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A Study of the effect of double diffusive instability and finger instability in heterogeneous fluid layer

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Abstract

The density of heterogeneous fluid is considered to be of the form $\rho_0 f(z)$ in such a way that df/dz is constant. The cumulative effect of the heterogeneity of the medium and the coupled molecular diffusion on the system is studied in detail. The results are presented through graphs for a wide range of the parameters and are discussed.

Introduction

In recent years, the study of the phenomenon of the convective process in a horizontal fluid/porous layer has received remarkable attention owing to its very wide applications in Science, Engineering and Industrial areas. Convection can be the dominant mode of heat and mass transport in many processes that involve the freezing and melting of the storage material. Natural convection is an omnipresent transport phenomenon in saturated porous geological structures. In fact, the study of convection is of most importance in geophysical, astrophysical and heat transfer problems. For example, the extraction of energy from geothermal sources is the most promising one among the other methods and it is believed that the fluid in these reservoirs arehighly permeable and consists of multi-components rather than a single component. Therefore, buoyancy driven convection in a porous medium with water as the working fluid is an important mechanism of energy transport. In fact, the key feature of a major geothermal system while on the land or beneath the sea floor is a high intrinsic heat transport. In fact, the local thermal conditions and the physical properties of media are directly of great importance on the characteristics of the heat and mass transfer in such real configurations. Moreover, the nature of the fluid flow (2D, 3D, pattern and range) is drastically dependent on the complexity of geophysical sites, i.e., the geometry, heterogeneity and anisotropy of the domains. Fluid motions induced by free convection have tangible effects in geothermal areas, on the diffusion of pollutants or on the mineral diagenesis processes.

Technically, the phenomenon is important as it may occur in porous insulation of buildings thereby increasing the loss of heat. In the stellar atmosphere also, certain heavenly bodies may be considered to be porous material and the study has relevance to that also. Also, convection in planetary cores and stellar interiors often occurs in the presence of strong rotational and magnetic constraints. Over the past four decades, there has been an increasing concern about soil and water contamination from industrial and agricultural chemicals. In such cases, thermal and chemical interactions between a rotating porous layer and an overlying fluid layer can be considered. Such a study has many engineering and environmental applications also.

The multi-component onset of convection is important in many naturally occurring phenomena and technological processes. In double diffusive convection, heat and solute diffuse at different rates, as a result of which complex flow structures may form which have no counterpart in buoyant flows driven by a single component. Extensive literature pertaining to this phenomenon is available (Fujita and Gosting (1956); Stern (1960); Miller (1966); Miller et al. (1967); Nield (1968); Hurle and Jakeman (1971); Huppert and Manins (1972); Schechter et al (1972); Velarde and Schechter (1972); Vitaliano et al. (1972); Caldwell (1974); Turner (1974,1985); Griffiths (1979); Antoranz and Venard (1979); Leaist and Lyons (1980); Placsek and Toomre (1980); Narusawa and Suzukawa (1981); Srimani (1981); Takao, Tsuchiya and Narasuwa (1982); McTaggart (1983); Srimani (1984, 1991); Torrones and Pearistein (1989); Anamika (1990); Chen and Su (1992); Zimmermann Muller and Davis (1992); Tanny, Chen and Chen (1994); Shivakumar (1997); Skarda, Jacqmin and McCaughan (1998);).But in Type Ill, considered in this review, an additional effect viz., the effect of the coupled fluxes of the two properties due to irreversible thermodynamic effects is considered. This is termed as the effect of Coupled diffusion or Cross-diffusion and Sort effect is an example of this cross- diffusion where a flux of salt is caused by a spatial gradient of temperature. In fact, Dufour effect is the corresponding flux of heat caused by a spatial gradient of temperature.

Some literature (Hurle and Jakeman (1971); Skarda, Jacqmin and McCaughan (1998); Zimmermann et al (1992); Chen and Chen (1994)) in this direction are useful.McDougall (1983) has made a detailed study of Double-diffusive convection caused by coupled molecular diffusion. The results of his linear stability analysis, predicts that for a sufficiently large coupled diffusion effect, fingers can form even when the concentrations of both the components make the fluid's density gradient statically stable. The conditions for the occurrence of finger as well diffusive instabilities are predicted. The results of his finite amplitude analysis reveal that for sufficiently large cross-diffusion effect, fingers do exist.

Absolutely very sparse literature is available in this direction and no literature is available for a heterogenous fluid layer.

The works discussed so far deals with penetrative convection in fluid and porous layers in absence of fixed-flux conditions. To the author's knowledge no literature pertaining to penetrative convection subject to nonlinear temperature / salinity profile with nonlinear density temperature and / are salinity relationship is available under different constraints in presence of fixed-flux conditions. Sparse literature with some common subject is available. Therefore, an attempt is made in this review to include all these effects with the object of providing the prevailing influences of the relevant physical parameters on the stability of the system as well as on the bifurcation and fluid patterns.

i) What similarities exist between the results pertaining to the study of double diffusive convection with coupled molecular diffusion in homogeneous and non-homogeneous fluid layers?

ii) Is it possible to recover the results pertaining to the case of homogeneous fluid layer from the present results?

iii) What additional features are exhibited by the double diffusive convective system in a heterogeneous fluid layer under the influence of cross-diffusion terms?

Mathematical formulation and the solution

The physical configuration considered here constitutes a heterogeneous in compressible fluid layer confined between two horizontal plates, which maintain a contrast in fluid properties between the plates ΔT and ΔS. In the initial state the gradients of T and S are uniform. In this paper, the analysis is carried out for a two-dimensional situation.

The equation of state

$$\rho = \rho_0[f(z) + \alpha T + \beta S] + \delta\rho$$

where the density of the heterogeneous fluid is of the form $\rho_0 f(z)$. Here $\rho_0$is the density at the lower boundary and $f(z)$ satisfies the following conditions

i) $f(0) = 1$ and

ii)$f(z)$ is a monotonic function of z such that df/dz is a constant

The other quantities have their predefined meanings and are the perturbed quantities. $D_{11}$and $D_{21}$are the two cross-diffusion terms such that $D_{11} > D_{22}$and z-axis is directed vertically upwards.

Linear stability analysis

In this article, we carry out the linear stability analysis of the system. The normal mode technique (Chandrasekar 1961) is employed. The form of the physical quantities is taken as

$$[u_x, u_z, T, S, \delta\rho] = [u_x(z), u_z(z), T(z), S(z), \delta\rho(z)]\exp\left(ik_x x + k_y y + pt\right)$$

where k_x and k_y are the horizontal wave numbers such that the wave number of the disturbance is

$$k^* = \sqrt{{k_x}^2 + {k_y}^2}$$

Also $p = p_1 + ip_2$ where $p_1$denotes the growth rate and p_2 denotes the frequency of the disturbances. We have the set of dimensionless equations

$$pT = (D^2 - a^2)T + \frac{D_{12}}{D_{11}}(D^2 - a^2)S\frac{\Delta S}{\Delta T} + w$$

$$pS = \tau(D^2 - a^2)S + \frac{D_{21}}{D_{11}}(D^2 - a^2)T\frac{\Delta T}{\Delta S} + w$$

$$\frac{\partial}{\partial t}(\delta\rho) + \rho_0 w\frac{df}{dz} = 0$$

$$\left[p(D^2 - a^2)\left(p - \sigma(D^2 - a^2)\right)\right]w = -a^2\sigma R_2 w + a^2\sigma p(RT + R_s S)$$

The following dimensionless parameters govern the problem

$R = \alpha g \Delta T d^3/\nu\, D_{11}$ is the Rayleigh number,

$R_s = \beta g \Delta S d^3/\nu\, D_{11}$ is the solute Rayleigh number,

$R_2 = g d^4 (df/dz)/\nu\, D_{11}$ is the heterogeneity parameter,

$\sigma = \nu/D_{11}$ is the Prandtl number,

$\tau = D_{22}/D_{11}$ is the diffusivity parameter.

The boundaries are assumed to be stress-free and hence we have

$w = D^2 w = S = T = 0$at $z = 0$and $z = 1$

In addition to the above dimensionless parameters, cross-diffusion parameters also appear.

Stationary convection

Let the instability set in as stationary convection. Then the marginal state is characterized by $p = 0$.On substituting $p = 0$ in (1) to (4) we obtain

$(D^2 - a^2)T + \frac{D_{12}}{D_{11}}(D^2 - a^2)S\frac{\Delta S}{\Delta T} = -w$ (5)

$\tau(D^2 - a^2)S + \frac{D_{21}}{D_{11}}(D^2 - a^2)T\frac{\Delta T}{\Delta S} = -w$ (6)

$a^2 \sigma R_2 w = 0$ (7)

From equation (7) we get $w = 0$. Substituting $w = 0$ in equation (5) and equation (6) and integrating we get the following result on using the boundary conditions

$$S = T = 0$$

From this, we conclude that for $p = 0$, the initial state is unperturbed which is a contradiction. Hence$p \neq 0$. In other words, stationary convection is not possible for the problem under consideration and also the principle of exchange of stabilities is not valid.

Oscillatory convection

In stationary convection, it is shown that the instability cannot set in as stationary convection. Therefore, it may be oscillatory. But the oscillatory marginal state is characterized by $p = ip_2$, where p_2 is real. Now substituting $p = ip_2$, in (1) to (4) yields the following

$ip_2 T = (D^2 - a^2)T + \frac{D_{12}}{D_{11}}(D^2 - a^2)S\frac{\Delta S}{\Delta T} + w$ (8)

$$ip_2T = \tau(D^2 - a^2)S + \frac{D_{21}}{D_{11}}(D^2 - a^2)T\frac{\Delta T}{\Delta S} + w \qquad (9)$$

$$[-p_0{}^2(D^2 - a^2) - ip_2\sigma(D^2 - a^2)^2w = a^2\sigma R_2 w + a^2\sigma ip_2(RT + R_sS)](10)$$

Following Chandrasekar (1961), we seek the solution of (8) to (10) (corresponding to the lowest mode) as

$$\left.\begin{aligned} T &= T_0 \sin\pi z \\ S &= S_0 \sin\pi z \\ w &= w_0 \sin\pi z \end{aligned}\right\} \qquad (11)$$

Substituting (11) into (8) and (9) and rearranging, we obtain

$$w_0 = T_0\left(k_0{}^2 + ip_2\right) + \left(\frac{D_{12}}{D_{11}}\frac{\Delta S}{\Delta T}k_0{}^2\right)S_0 \qquad (12)$$

$$w_0 = S_0\left(\tau k_0{}^2 + ip_2\right) + \left(\frac{D_{21}}{D_{11}}\frac{\Delta T}{\Delta S}k_0{}^2\right)T_0 \qquad (13)$$

Solving we obtain

$$T_0 = \frac{(B_2 - B_1)}{(A_1B_2 - A_2B_1)}w_0,\ S_0 = \frac{(A_1 - A_2)}{(A_1B_2 - A_2B_1)}w_0 \quad (14)$$

where

$$A_1 = k_0{}^2 + ip_2,\quad A_2 = \frac{D_{21}}{D_{11}}\frac{\Delta T}{\Delta S}k_0{}^2,\quad B_1 = \frac{D_{12}}{D_{11}}\frac{\Delta S}{\Delta T}k_0{}^2,\quad B_2 = \tau k_0{}^2 + ip_2,\quad k_0{}^2 = \pi^2 + a^2$$

(15)Substitute (14) and (15) into (8) so that

$$[p_2{}^2k_0{}^2 - ip_2\sigma k_0{}^4 - a^2\sigma R_2](A_1B_2 - A_2B_1) = ia^2\sigma p_2[R(B_2 - B_1) - R_s(A_1 - A_2)]$$

which on simplification yields

$$R = \frac{(a_1 + ia_2) + R_s(b_1 + ib_2)}{(c_1 + ic_2)}$$

$$R = \frac{[(a_1 + ia_2) + R_s(b_1 + ib_2)](c_1 - ic_2)}{c_1{}^2 + c_2{}^2}$$

where

$$a_1 = p_2{}^4k_0{}^2 - p_2{}^2\left[k_0{}^6(\sigma\tau + \tau + \sigma) + a^2\sigma R_2 - \frac{D_{12}}{D_{11}}\frac{D_{21}}{D_{11}}k_0{}^6\right] + a^2\sigma R_2k_0{}^4\left[\tau - \frac{D_{12}}{D_{11}}\frac{D_{21}}{D_{11}}\right]$$

$$a_2 = -{p_2}^3{k_0}^4(\sigma+\tau+1) + p_2\sigma\left[a^2R_2(\tau+1){k_0}^2 + \tau{k_0}^8 - \frac{D_{12}}{D_{11}}\frac{D_{21}}{D_{11}}{k_0}^8\right]$$

$$b_1 = -{p_2}^2a^2\sigma$$

$$b_2 = p_2a^2\sigma{k_0}^2\left[1-\frac{D_{21}}{D_{11}}\frac{\Delta T}{\Delta S}\right]$$

$$c_1 = {p_2}^2a^2\sigma$$

$$c_2 = p_2a^2\sigma{k_0}^2\left[\frac{D_{12}}{D_{11}}\frac{\Delta S}{\Delta T}-\tau\right]$$

