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FUZZY NUMERICAL DIFFERENTIATION AND INTEGRATION

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ABSTRACT

The idea of a fuzzy set was initially presented to the general public by Lotfi A. Zadeh, who was the pioneer in carrying out this endeavor. During the year 1965, this particular event took occurred. In the realms of business, nature, and humanity, it is feasible to depict uncertain systems by employing complicated mathematical methodologies such as fuzzy sets and fuzzy logic. In order to fully comprehend the fact that this membership grade can take on values that fall anywhere between 0 and 1, it is essential to be aware of the fact that this is the case. The assignment of a value of 0 to u(x) indicates that the person is not a member, but the assignment of a value that falls within the range of 0 to 1 indicates that the individual is a partial member. Furthermore, the fact that the variable u(x) has been assigned the value 1 demonstrates that the individual is a complete member of the group. In contrast to ordinary differential equations (ODE), delay differential equations (DDE) entail the development of the state variable based on its previous values. In order to complete the DDE problem, it is necessary to have information not only of the current state but also of the state at a certain point in time in the past. In the fields of science and engineering, DDE have a wide range of applications.

Keywords: *fuzzy, assignment, humanity, methodologies.*

INTRODUCTION

The idea of a fuzzy set was initially presented to the general public by Lotfi A. Zadeh, who was the pioneer in carrying out this endeavour. During the year 1965, this particular event took occurred. In the realms of business, nature, and humanity, it is feasible to depict uncertain systems by employing complicated mathematical methodologies such as fuzzy sets and fuzzy logic. The modelling of the relationships that exist between these systems is one way to accomplish this goal. The capacity to cope with imprecise concepts is of the highest significance as a result of the creation of new areas such as robotics, artificial intelligence, language theory, and general system theory. This is because these new fields are a consequence of the extension of existing fields.

When considering fuzzy sets, it is vital to take into consideration a base set X that is not empty and that has components that are of interest. This is because fuzzy sets are a type of set that is not empty. Each and every element x that is a component of X is deserving of a membership grade denoted by the notation u(x). In order to fully comprehend the fact that this membership grade can take on values that fall anywhere between 0 and 1, it is essential to be aware of the fact that this is the case. The assignment of a value of 0 to u(x) indicates that the person is not a member, but the assignment of a value that falls within the range of 0 to 1 indicates that the individual is a partial member. Furthermore, the fact that the variable u(x) has been assigned the value 1 demonstrates that the individual is a complete member of the group. We consider a fuzzy subset of X to be any nonempty subset $\{(x, u(x)): x \in X\}$ of $X \times [0, 1]$ that contains elements that are not empty for the purpose of this discussion. This particular subset is valid for particular functions, such as the function $u: X \to [0, 1]$. According to Zadeh, who offered an explanation of the concept, the following is stated: This was something that was brought to my attention while I was considering a fuzzy subset of X. I am grateful for the information. There is a common practice of using the fuzzy set-in place of the function u itself. This kind of behaviour is rather typical. It is true that this is the case in a variety of different circumstances. The following is an illustration of a fuzzy set that serves as an example of a fuzzy set that depicts the notion of being youthful:

$$u(x) = \begin{cases} 1 & \text{if} \quad x < 25, \\ \frac{40-x}{15} & \text{if} \quad 25 \leq x \leq 40, \\ 0 & \text{if} \quad 40 < x, \end{cases}$$

It is of the utmost importance that this matter be taken seriously. It is a well-known fact that there are so many various memberships grade functions that are feasible options. There are also a lot of different membership grade functions. In the event that an ordinary subset A of X is taken into consideration, there are two distinct membership alternatives that are available for selection. Full membership and non-membership are the two choices that are being considered in this aspect of the situation. It is feasible to discern between this particular group of items. An example of the source of the fuzzy set is the fuzzy set on X, which may be identified by its characteristic function $\mu A: X \to$. The set is established as [0, 1], which is established when the set is taken into consideration.

