in_i} *and* R_{jm_j} are the average rates at which the customers renege after a wait of certain time T_{0i} *and* T_{0j} whenever there are n_i and m_j customers in the Q_i and Q_{1j} service phases respectively and $P(\tilde{m},\tilde{n};t) = 0$ if any of the arguments is negative

Steady-State Equations:

We write the steady-state equations of the queuing model by equating the time derivative to zero in the equation (1)

$$\left[\sum_{i=1}^{M}\lambda_i+\sum_{j=1}^{N}\frac{\lambda_{1j}}{m_j+1}+\sum_{i=1}^{M}\delta(n_i)(\mu_{in_i}+\delta_{n_i-c_i}r_{in_i})+\sum_{j=1}^{N}\delta(m_j)\{(\mu_{1j})+R_{jm_j}\}\right]P(\tilde{n},\tilde{m})$$

$$=\sum_{i=1}^{M}\lambda_i P(T_{i\,.}(\tilde{n}),\tilde{m}))+\sum_{j=1}^{N}\frac{\lambda_{1j}}{m_j}P(\tilde{n};T_{j\,.}(\tilde{m}))$$

$$+\sum_{i=1}^{M}\delta_{n_i-c_i}(r_{in_{i+1}})P(T_{.i}(\tilde{n}),\tilde{m})+\sum_{i=1}^{M-1}q_i\mu_{in_{i+1}}P(T_{.i\,i+1.}(\tilde{n}),\tilde{m})$$

$$+\sum_{i=1}^{M}p_i\mu_{in_{i+1}}P(T_{.i}(\tilde{n}),\tilde{m})+\sum_{j=1}^{N}\mu_{Mn_M+1}\frac{q_{Mj}}{m_j}P(n_1,n_2,.....n_M+1,T_{j\,.}(\tilde{m}))$$

$$+\sum_{j=1}^{N}(\mu_{1j}+R_{jm_j+1})P(\tilde{n};T_{.j}(\tilde{m})) \qquad ....(2)$$

for $n_i \geq 0$, $mj \geq 0$; (i = 1,2,….,M) ; (j= 1,2,….,N) .

Case I:– When $n_i < c_i$:

For $n_i < c_i$,then the resulting equations (2) reduce to as under:

$$\left[\sum_{i=1}^{M}\lambda_i+\sum_{j=1}^{N}\frac{\lambda_{1j}}{m_j+1}+\sum_{i=1}^{M}\delta(n_i)(n_i\mu_i)+\sum_{j=1}^{N}\delta(m_j)\{(\mu_{1j})+R_{jm_j}\}\right]P(\tilde{n},\tilde{m})$$

$$=\sum_{i=1}^{M}\lambda_i P(T_{i.}(\tilde{n}),\tilde{m}))+\sum_{j=1}^{N}\frac{\lambda_{1j}}{m_j}P(\tilde{n};T_{j.}(\tilde{m}))$$

$$+\sum_{i=1}^{M-1}q_i\mu_i(n_i+1)P(T_{.i\,i+1.}(\tilde{n}),\tilde{m})$$

$$+\sum_{i=1}^{M}p_i\mu_i(n_i+1)P(T_{.i}(\tilde{n}),\tilde{m})+\sum_{j=1}^{N}\mu_M(n_M+1)\frac{q_{Mj}}{m_j}P(n_1,n_2,.....n_M+1,T_{j.}(\tilde{m}))$$

$$+\sum_{j=1}^{N}(\mu_{1j}+R_{jm_j+1})P(\tilde{n};T_{.j}(\tilde{m})) \qquad ....(3)$$

The solutions of the steady state equations (3) can be verified to be:

$$P(\tilde{n},\tilde{m}) = P(\tilde{0},\tilde{0})\left[\left(\frac{1}{n_1!}\right)\left(\frac{\lambda_1}{\mu_1}\right)^{n_1}\right]\left[\left(\frac{1}{n_2!}\right)\left(\frac{\lambda_2+q_1\alpha_1^{'}}{\mu_2}\right)^{n_2}\right]\left[\left(\frac{1}{n_3!}\right)\left(\frac{\lambda_3+q_2\alpha_2^{'}}{\mu_3}\right)^{n_3}\right]$$

$$..............\left[\left(\frac{1}{n_M!}\right)\left(\frac{\lambda_M+q_{M-1}\alpha_{M-1}^{'}}{\mu_M}\right)^{n_M}\right]\left[\left(\frac{1}{m_1!}\right)\left(\frac{(\lambda_{11}+\mu_M q_{M1}\rho_M)^{m_1}}{\prod_{j=1}^{m_1}(\mu_{11}+R_{1j})}\right)\right]$$

$$\left[\left(\frac{1}{m_2!}\right)\left(\frac{(\lambda_{12}+\mu_M q_{M2}\rho_M)^{m_2}}{\prod_{j=1}^{m_2}(\mu_{12}+R_{2j})}\right)\right]................\left[\left(\frac{1}{m_N!}\right)\left(\frac{(\lambda_{1N}+\mu_M q_{MN}\rho_M)^{m_N}}{\prod_{j=1}^{m_N}(\mu_{1M}+R_{Mj})}\right)\right] \quad ....(4)$$

for $n_i \geq 0$, $m_j \geq 0$; (i = 1,2,….,M) ; (j= 1,2,….,N) .
where

$$\rho_M = \frac{\lambda_M + q_{M-1}\alpha_{M-1}^{'}}{\mu_M}$$

$$\alpha_1^{'} = \lambda_1$$

$$\alpha_k^{'} = \lambda_k + q_{k-1}\alpha_{k-1}^{'} \qquad k = 2,3,........,M-1$$

We obtain $P(\tilde{0},\tilde{0})$ from the normalizing condition $\sum_{\tilde{m}=\tilde{0}}^{\infty}\sum_{\tilde{n}=\tilde{0}}^{\infty} P(\tilde{n},\tilde{m}) = 1$ and with the restrictions that traffic intensity of each service channel of the system is less than unity.