Separating real and imaginary part we have

$$R = \frac{[(a_1c_1+a_2c_2)+R_s(b_1c_1+b_2c_2)] + i[(a_2c_1-a_1c_2)+R_s(b_2c_1-b_1c_2)]}{{c_1}^2+{c_2}^2}$$

Setting the imaginary part to be zero, we obtain a quadratic equation is p_2

$$A^*{p_2}^4 + B^*{p_2}^2 + C^* = 0 \qquad (16)$$

so that $${p_2}^2 = \frac{-B^*\pm\sqrt{{B^*}^2-4A^*C^*}}{2A^*} \qquad (17)$$

where

$$A^* = {k_0}^2\left[1+\sigma+\frac{D_{12}}{D_{11}}\frac{\Delta S}{\Delta T}\right]$$

$$B^* = \sigma{k_0}^6\left[\frac{D_{12}}{D_{11}}\frac{D_{21}}{D_{11}}-\tau\right] + {k_0}^6\left[\frac{D_{12}}{D_{11}}-\tau\right]\left[\frac{D_{12}}{D_{11}}\frac{D_{21}}{D_{11}}-(\sigma\tau+\tau+\sigma)\right] - a^2\sigma R_2\left[1+\frac{D_{12}}{D_{11}}\frac{\Delta S}{\Delta T}\right] + a^2\sigma R_s\left[\tau-1+\frac{D_{21}}{D_{11}}\frac{\Delta T}{\Delta S}-\frac{D_{12}}{D_{11}}\frac{\Delta S}{\Delta T}\right]$$

$$C^* = a^2\sigma R_2{k_0}^4\left[\tau-\frac{D_{12}}{D_{11}}\frac{D_{21}}{D_{11}}\right]\left[\frac{D_{12}}{D_{11}}\frac{\Delta S}{\Delta T}-\tau\right]$$

Again, the real part yields

$$R = \sum_{i=1}^{5}\frac{{A_i}^*}{{A_6}^*}$$

It is not necessary that we always have a real $p_2$from (16) which is required for oscillatory marginal state. Since (17) is a quadratic equation in $p_2^{\,2}$, the roots of (16) may be both real, both complex or may be real and coincident, depending upon the nature of the discriminant $B^{*2} - 4A^* C^*$.

The conditions for the existence of marginal state can be derived for the marginal state can be derived for the following two cases:

(i) $R_2 > 0$i.e., $df\ / dz\ > 0$

(ii) $R_2 < 0$ i.e., $df\ / dz\ < 0$

A careful glance at (16) and (17a) reveals the following results:

Case(i) $R_2 > 0$, in this case, if $\tau > \frac{D_{12}}{D_{11}} \frac{D_{21}}{D_{11}}$ and $\tau < \frac{D_{12}}{D_{11}} \frac{\Delta S}{\Delta T}$ then $p_2^{\,2} < 0$

i.e., p_2 will not be real and the over stability cannot occur and hence the marginal state cannot exist.

On the other hand, if one of the conditions on τ is violated, then $p_2^{\,2} > 0$ and $p_2$will be real and over stability occurs. Clearly marginal state exists.

Case(ii) $R_2 < 0$

In this case, if the conditions on τ specified above are satisfied, then $p_2^{\,2} > 0$ and $p_2$will be real. On the other hand, if one of the conditions on τ is violated, then once again $p_2^{\,2} < 0$ and marginal state cannot exist.

The non-oscillatory modes are characterized by $p = p_1$ ($p_1$is real) and $p_2 = 0$. Hence, the substitution of $p = p_1$ and (11) into (1) to (4) results in a quadratic equation and it is found that for $R_2 > 0$ and

$$\frac{D_{12}}{D_{11}} \frac{D_{21}}{D_{11}} < \tau < \frac{D_{12}}{D_{11}} \frac{\Delta S}{\Delta T}$$

Only non-oscillatory modes can exist. Finally, we can write the condition for non-oscillatory modes as $R_2 > 0$ and

$$\frac{D_{12}}{D_{11}}\frac{D_{21}}{D_{11}} < \frac{D_{22}}{D_{11}} < \frac{D_{12}}{D_{11}}\frac{\Delta S}{\Delta T}$$

The above results suggest that both the heterogeneity parameter and the cross-diffusion parameters have a dominating influence on the prediction of oscillatory as well as non-oscillatory modes.

Results and Discussion

i)In figures (1) to (8) the graphs of the profiles T_0, S_0 vs D and T_1,S_1 vs D are presented for $(RT_1, RT_2) = (3, 1)$, $\sigma = 10$, $\tau = 0.1$ and $R_2 = -10, 0, 10$ respectively. A comparison of figures (2) and (26) shows if $RT_1 > RT_2$, then there is a considerable reduction in the values of the T_0,S_0 etc. when compared to the case of $RT_1 = RT_2$.

ii)In figures (9) to (16), the graphs of T_0, S_0, T_1 and S_1, are presented for$(RT_1, RT_2) = (1, 1)$, $\sigma = 10$, $\tau = 0.707101$ and $R_2 = -10, 0, 10$respectively. It is interesting to observe the behaviour of the profiles for a wide range of the parameters under different combinations.

iii) In figures(17) to (36) the profiles of T_0, S_0 vs$D$ and T_1, S_1 vs$D$are presented for the parametric values specified. Here$RT_1 > RT_2$ and$\tau = 0.707101$. The behaviour of the profiles is interesting.

From the above results, it is concluded that the behaviour of the profiles is in accordance with the conditions discussed in sec oscillatory convection and by a suitable choice of the parameters τ,RT_1 and RT_2, the behaviour the profiles for $R_2 > 0$ or $R_2 < 0$, can be discussed for both oscillatory and non- oscillatory modes. It is interesting to note that the cumulative effect of the heterogeneity and the diffusion parameters is remarkable. The results are in excellent agreement with the available results Mcdougal (1983) in the limiting cases.

References

1. J.C. ANTORANZ and VEGIAR DE. MIC, (1979), Thermal diffusion and convective stability: The role of uniform rotation of the container. *Phys. Fluids* 22, 1038

2. B.M. ANAMIKA, (1990) Dispersion effect on nonlinear double diffusive convection in an anisotropic porous layer, *Ph.D. Thesis, Bangalore Univ.*, India

3. D.R. CALDWELL,(1974), Experimental studies on the onset of thermohaline convection, *J. Fluid Mech.*64,347

4. C.F. CHEN and T.F. SU, (1992), Effect of surface tension on the onset of convection in a double-diffusive layer, *Phys. Fluids*, 4 (11), 2360

5. C.F.CHEN and C.C.CHEN, (1994), Effect of surface tension on the stability of a binary fluid layer under reduced gravity, *Phys. Fluids* ,6, 1482.

6. H.FUJITA and L.J. GOSTING, (1956), An exact solution of the equations of free diffusion in three-component systems with interacting flows, and its use in evaluation of the diffusion coefficients, *J.Chem.Soc.Am*,78,1098

7. R.W. GRIFFITHS, (1979), The influence of a third diffusive component upon the onset of convection, *J. Fluid Mech*. 92, 659

8. D.T. HURLE and E. JAKEMAN, (1971), Soret-driven thermosolutal convection, *J. Fluid Mech*. 47, 667

9. H. E. HUPPERT and P.C. MANIS, (1973), Limiting conditions for salt-fingering at an interface, *Deep sea Res*. 20, 315

10. D.G.LEAIST and P.A.LYONS, (1980), multi-component diffusion in dilute solutions of mixed electrolytes, *Aust.J.Chem*.33,1869

11. C.L. MC TAGGART, (1983), Convection driven by concentration and temperature dependent surface tension, *J. Fluid Mech*. 134, 301

12. L. MILLER, (1966), Instabilities in multicomponent liquid diffusion, *J. South African Chem. Inst*. 19, 125

13. L. MILLER, T.H. SPURLING and E.A. MASON, (1967), Instabilities in ternary diffusion, *Phys. Fluid* .10, 1809

14. U. NARUSAWA and A.Y. SUZUKAWA, (1981), Experimental study of double-diffusive cellular convection due to uniform lateral heat-flux, *J. Fluid Mech.* 113, 387

15. D. A.NIELD, (1968), Onset of thermohaline convection in a porous medium, *Water Resources Res*.4,553

16. S.A. PLACSEK and J. TOOMRE, (1980), Nonlinear evolution and structure of salt fingers. In Marine Turbulence (ed. J.C.J. NIHOUL), *Elsevier* ,193

17. R.S. SCHECHTER, I. PRIGOGINE and J.R. HAMM, (1972), Thermal diffusion and convective stability, *Phys. Fluids.* 15, 371

18. A. V. SHIVAKUMARA, (1997), Asymptotic solutions for double diffusive and magneto-convection problems in fluid and porous layers, *Ph.D. Thesis, Bangalore Univ. India*

19. J.R.L. SKARDA, D. JACQMIN and F.E. Mc CAUGHAN, (1998), Exact & approximate solution to the double-diffusive Marangoni Benard problem with cross-diffusive, *J. Fluid Mech*. 366, 109

20. M.E. STERN, (1960), The salt-fountain and Thermohaline convection, *Tellus* 12, 599

21. P. K. SRIMANI, (1981), Finite amplitude cellular convection in a rotating and non-rotating fluid saturated porous layer, *Ph.D. Thesis*, *Bangalore Univ. India*

22. P. K. SRIMANI, (1984), The effect of Coriolis force on nonlinear thermal convection in an anisotropic porous layer, *Proc. 29th ISTAM Conf. India*

23. P. K. SRIMANI, (1990), Marangoni porous convection in a cylindrical geometry. Part 1 & Part 2, *Ind. J. Pure & Appl. Math*., 21 (8), 763

24. J. TANNY, C. C. CHEN and C.F. CHEN, (1995), Effects of interaction between Marangoni and double-diffusive instabilities, *J. Fluid Mech.* 303, 1

25. TORRONES and PERLSTEIN, (1989), The onset of convection in a multi component fluid layer, *Phys. Fluids A* 1 (5) 845.

26. J.S. TURNER, (1985), Multicomponent convection, *Ann. Rev. Fluid Mech.* 17, 11

27. M.G. VELARDE and R.S. SCHECHTER, (1972), Thermal diffusion and convective stability, *Phys. Fluids*. 15, 379

28.V. VITAGLIANO, A.S. ZAGAR, R. SARTORIO and M. CORCIONE, (1972), Dissipative structures and diffusion in ternary systems, *J, Phys. Chem.* 76, 2050

29. G. ZINMMERMANN, U. MULLER and S. A. DAVIS, (1992), Benard convection in binary mixtures with Soret effect and solidification, *J. Fluid Mech.* 238, 65

Figures:

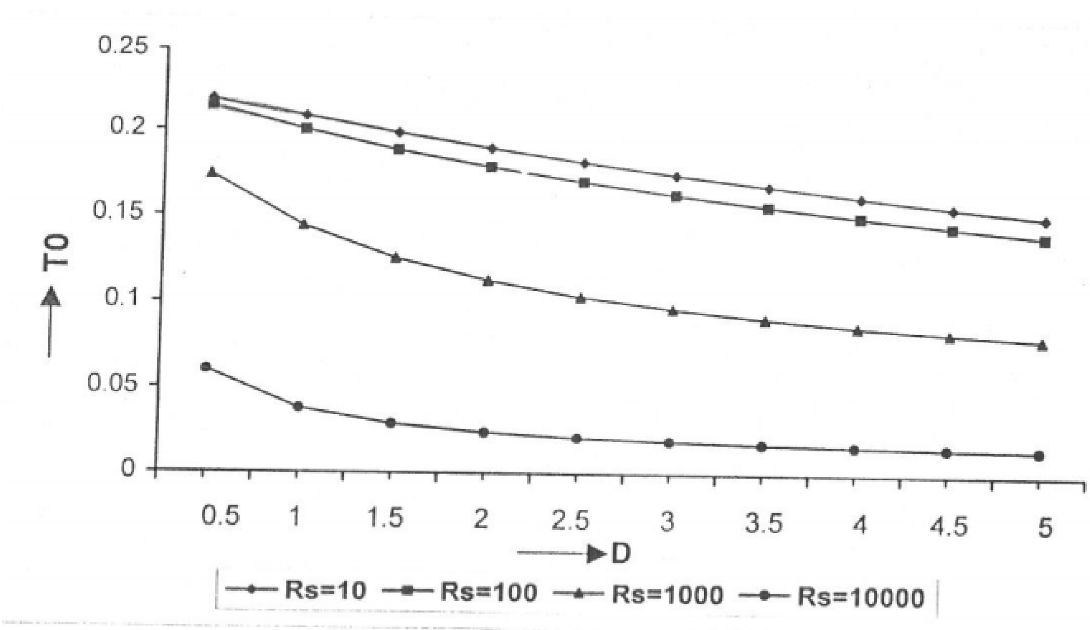

Figure (1): T_0 v/s D for $R_2 = -10$, $\sigma = 10$, $\tau = 0.1$ and $(RT_1, RT_2) = (3, 1)$