Electric Circuit

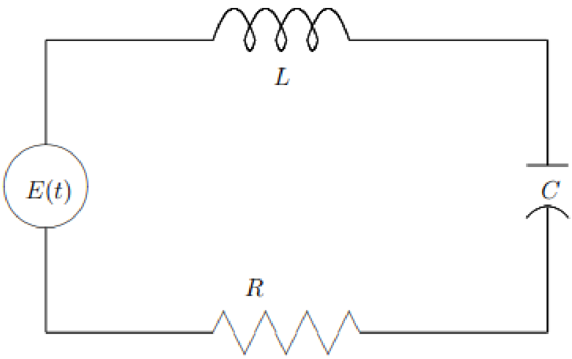

Figure 1 Electrical framework

In Figure 1.2, the circuit is shown with L equal to 1 h, R equal to 2 Ω, C equal to 0.25 f, and E(t) equal to 50 the bill. If Q(t) is the capacitor's charge at a time t larger than zero, then

$$Q''(t) + 2Q'(t) + 4Q(t) = 50\cos t, \tag{1.1}$$

While Q (0) is equal to γ0, Q′(0) is equal to γ1 respectively. It is the answer that stands out the most.

$$Q(t) = \gamma_0 \frac{2e^{-t}}{\sqrt{3}} \sin\left(\sqrt{3}t + \frac{\pi}{3}\right) + \gamma_1 \frac{e^{-t}}{\sqrt{3}} \sin\left(\sqrt{3}t\right) + \Psi(t), \tag{1.2}$$

Were

$$\Psi(t) = -\Delta_0 e^{-t}\cos(\sqrt{3}t) - \frac{\Delta_0 + \Delta_1}{\sqrt{3}} e^{-t} \sin(\sqrt{3}t) + G(t), \tag{1.3}$$

With

$$G(t) = \frac{100}{13}\sin t + \frac{150}{13}\cos t. \tag{1.4}$$

In order to ensure that γ01(α) equals 4+α, γ02(α) equals 6−α, γ11(α) equals α, and γ12(α) equals 2 − α, where 0 ≤ α < 1, let us assume that γ0 is equal to (4/5/6), and γ1 is equal to (0/1/2). The intervals Ik = [δk−1, δk], where k is a number between 1 and 3, are specified. The values of the δk are as follows: δ1 = (2 3) π/√ 3, δ2 = π/√ 3, δ3 = (5 3) π/√ 3, δ4 = 2 π/√ 3, δ5 = (8 3) π/√ 3, and so on. Let the fuzzy solution of Equation (1.1) be denoted by the representation Q(t) is equal to the product of q (t, α) and q (t, α). To determine the value of the variable q (t, α), the equation (1.2) is utilised in the following method.

OBJECTIVE OF THE STUDY

1. To study the fuzzy numerical differentiation and integration.

2. To examine the fuzzy Hermite interpolation with triangular fuzzy numbers.

RESEARCH METHODOLOGY

While generalized differentiability is being taken into consideration, the numerical solution of the FDE was carried out using the Euler approach. Using the help of generalized differentiability, the knowledge that there is a solution for FDDE, and the fact that this solution is the only one that is really accessible. By employing the Euler method and the Improved Euler application, the numerical solution of the FDDE is achieved in this chapter. This is accomplished by the usage of both systems. In accordance with the principles of generalized differentiability, this is effectively done.

Euler technique

Let the FDDE (1.24) be considered, let Δ = {t0, t1, . . . , tN = T} be given grid points and let hn+1 = tn+1 − tn, n = 0, 1, 2, . . . , N − 1 denote the corresponding step sizes at which the exact solutions [U1] α = [U1(t; α), U1(t; α)] and [U2] α = [U2(t; α), U2(t; α)] are approximated by some [u1] α = [u1(t; α), u1(t; α)] and [u2] α = [u2(t; α), u2(t; α)] respectively. Each of the four symbols that may be located at tn serves as a representation of the potential solutions that can be discovered there through the process of discovery.

Lemma 1: If W is a member of the set

C ([σ, b], En) and {Wn} N n=0, then the inequality is satisfied.

$$\|W_{n+1}\| \leq A\,\|W_n\|_{[\sigma,t_n]} + B, \quad 0 \leq n \leq N-1, \quad t \in (t_n, t_{n+1}],$$

For the purpose of this discussion, let us assume that both There are two positive constants, A and B.

$$\|W_n\| \leq A^n\,\|W_0\|_{[\sigma,t_n]} + B\frac{A^n - 1}{A - 1}, \quad 0 \leq n \leq N-1.$$

For every W and V that belong to the set C([σ, b], En), and if if {Wn} N n equals zero, then {Vn} N n equals zero, then the inequality is met.

$$\|W_{n+1}\| \leq \|W_n\|_{[\sigma,t_n]} + A\max\{\|W_n\|_{[\sigma,t_n]}, \|V_n\|_{[\sigma,t_n]}\} + B,$$
$$\|V_{n+1}\| \leq \|V_n\|_{[\sigma,t_n]} + A\max\{\|W_n\|_{[\sigma,t_n]}, \|V_n\|_{[\sigma,t_n]}\} + B,$$
$$0 \leq n \leq N-1, \quad t \in [t_n, t_{n+1}],$$

for a set of positive constants, A and B that are already known, and define

$$\eta_n = \|W_n\| + \|V_n\|, \quad 0 \leq n \leq N,$$

Then

$$\|\eta_n\| \leq \overline{A}^n \|\eta_0\|_{[\sigma,t_n]} + \overline{B}\frac{\overline{A}^n - 1}{\overline{A} - 1}, \quad 1 \leq n \leq N,$$

where A is equal to 1 plus 2A and B is equal to 2B.