Case II:– When $n_i \geq c_i$

for $n_i \geq c_i$, then the resulting equations (2) will reduce to as under:

$$\left[\sum_{i=1}^{M}\lambda_i + \sum_{j=1}^{N}\frac{\lambda_{1j}}{m_j+1} + \sum_{i=1}^{M}\delta(n_i)(c_i\mu_i + r_{in_i}) + \sum_{j=1}^{N}\delta(m_j)\{(\mu_{1j}) + R_{jm_j}\}\right]P(\tilde{n},\tilde{m})$$

$$= \sum_{i=1}^{M}\lambda_i P(T_{i.}(\tilde{n}),\tilde{m})) + \sum_{j=1}^{N}\frac{\lambda_{1j}}{m_j}P(\tilde{n};T_{j.}(\tilde{m})) + \sum_{i=1}^{M}(p_i c_i\mu_i + r_{in_{i+1}})P(T_{.i}(\tilde{n}),\tilde{m})$$

$$+ \sum_{i=1}^{M-1} q_i c_i \mu_i P(T_{.i\,i+1.}(\tilde{n}),\tilde{m}) + \sum_{j=1}^{N}\mu_M c_M \frac{q_{Mj}}{m_j}P(n_1,n_2,.....n_M+1,T_{j.}(\tilde{m}))$$

$$+ \sum_{j=1}^{N}(\mu_{1j} + R_{jm_j+1})P(\tilde{n};T_{.j}(\tilde{m})) \qquad ....(5)$$

The solutions of the steady state equations can be verified to be:

$$P(\tilde{n},\tilde{m}) = P(\tilde{0},\tilde{0}) \left[\frac{(\lambda_1)^{n_1}}{\prod_{i=1}^{n_1}(c_1\mu_1 + r_{1i})} \right] \left[\frac{\{\lambda_2(c_1\mu_1 + r_{1n_1+1}) + c_1q_1\mu_1\alpha_1\}^{n_2}}{\prod_{i=1}^{n_2}(c_2\mu_2 + r_{2i})(c_1\mu_1 + r_{1n_1+1})^{n_2}} \right]$$

$$\left[\frac{\{\lambda_3\prod_{i=1}^{2}(c_i\mu_i + r_{in_i+1}) + c_2q_2\mu_2\alpha_2\}^{n_3}}{\prod_{i=1}^{n_3}(c_3\mu_3 + r_{3i})\left[\prod_{i=1}^{2}(c_i\mu_i + r_{in_1+1})\right]^{n_3}} \right] \ldots\ldots\ldots\ldots\ldots\ldots$$

$$\left[\frac{\{\lambda_M\prod_{i=1}^{M-1}(c_i\mu_i + r_{in_i+1}) + c_{M-1}q_{M-1}\mu_{M-1}\alpha_{M-1}\}^{n_M}}{\prod_{i=1}^{n_M}(c_M\mu_M + r_{Mi})\left[\prod_{i=1}^{M-1}(c_i\mu_i + r_{in_I+1})\right]^{n_M}} \right] \left[\frac{(\lambda_{11} + \mu_M c_M \rho_M' q_{M1})^{m_1}}{\lfloor m_1 \prod_{j=1}^{m_1}(\mu_{11} + R_{1j})} \right]$$

$$\left[\frac{(\lambda_{12} + \mu_M c_M \rho_M' q_{M2})^{m_2}}{\lfloor m_2 \prod_{j=1}^{m_2}(\mu_{12} + R_{2j})} \right] \ldots\ldots\ldots\ldots \left[\frac{(\lambda_{1N} + \mu_M c_M \rho_M' q_{MN})^{m_N}}{\lfloor m_N \prod_{j=1}^{m_N}(\mu_{1M} + R_{Mj})} \right] \qquad (6)$$

Where $$\rho_M' = \frac{\lambda_M \prod_{i=1}^{M-1}(c_i\mu_i + r_{in_i+1}) + c_{M-1}\mu_{M-1}q_{M-1}\alpha_{M-1}}{(c_M\mu_M + r_{Mn_M+1})\prod_{i=1}^{M-1}(c_i\mu_i + r_{in_i+1})} \quad i$$

$$\alpha_1 = \lambda_1$$

$$\alpha_k = \lambda_k \prod_{i=1}^{k-1}(c_iu_i + r_{in_i+1}) + q_{k-1}\alpha_{k-1}u_{k-1}c_{k-1}\,.$$

We obtain $P(\tilde{0},\tilde{0})$ from the normalizing condition $\sum_{\tilde{m}=\tilde{0}}^{\infty}\sum_{\tilde{n}=\tilde{0}}^{\infty} P(\tilde{n},\tilde{m}) = 1$ and with the restrictions that traffic intensity of each service channel of the system is less than unity

Thus $P(\tilde{n},\tilde{m})$ is completely determined.

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