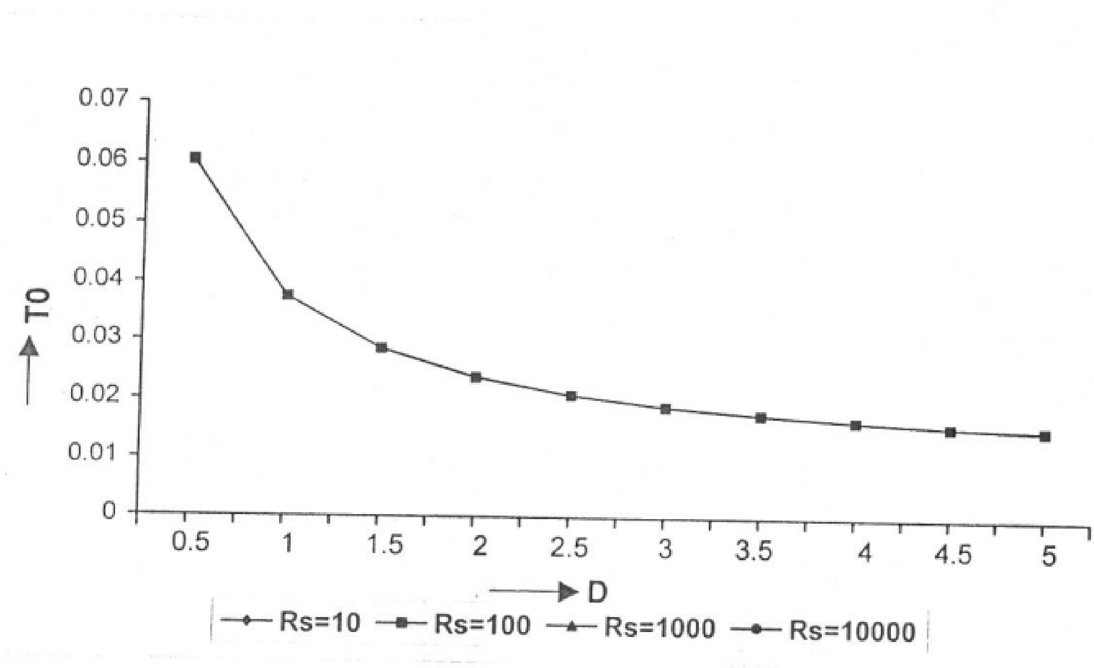

Figure (2): T_0 v/s D for $R_2 = 0\, \&\, 10$, $\sigma = 10$, $\tau = 0.1$ and $(RT_1, RT_2) = (3, 1)$

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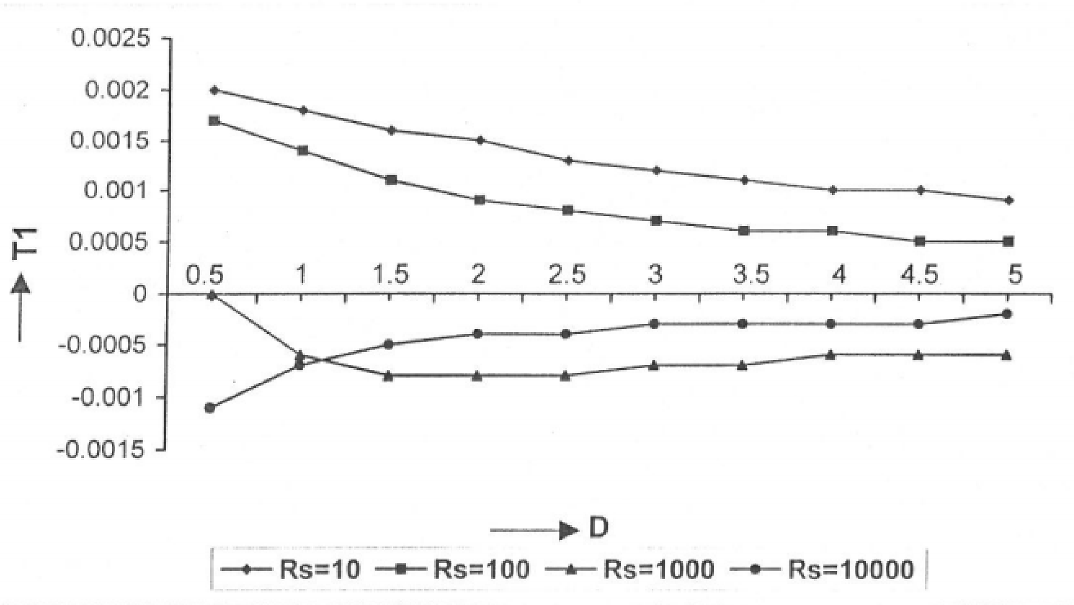

Figure (3): T_1 v/s D for $R_2 = -10$, $\sigma = 10$, $\tau = 0.1$ and $(RT_1, RT_2) = (3, 1)$

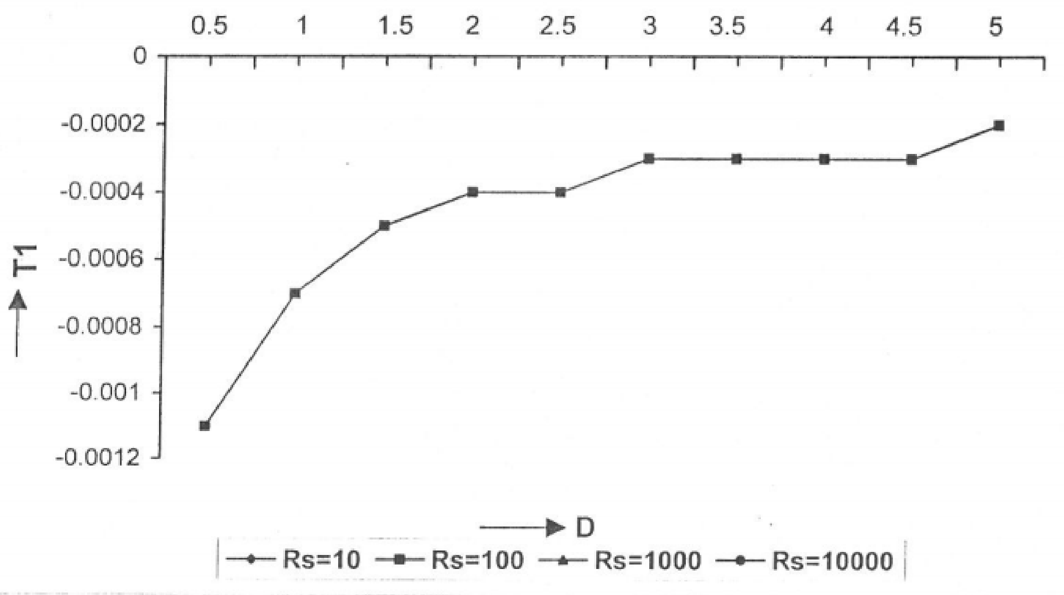

Figure (4): T_1 v/s D for $R_2 = 0 \& 10$, $\sigma = 10$, $\tau = 0.1$ and $(RT_1, RT_2) = (3, 1)$

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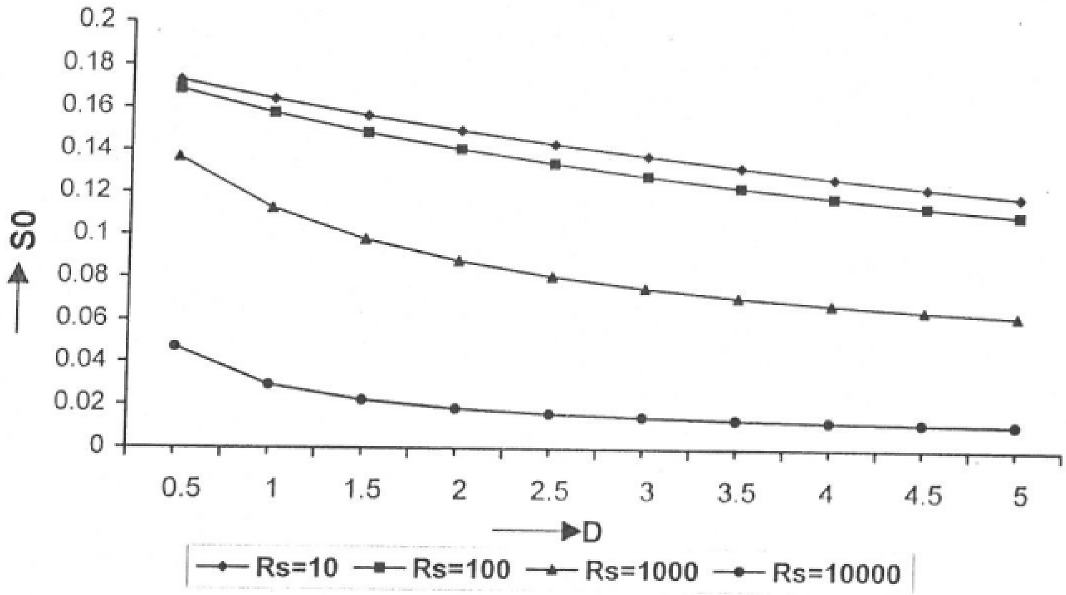

Figure (5): S_0 v/s D for $R_2 = -10$, $\sigma = 10$, $\tau = 0.1$ and $(RT_1, RT_2) = (3, 1)$

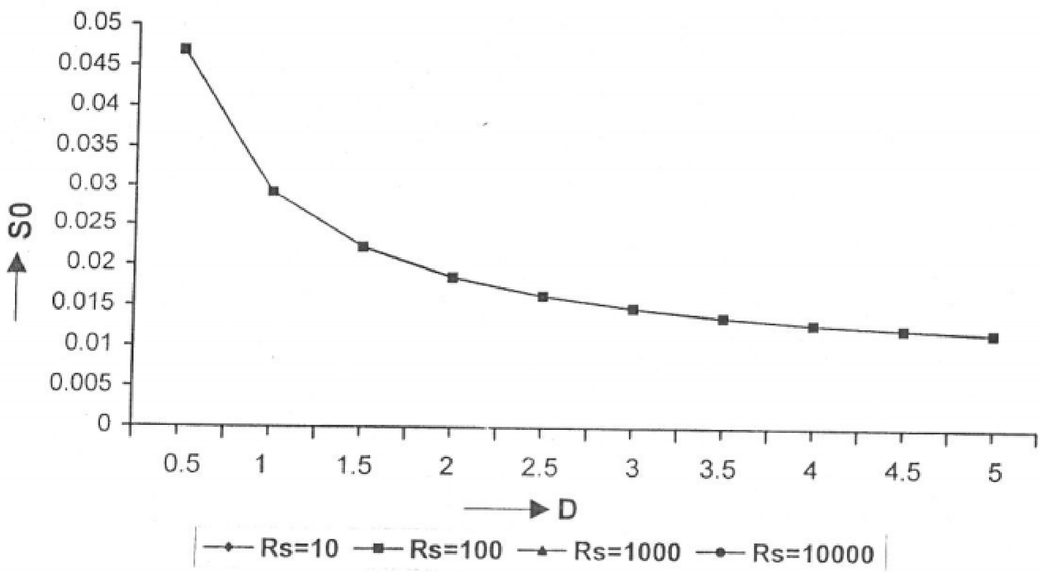

Figure (6): S_0 v/s D for $R_2 = 0 \& 10$, $\sigma = 10$, $\tau = 0.1$ and $(RT_1, RT_2) = (3, 1)$

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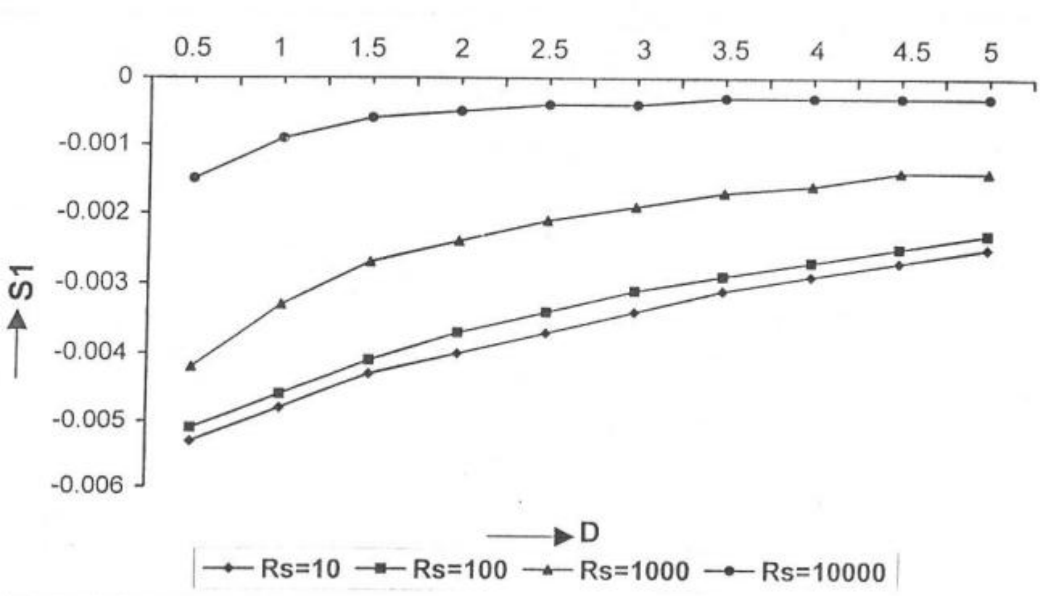

