A study on Enumerative, Coding and lexicodes in Grassmannian

Ashwani Kumar, Research Schlor, Department Mathematics, CMJ University, Meghalya

ABSTRACT

Codes in the Grassmannian space have found recently application in network coding Representation of dimensional subspaces of F_q^n has generally an essential role in solving coding problem in the Grassmannian and the particular in encoding subspaces of the Grassmannian are presented.

1. Introduction

In this paper we consider enumerative coding and lexicodes in the Grassmannian. Two different lexicographic orders for the Grassmannian induced by different representations of k-dimensional subspaces of F_q^n are given in the section. One enumerative coding method is based on a Ferrers diagram representation and on an order for Gq (n,k) based on representation. The complexity of this enumerative coding is o $(K\frac{5}{2}$ (n-k)$\frac{5}{2})$ digit operations. Constant dimension lexicodes are considered in section. A computer search for large constant dimension codes is usually inefficient since the search space domain is extremely large.

2. Lexicographic orders for the Grassmannian:

Inthis section we present two different lexicographic orders for the Grassmannian. First is based on the extended representation of a subspace in Gq (n, k) and the second one is based on the ferrers tableaus form representation of a subspaces in Gq (n, k). We will see in the sequel that the first order will result in more efficient enumerative coding in Gq (n, k), while the second order will lead to large constant dimension lexicodes.

2.1.1 Order for Gq (n, k) based on extended representation

Let X and Y Є Gq (n, k) be two k-dimensional subspaces and EXT(X) and EXT(Y) be the extended representations of X and Y.

Let I be the least index such that EXT(X) and EXT(Y) have different columns. We say that X<Y if

$$\{\genfrac{}{}{0pt}{}{v(x)_i}{x_i}\} < \{\genfrac{}{}{0pt}{}{v(y)_i}{y_i}\}$$

Example for X, Y, Z Є G_2 (6, 3) whose extended representation are given by

$$\text{EXT(X)} = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$

$$\text{EXT (Y)} = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

And

$$\text{EXT (Z)} = \begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

We have $y<x,z$

2.1.2 Order for Gq (n, k) Based on ferrers Tableaus form

Let F_x be a ferrers diagram of a subspace X Є Gq (n, k), F_x can be embedded in a k (n-k) box. We represent F_x by an integer vector of length n-k, $(F_{n-k},\ldots\ldots F_2, F_1)$, where F_i is equal to the number of dats in the i^{th} column of $F_{x,}$ $|<i\leq n-k$, where we number the column from right to left for two ferrers diagram F and $\tilde{F}$, we say that $F<\tilde{F}$ if one of the following two condition holds.

- $|F|>|\tilde{F}|$
- $|F|=|\tilde{F}|$, and $F_i>\tilde{F}_{i,}$ For the least index I where the two diagrams F and $\tilde{F}$ have a different numbers of dates.

Example If three ferrers diagram are given by

$$F_1 = \begin{matrix} \bullet & \bullet & \bullet \\ & \bullet & \bullet \\ & \bullet & \bullet \end{matrix}\ , \quad F_2 = \begin{matrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ & & \bullet \end{matrix}\ , \quad F_3 = \begin{matrix} \bullet & \bullet & \bullet \\ & \bullet & \bullet \\ & & \bullet \end{matrix}$$

Then $F_1<F_2<F_3$

Now we define the following order of subspaces in the Grassmannian based on the ferrers tableaux form representations. We say that X<Y if one of the following two condition holds.

- $F_x<F_{y,}$
- $F_x=F_y$ and $(x_1,x_2,\ldots\ldots\ldots x|\ F_X|)<(y_1,y_2,\ldots\ldots y|\ F_y|)$

Example: Let X, Y, Z, W Є G_2 (6, 3) be given by

$$F(x) = \begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ & & 1 \end{matrix}\ , \quad F(y) = \begin{matrix} 1 & 0 & 1 \\ & 0 & 0 \\ & 1 & 1 \end{matrix}, \quad F(z) = \begin{matrix} 1 & 1 & 1 \\ & 1 & 1 \\ & & 0 \end{matrix}, \quad F(w) = \begin{matrix} 1 & 1 & 1 \\ & 1 & 1 \\ & & 1 \end{matrix}$$

By the defination, we have that Fy< Fx< Fz= Fw

2.1.3 Enumerative coding for Grassmannian:

In this section we consider the problem for encoding/ decoding of subspaces in the Grassmannian in an efficient way. By encoding we mean a transformation of an information word into a k-dimensional subspace. To solve this coding problem, we will use the general enumerative coding method which was presented by cover.

Let $\{0, 1\}^n$. Denote by n_s ($x_1, x_2,\ldots\ldots, x_k$), where x_1 is the most significant bit.

3. Conclusion

The above result is obtain by a study on enumerative coding and lexicodes.

4. References

I. A. J. Van Zantan ,"Lexicographic Order and linearity" designs, codes and cryptography, vol.10,85-97,1997.

II. S. T. Xia and F.W. Fu, "Johnson type bands on constant dimension codes", designs, Codes,Crypto., Vol 50 no. 2, PP.163-172 February 2009.

III. S. Topalova and S. Zhelezova, "2-Spreads and transtive and orthogonal 2 – parallelisms of PG (5, 2)", Graphs and Comb. Vol.26, pp. 727-735, 2010.

IV. R. P. Stanely, Enumerative Combinatorics, Monterey,CA: Wadsworth, 1986, Vol.1,

V. N. Silberstein and T. Etzion, "Enumerative Coding for Grassmannian Space", IEEE Trans, Inform. Theory. Vol. 57, no. 1 Jan. 2011.

VI. T.Richardson,"Error Floors of LDPC Codes", Proc. Of the I[st]Annual Allerton conf. Commun. Monticello, IL, PP. 1426-1435, October 2003.

VII. F.J. Mac. Williams and N. J. A. Sloane, The theory of error-correcting codes, North-Holland, 1978.

VIII. R. W. Young and N. Cai." Network error Correction, Part I! Basic Concepts and upper bounds ", Commun. Information, syst. Vol. 6,PP- 19-36, May 2006.