

A study on Enumerative, Coding and lexicodes in Grassmannian

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ABSTRACT

Codes in the Grassmannian space have found recently application in network coding Representation of dimensional subspaces of F_q^n has generally an essential role in solving coding problem in the Grassmannian and the particular in encoding subspaces of the Grassmannian are presented.

1. Introduction

In this paper we consider enumerative coding and lexicodes in the Grassmannian. Two different lexicographic orders for the Grassmannian induced by different representations of k -dimensional subspaces of F_q^n are given in the section. One enumerative coding method is based on a Ferrers diagram representation and on an order for $G_q(n, k)$ based on representation. The complexity of this enumerative coding is $O(K \frac{5}{2} (n-k) \frac{5}{2})$ digit operations. Constant dimension lexicodes are considered in section. A computer search for large constant dimension codes is usually inefficient since the search space domain is extremely large.

2. Lexicographic orders for the Grassmannian:

In this section we present two different lexicographic orders for the Grassmannian. First is based on the extended representation of a subspace in $G_q(n, k)$ and the second one is based on the ferrers tableaux form representation of a subspaces in $G_q(n, k)$. We will see in the sequel that the first order will result in more efficient enumerative coding in $G_q(n, k)$, while the second order will lead to large constant dimension lexicodes.

2.1.1 Order for $G_q(n, k)$ based on extended representation

Let X and $Y \in G_q(n, k)$ be two k -dimensional subspaces and $EXT(X)$ and $EXT(Y)$ be the extended representations of X and Y .

Let I be the least index such that $EXT(X)$ and $EXT(Y)$ have different columns. We say that $X < Y$ if

$$\left\{ \begin{matrix} v(x)_i \\ x_i \end{matrix} \right\} < \left\{ \begin{matrix} v(y)_i \\ y_i \end{matrix} \right\}$$

Example for $X, Y, Z \in G_2(6, 3)$ whose extended representation are given by

$$EXT(X) = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$

$$\text{EXT (Y)} = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

And

$$\text{EXT (Z)} = \begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

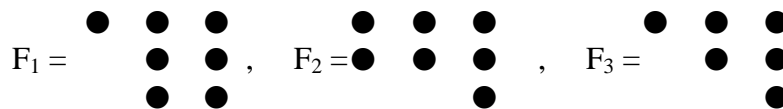
We have $y < x, z$

2.1.2 Order for $G_q(n, k)$ Based on ferrers Tableaus form

Let F_x be a ferrers diagram of a subspace $X \in G_q(n, k)$, F_x can be embedded in a k $(n-k)$ box. We represent F_x by an integer vector of length $n-k$, $(F_{n-k}, \dots, F_2, F_1)$, where F_i is equal to the number of dots in the i^{th} column of F_x , $|i \leq n-k$, where we number the column from right to left for two ferrers diagram F and \tilde{F} , we say that $F < \tilde{F}$ if one of the following two condition holds.

- $|F| > |\tilde{F}|$
- $|F| = |\tilde{F}|$, and $F_i > \tilde{F}_i$, For the least index i where the two diagrams F and \tilde{F} have a different numbers of dots.

Example If three ferrers diagram are given by



Then $F_1 < F_2 < F_3$

Now we define the following order of subspaces in the Grassmannian based on the ferrers tableaux form representations. We say that $X < Y$ if one of the following two condition holds.

- $F_x < F_y$,
- $F_x = F_y$ and $(x_1, x_2, \dots, x_i | F_x|) < (y_1, y_2, \dots, y_i | F_y|)$

Example: Let $X, Y, Z, W \in G_2(6, 3)$ be given by

$$F(x) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ & & 1 \end{pmatrix}, \quad F(y) = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ & & 1 & 1 \end{pmatrix}, \quad F(z) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ & & 0 \end{pmatrix}, \quad F(w) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ & & 1 \end{pmatrix}$$

By the definition, we have that $Fy < Fx < Fz = Fw$

2.1.3 Enumerative coding for Grassmannian:

In this section we consider the problem for encoding/ decoding of subspaces in the Grassmannian in an efficient way. By encoding we mean a transformation of an information word into a k -dimensional subspace. To solve this coding problem, we will use the general enumerative coding method which was presented by Cover.

Let $\{0, 1\}^n$. Denote by $n_s(x_1, x_2, \dots, x_k)$, where x_1 is the most significant bit.

3. Conclusion

The above result is obtained by a study on enumerative coding and lexicons.

4. References

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