Figure (7): S_1 v/s D for $R_2 = -10$, $\sigma = 10$, $\tau = 0.1$ and $(RT_1, RT_2) = (3, 1)$

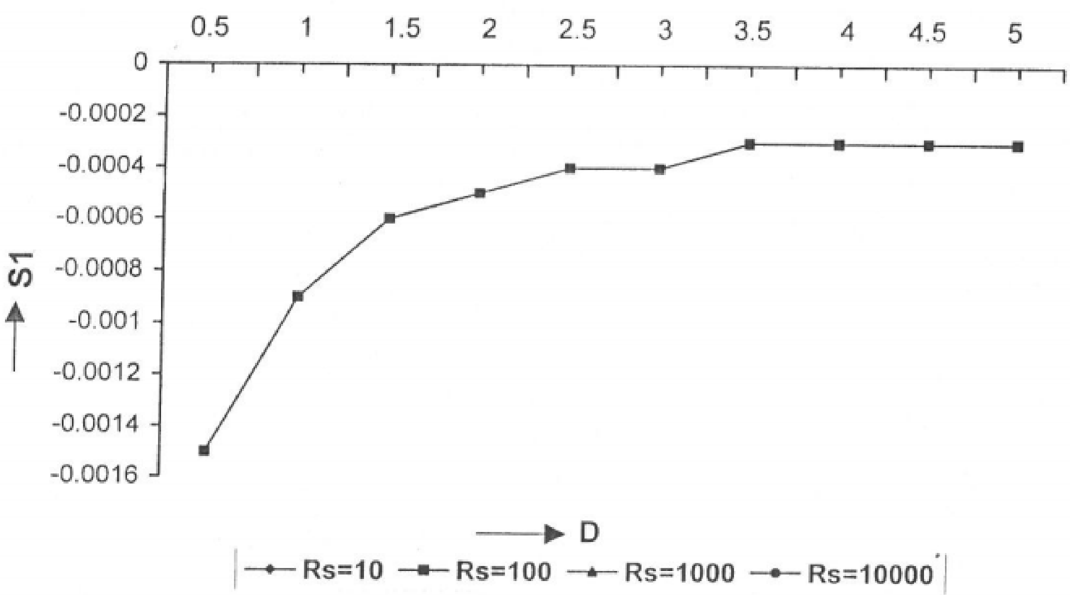

Figure (8): S_1 v/s D for $R_2 = 0$ &10 , $\sigma = 10$, $\tau = 0.1$ and $(RT_1, RT_2) = (3, 1)$

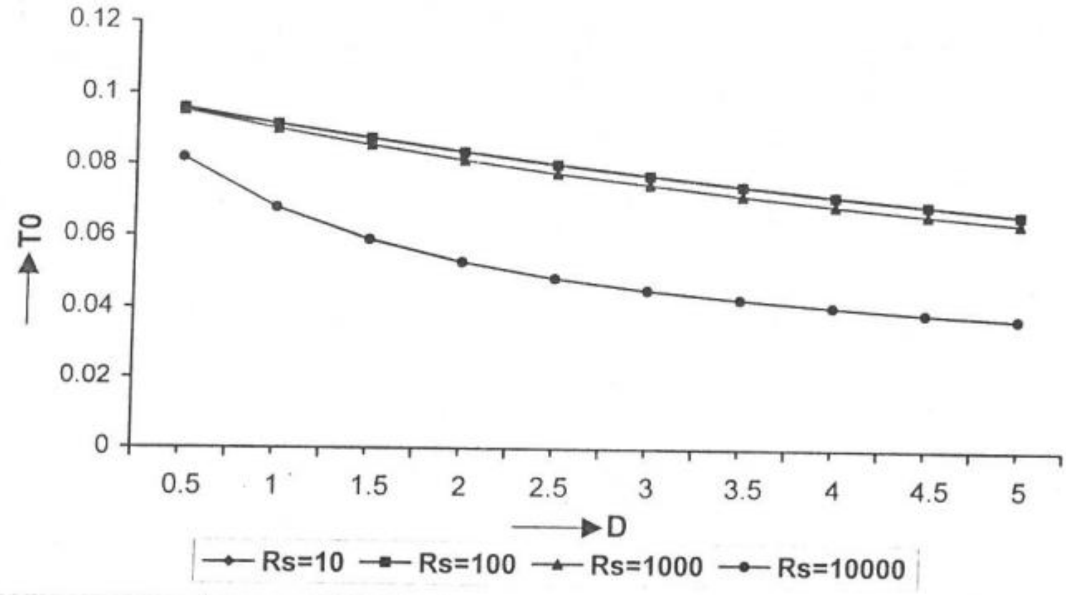

Figure (9): T_0 v/s D for $R_2 = -10$, $\sigma = 10$, $\tau = 0.707107$ and $(RT_1, RT_2) = (1, 1)$

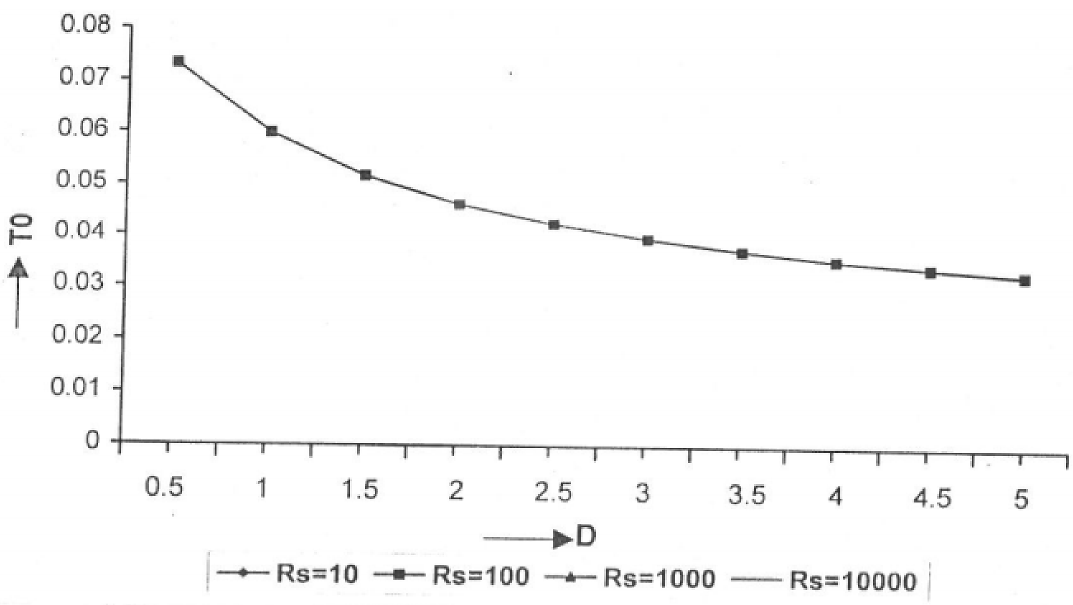

Figure (10): T_0 v/s D for $R_2 = 0 \& 10$, $\sigma = 10$, $\tau = 0.707107$ and $(RT_1, RT_2) = (1, 1)$

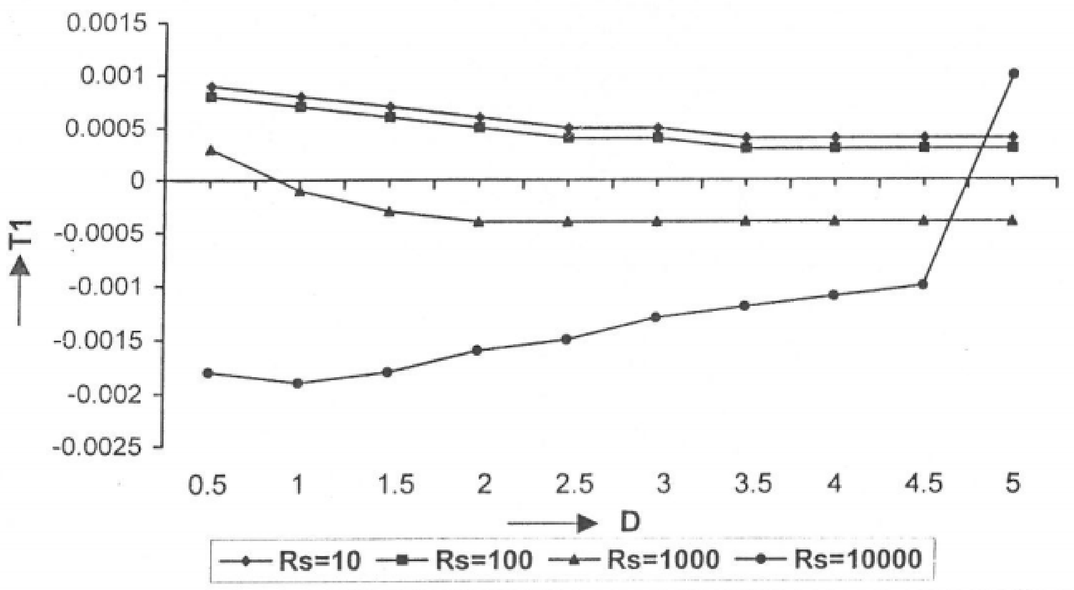

Figure (11): T_1 v/s D for $R_2 = -10$, $\sigma = 10$, $\tau = 0.707107$ and $(RT_1, RT_2) = (1, 1)$

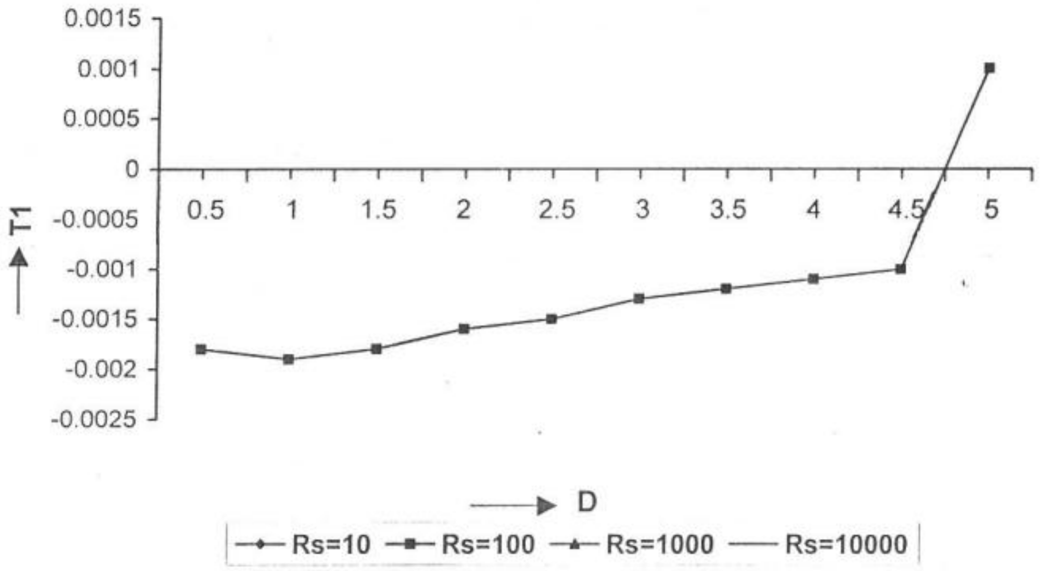

Figure (12): T_1 v/s D for $R_2 = 0$ & 10 , $\sigma = 10$, $\tau = 0.707107$ and $(RT_1, RT_2) = (1, 1)$

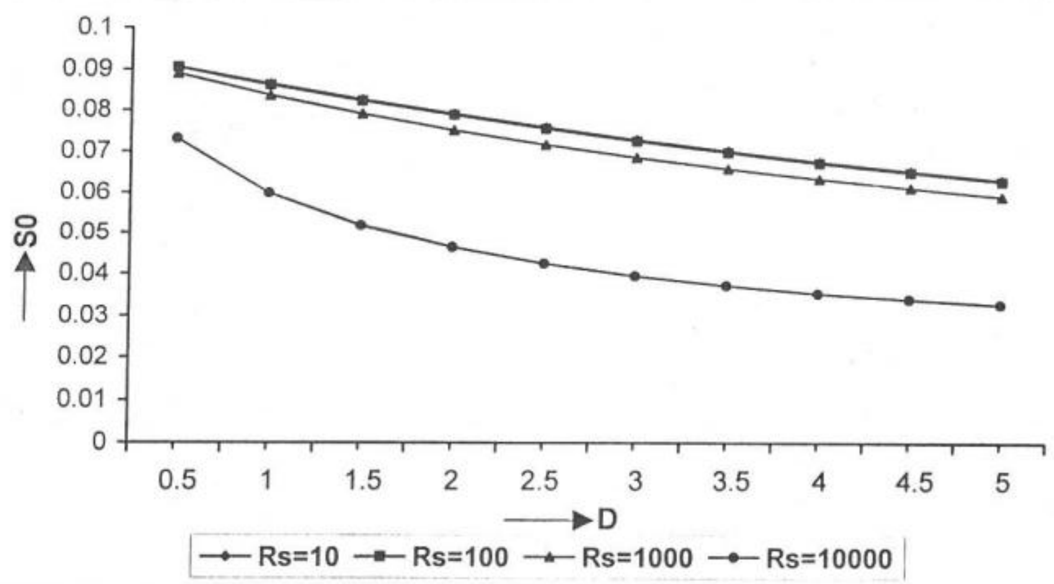

Figure (13): S_0 v/s D for $R_2 = -10$, $\sigma = 10$, $\tau = 0.707107$ and $(RT_1, RT_2) = (1, 1)$

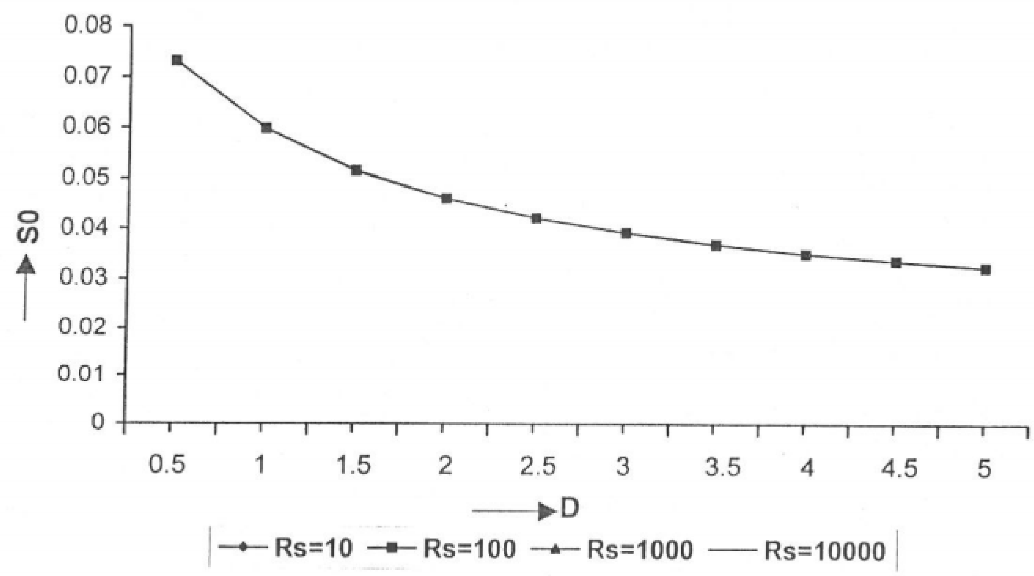

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Figure 14: S_0 v/s D for $R_2 = 0 \,\&\, 10$, $\sigma = 10$, $\tau = 0.707107$ and $(RT_1, RT_2) = (1, 1)$

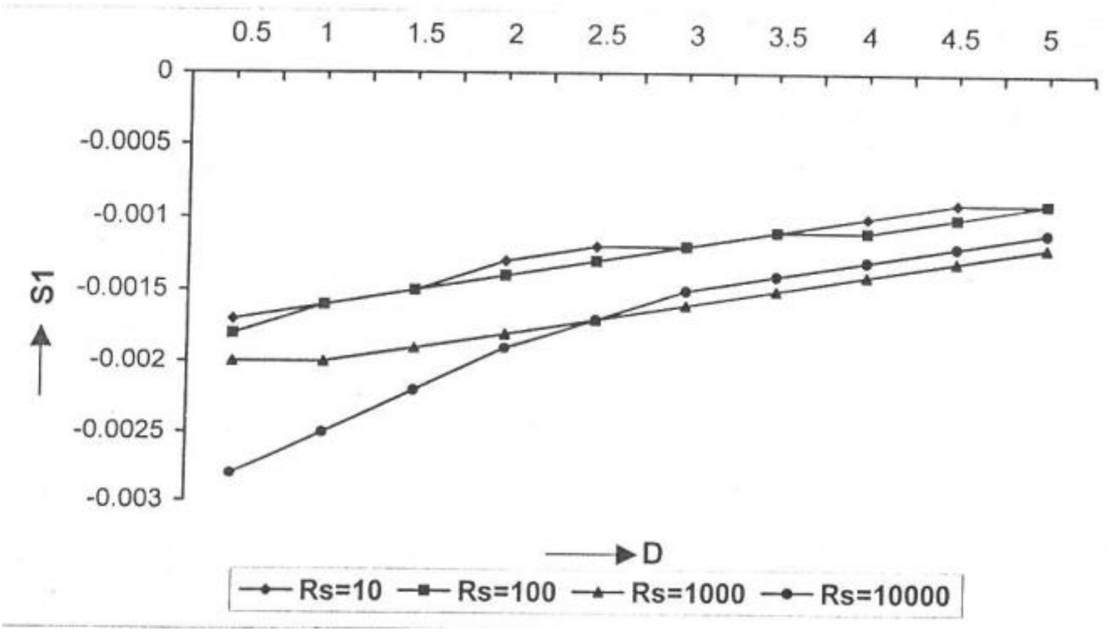

Figure (15): S_1 v/s D for $R_2 = -10$, $\sigma = 10$, $\tau = 0.707107$ and $(RT_1, RT_2) = (1, 1)$

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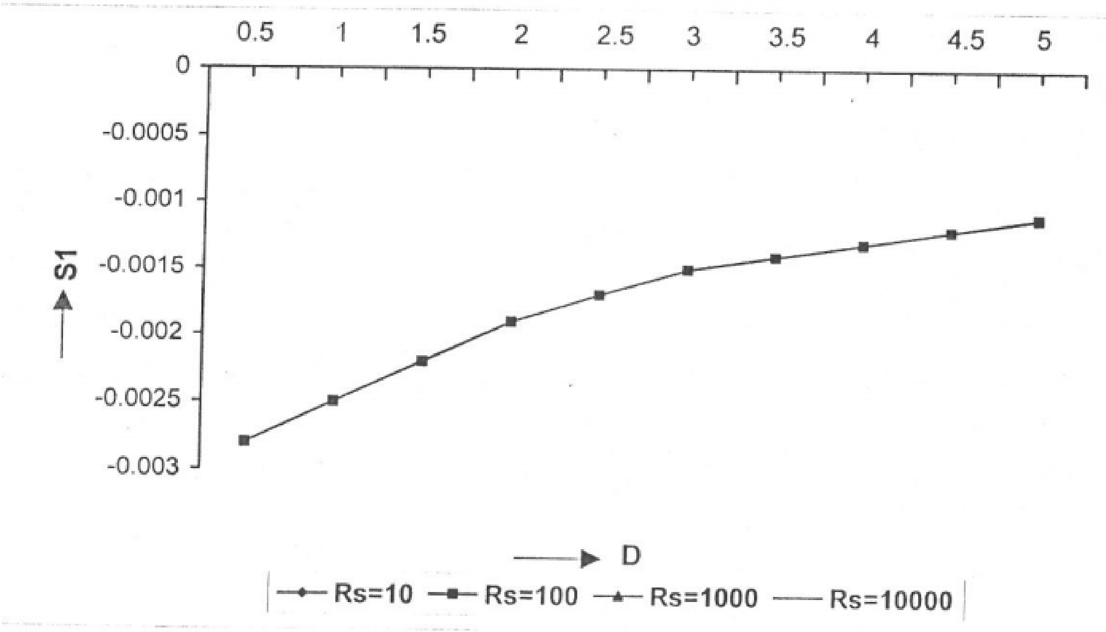

Figure (16): S_1 v/s D for $R_2 = 0 \,\&\, 10$, $\sigma = 10$, $\tau = 0.707107$ and $(RT_1, RT_2) = (1,1)$

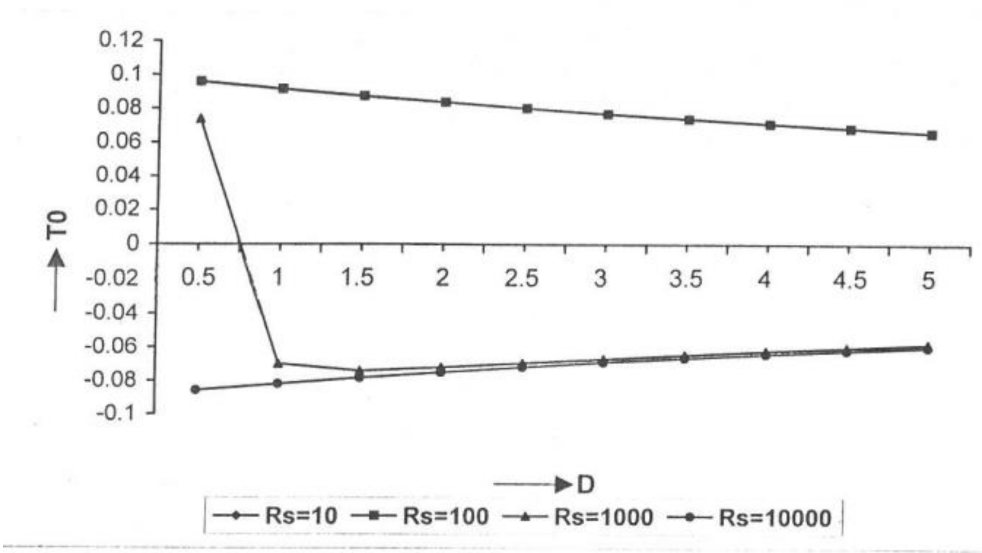

Figure (17): T_0 v/s D for $R_2 = -10$, $\sigma = 10$, $\tau = 0.707107$ and $(RT_1, RT_2) = (1,3)$

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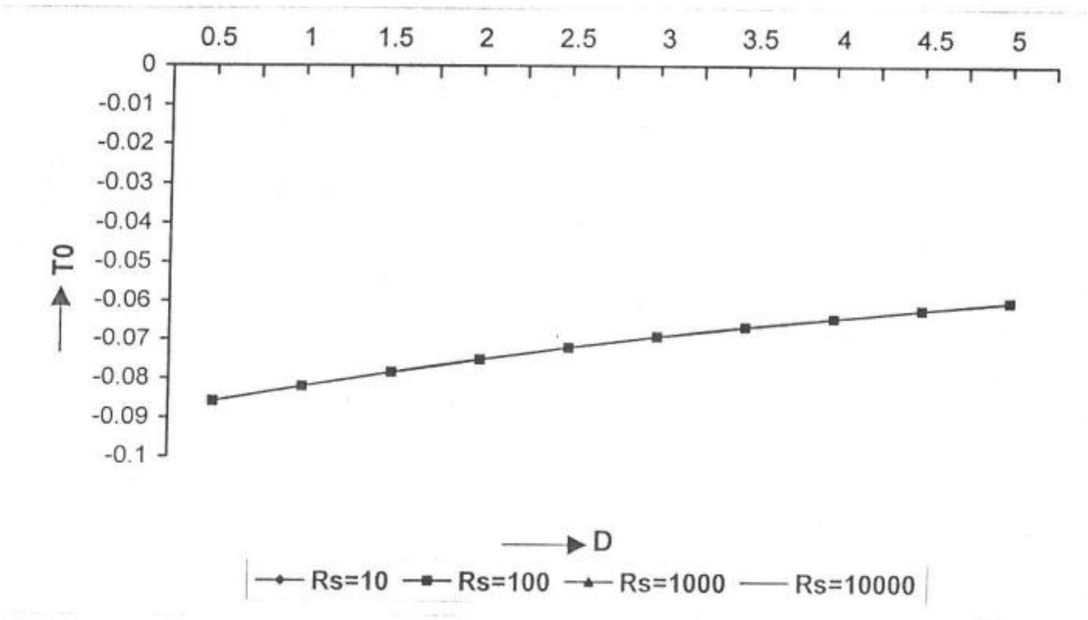

Figure (18): T_0 v/s D for $R_2 = 0 \,\&\, 10$, $\sigma = 10$, $\tau = 0.707107$ and $(RT_1, RT_2) = (1, 3)$

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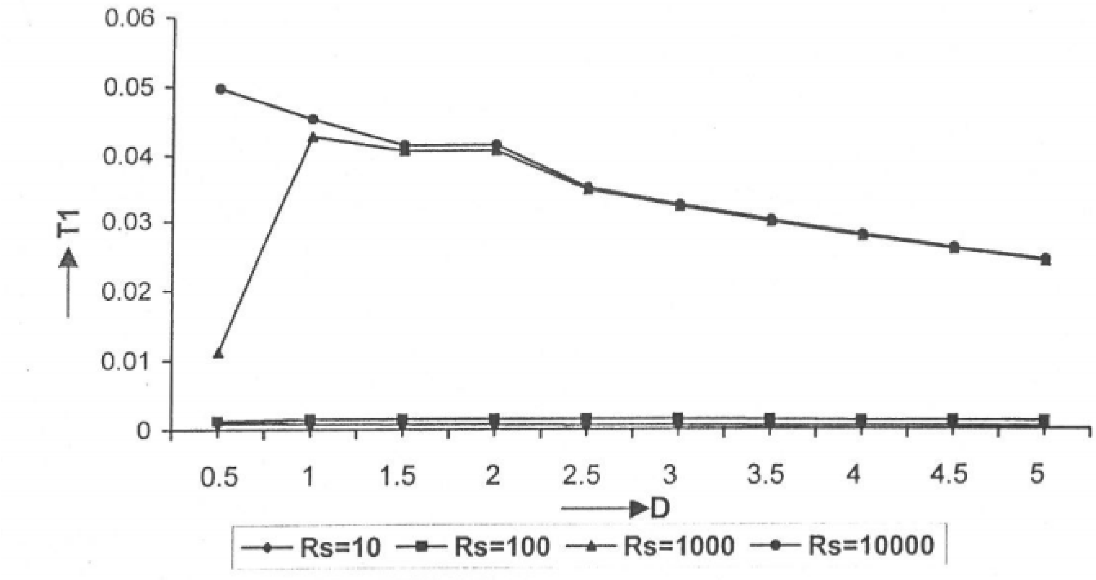


Figure(19): T_1 v/s D for $R_2 = -10$, $\sigma = 10$, $\tau = 0.707107$ and $(RT_1, RT_2) = (1, 3)$

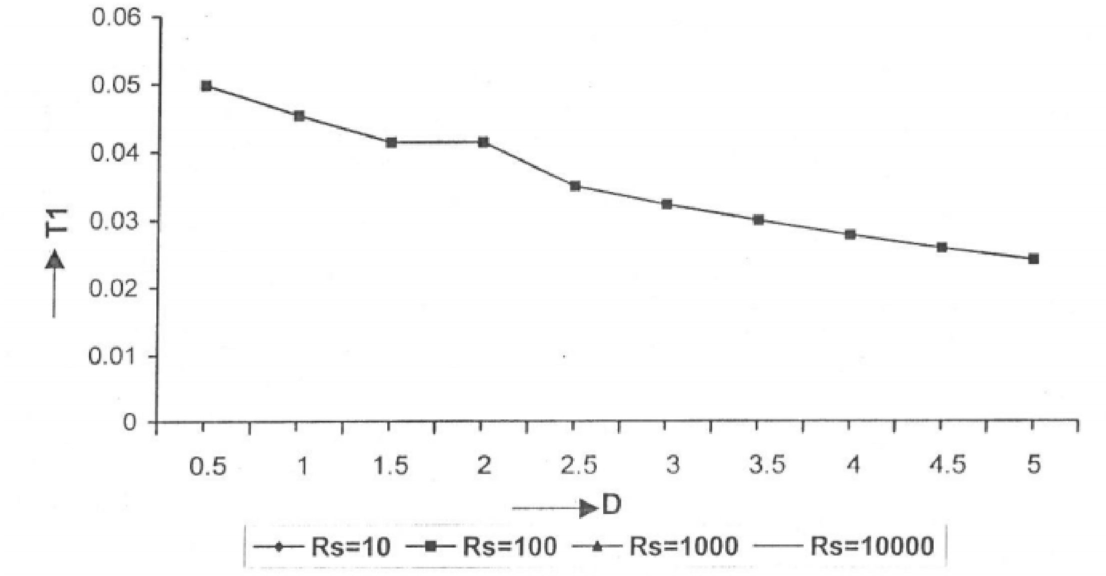
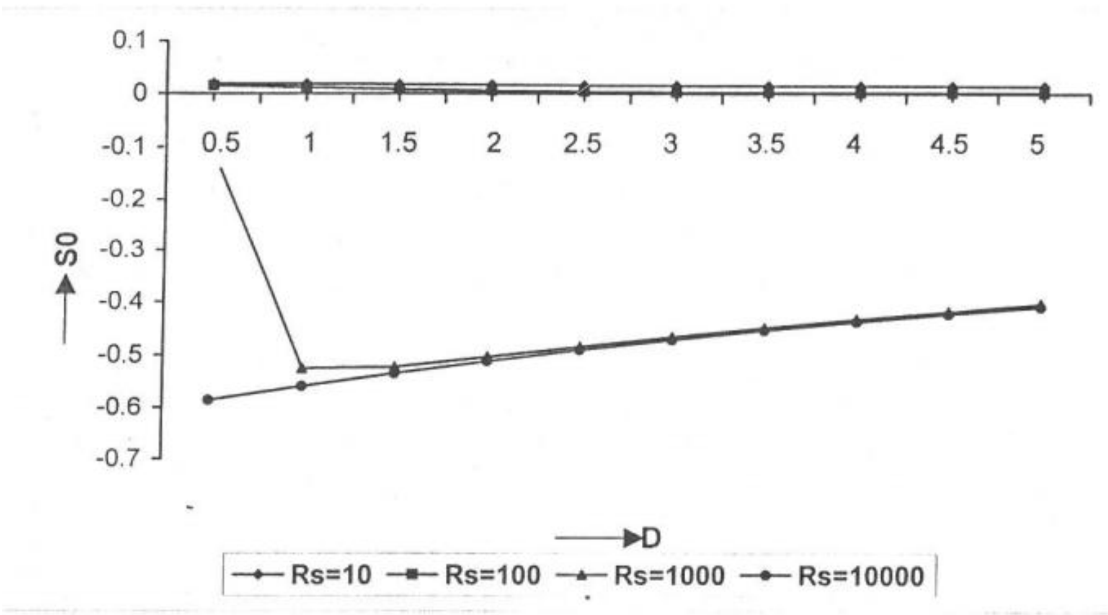


Figure (20): T_1 v/s D for $R_2 = 0$ & 10, $\sigma = 10$, $\tau = 0.707107$ and $(RT_1, RT_2) = (1, 3)$



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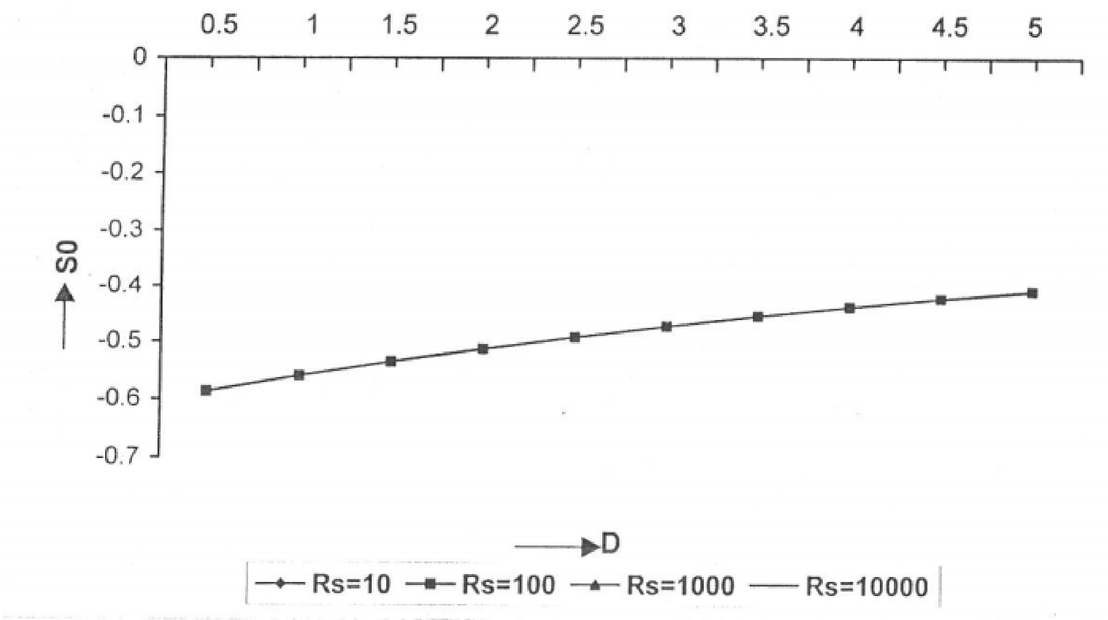

Figure (22): S_0 v/s D for $R_2 = 0$ & 10, $\sigma = 10$, $\tau = 0.707107$ and $(RT_1, RT_2) = (1, 3)$

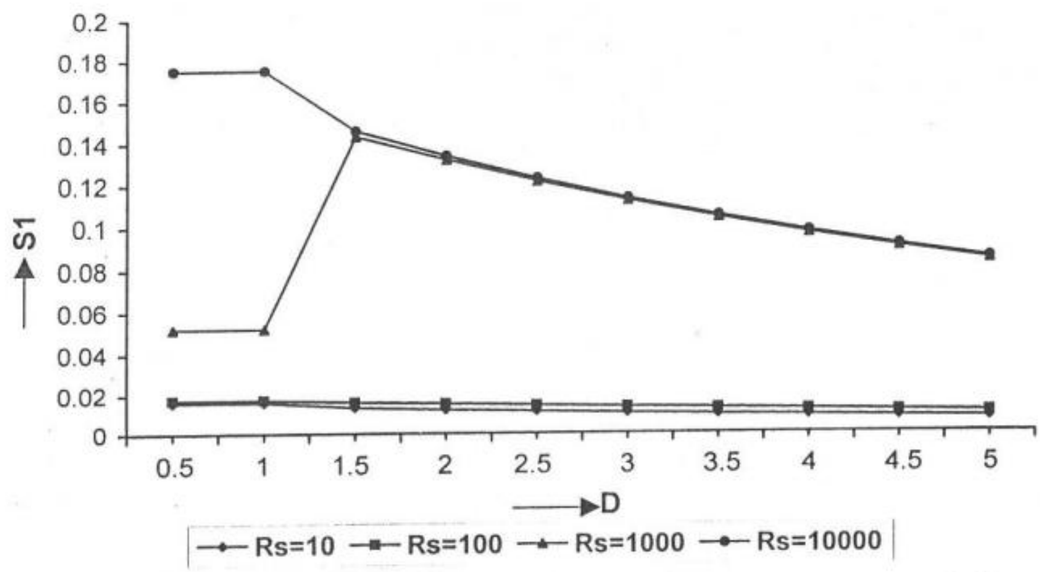

Figure (23): S_1 v/s D for $R_2 = -10$, $\sigma = 10$, $\tau = 0.707107$ and $(RT_1, RT_2) = (1, 3)$

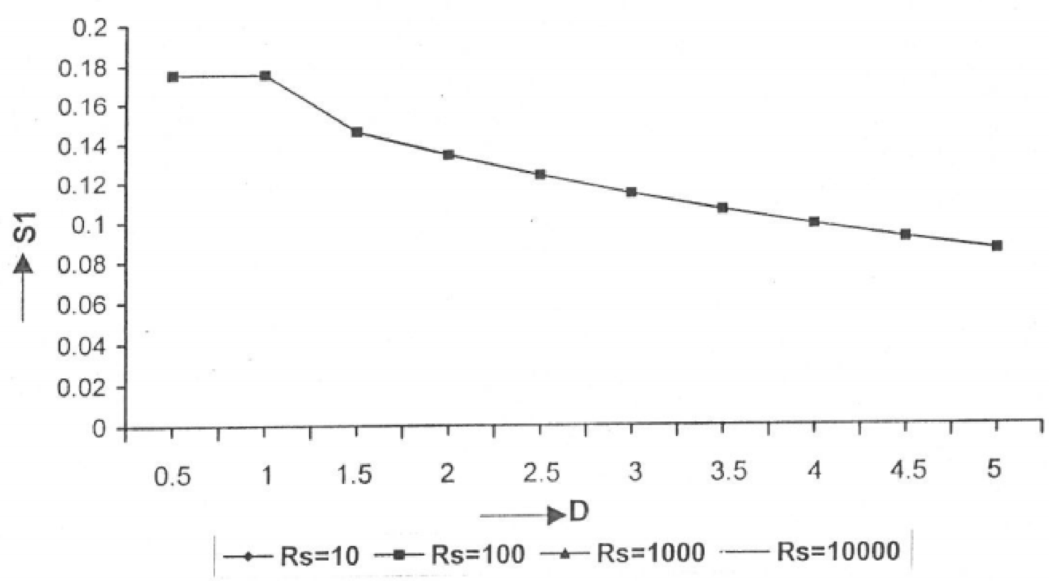

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Figure (24): S_1 v/s D for $R_2 = 0\ \&10$, $\sigma = 10$, $\tau = 0.707107$ and $(RT_1, RT_2) = (1, 3)$

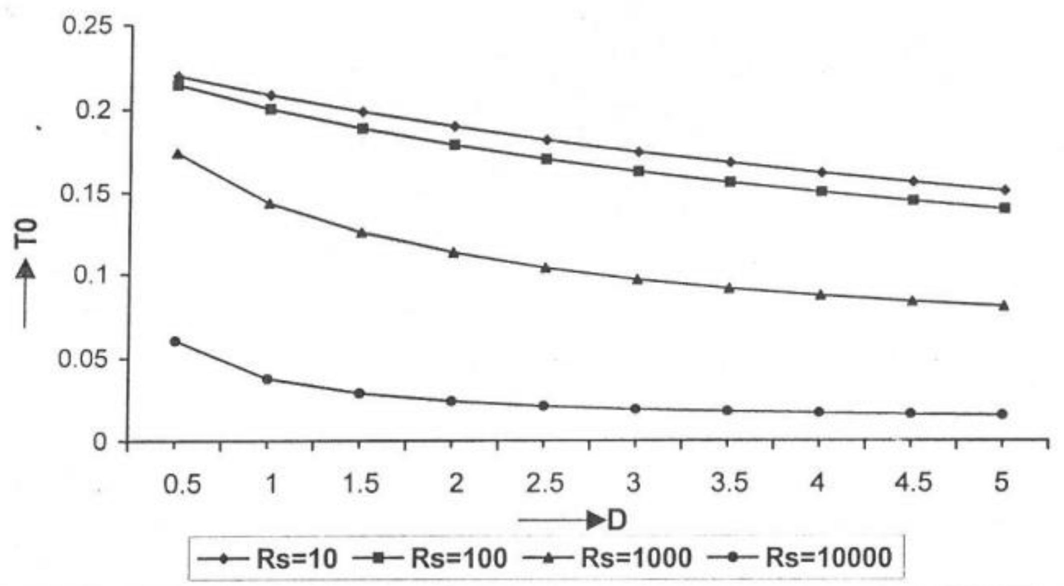
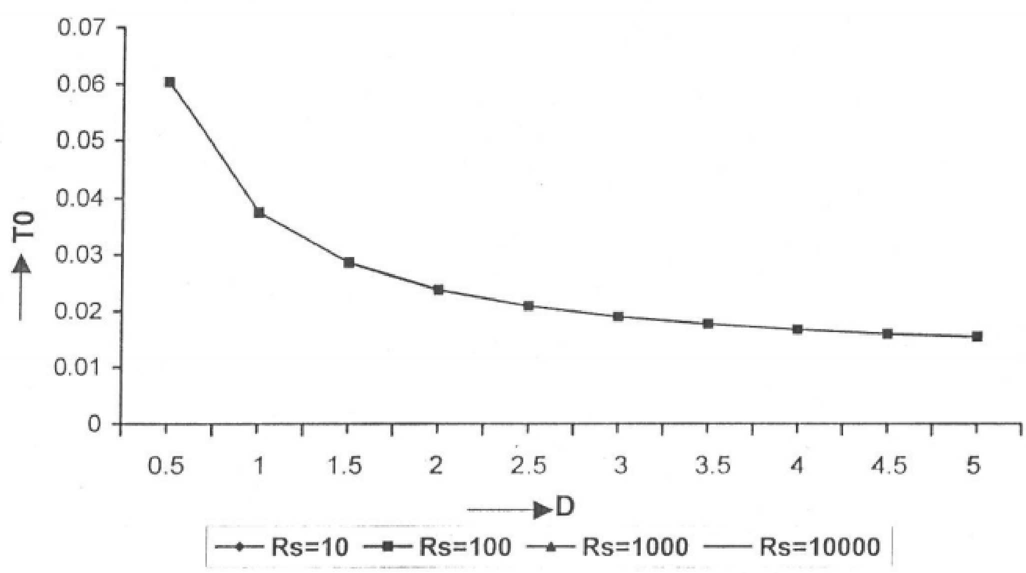


Figure (25): T_0 v/s D for $R_2 = -10$, $\sigma = 10$, $\tau = 0.707107$ and $(RT_1, RT_2) = (3, 1)$



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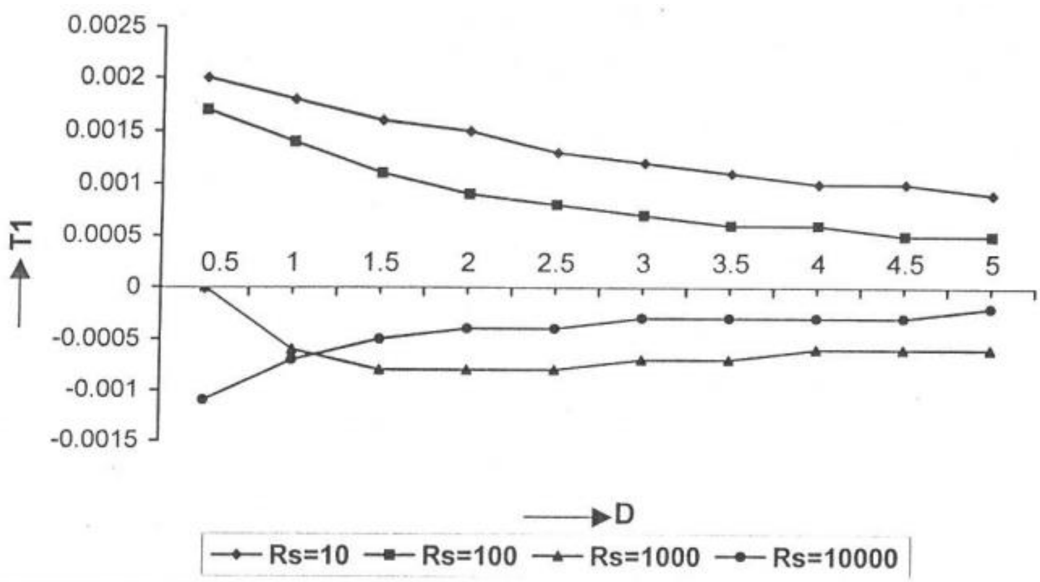

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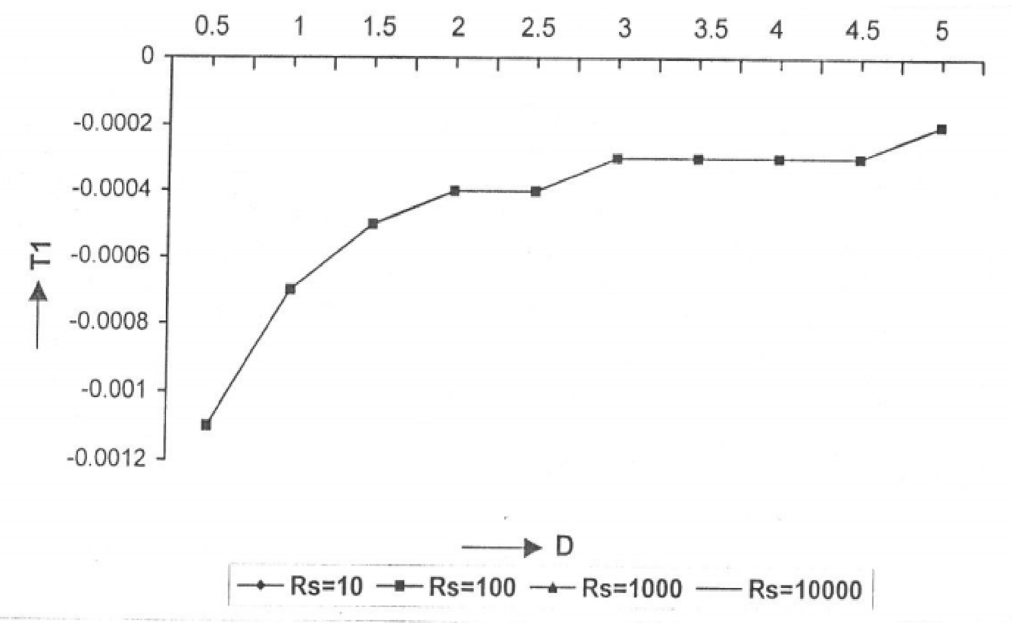

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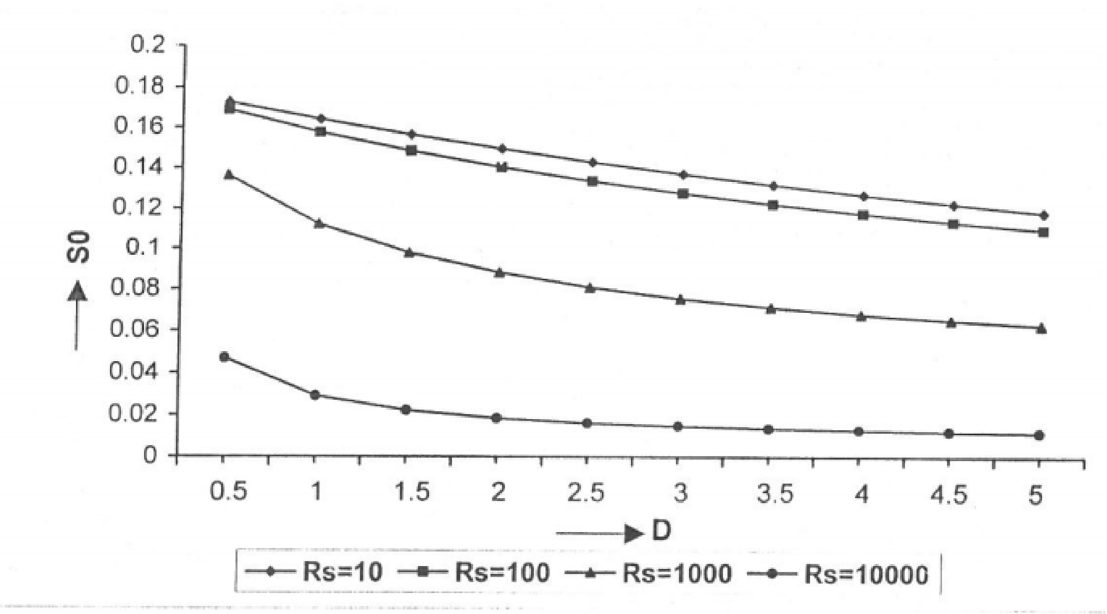

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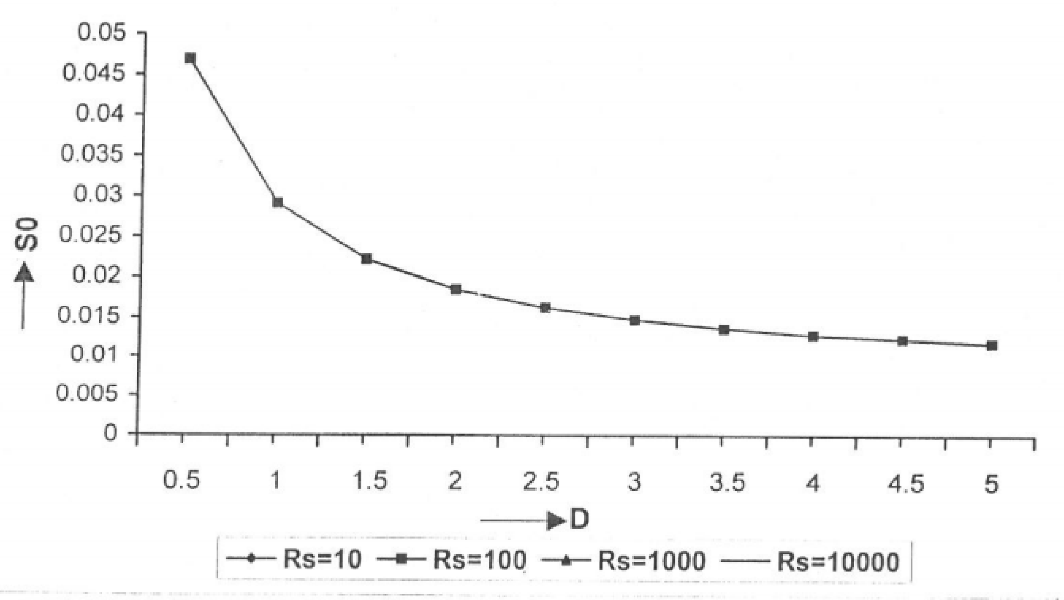

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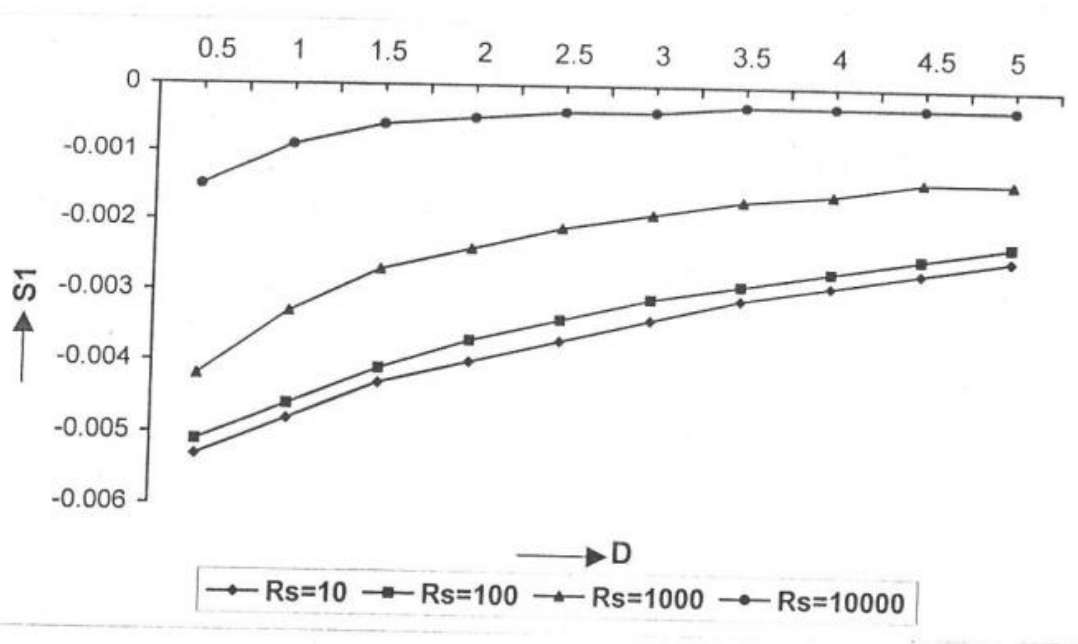

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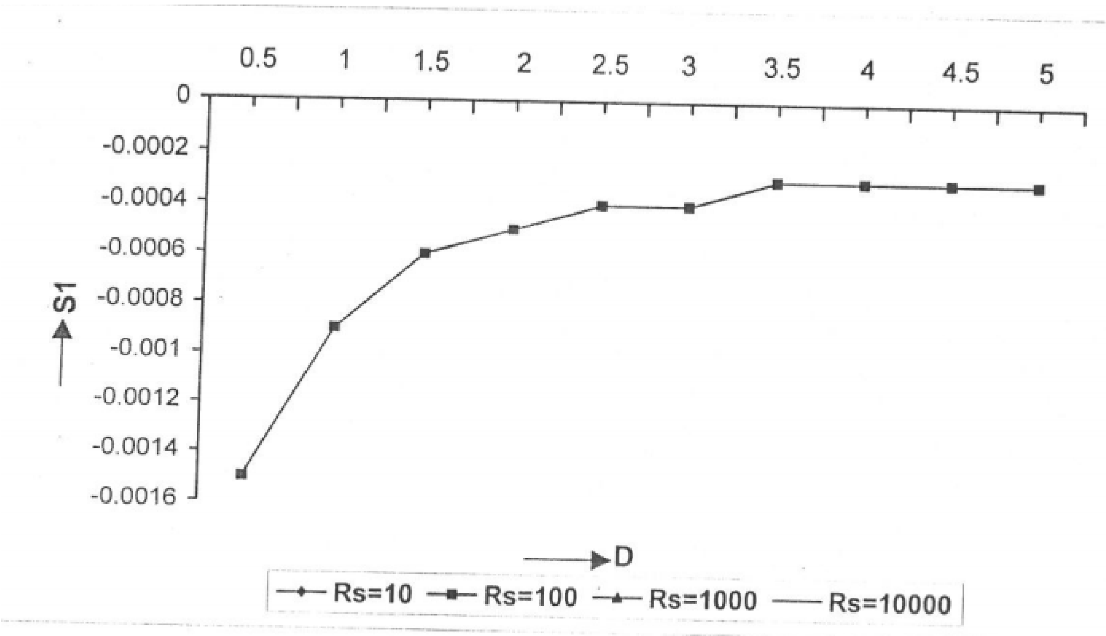

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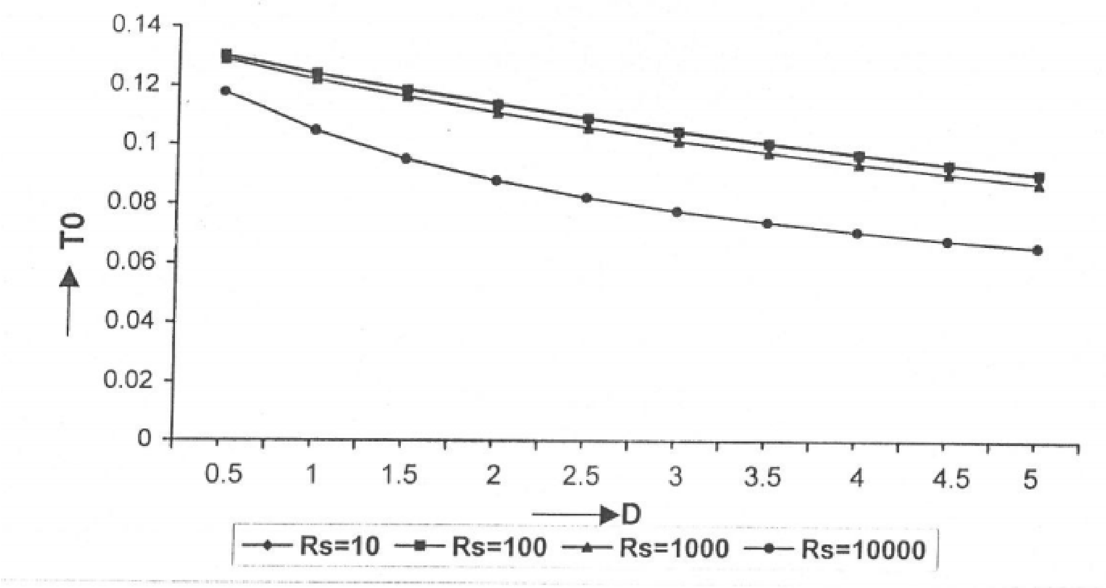

Figure (33): T_0 v/s D for $R_2 = -10$, $\sigma = 10$, $\tau = 0.707107$ and $(RT_1, RT_2) = (4, 4)$

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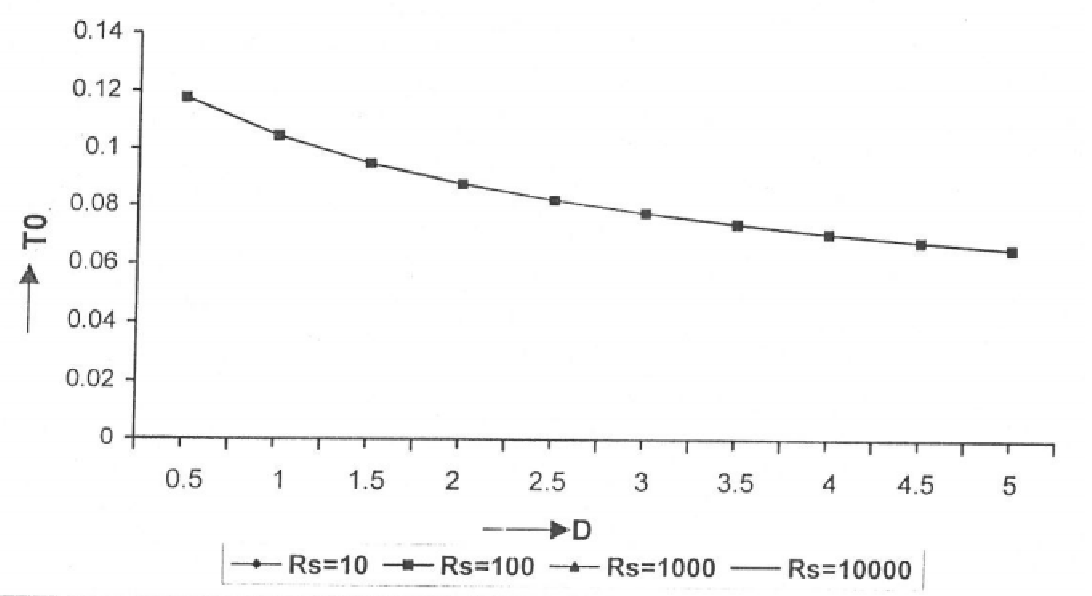

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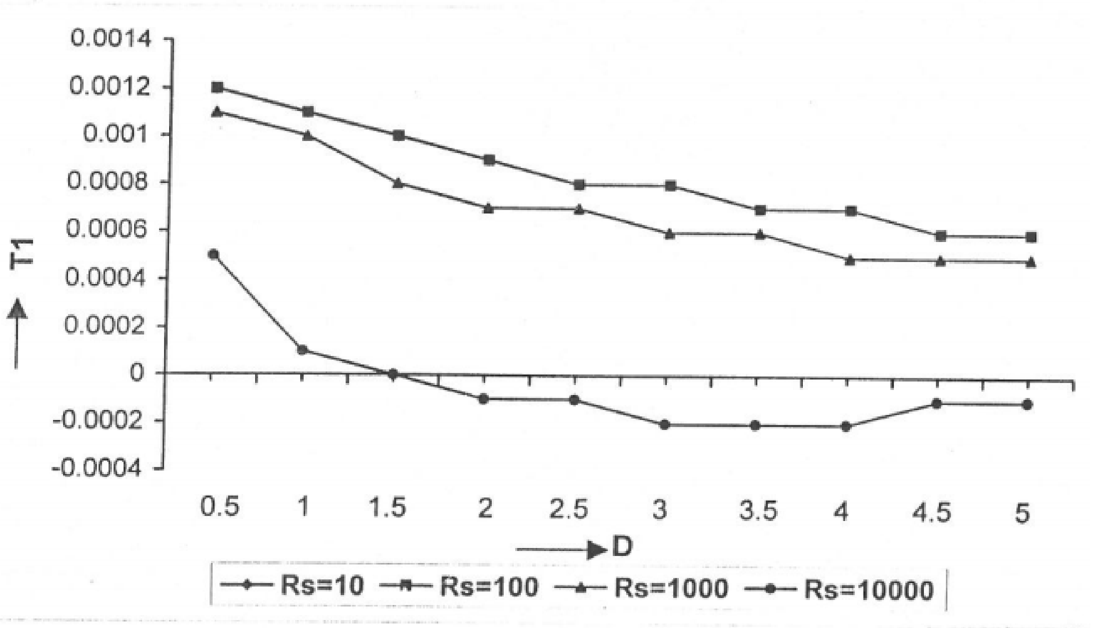

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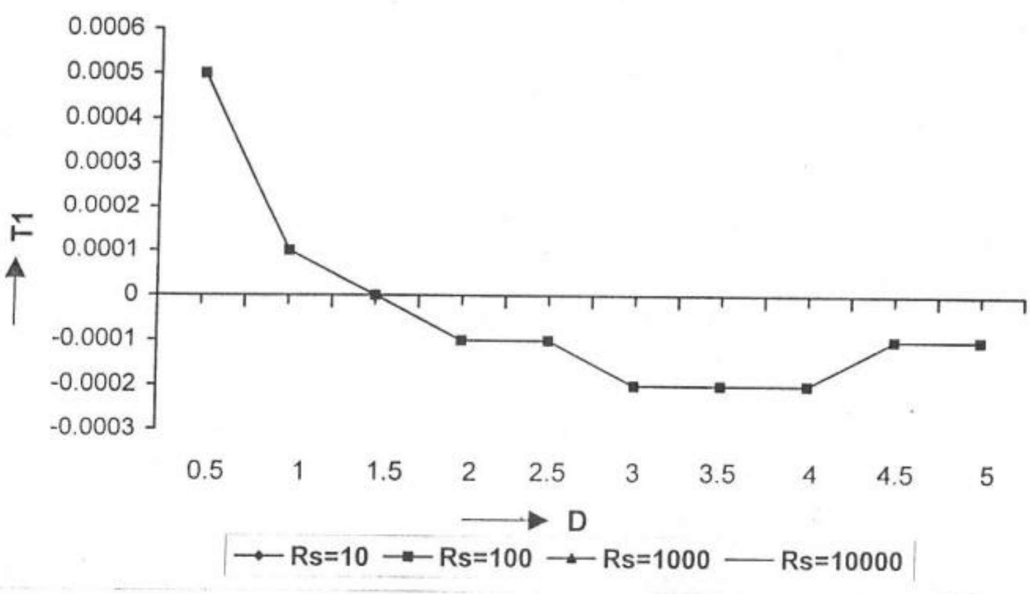

Figure (36): T_1 v/s D for $R_2 = 0$ & 10, $\sigma = 10$, $\tau = 0.707107$ and $(RT_1, RT_2) = (4, 4)$