When the starting condition u0 is considered, the expression of the solution may be found on the interval, which corresponds to the interval. Figure 2 displays the approximate solutions, while provides the numerical values. Both figures may be seen in the same document.

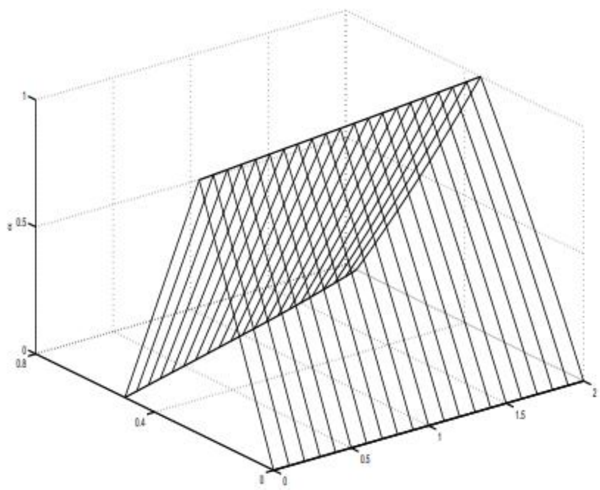

Figure 2. Approximation solutions for (1)-differentiability at t = 2 using the Euler technique

DATA ANALYSIS

Four-dimensional translation

Assuming that the values of $f(\tilde{x})$ among a collection of hazy spots $\tilde{x}_0, \tilde{x}_1, \dots \dots \dots . \tilde{x}_n,$ It is common practice to begin the process of generating the numerical differentiation techniques by first obtaining the using polynomial interpolation $P_n(\tilde{x})$ Next, differentiate this polynomial r times using the formula. $(\mathrm{n} \geq r)$ (to acquire) $P_n^{(r)}(\tilde{x}_k)$ The value of is approximately determined by this. $f^{(r)}(\tilde{x})$ inside the fuzzy points of the nodes $(\tilde{x}_k)$. While it is true that this is the case, $P_n(\tilde{x})$ and $f(\tilde{x})$ even if the values at the nodal fuzzy points are same, the derivatives may be quite different when those fuzzy points are severed. There is a possibility that the approximation will perform even worse as the order of derivative increases.

Assume that

$$
\begin{aligned}
&P_2^{(2)}(\tilde{x}) \\
&= \frac{1}{2h^2}\Big\{\Big[(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3) - \left(\tilde{x}_{11}^{(2)}, \tilde{x}_{12}^{(2)}, \tilde{x}_{13}^{(2)}\right)(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3) \\
&- (\tilde{x}_{21}^{(2)}, \tilde{x}_{22}^{(2)}, \tilde{x}_{23}^{(2)})\Big] \times f\left(\tilde{x}_{01}^{(2)}, \tilde{x}_{02}^{(2)}, \tilde{x}_{03}^{(2)}\right) \\
&- 2\Big[\Big[(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3) - \left(\tilde{x}_{01}^{(2)}, \tilde{x}_{02}^{(2)}, \tilde{x}_{03}^{(2)}\right)(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3) \\
&- (\tilde{x}_{21}^{(2)}, \tilde{x}_{22}^{(2)}, \tilde{x}_{23}^{(2)})\Big] \times f\left(\tilde{x}_{11}^{(2)}, \tilde{x}_{12}^{(2)}, \tilde{x}_{13}^{(2)}\right)\Big] \\
&+ \Big[\Big[(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3) - \left(\tilde{x}_{01}^{(2)}, \tilde{x}_{02}^{(2)}, \tilde{x}_{03}^{(2)}\right)(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3) \\
&- (\tilde{x}_{11}^{(2)}, \tilde{x}_{12}^{(2)}, \tilde{x}_{13}^{(2)})\Big] \times f\left(\tilde{x}_{21}^{(2)}, \tilde{x}_{22}^{(2)}, \tilde{x}_{23}^{(2)}\right)\Big]\Big\}
\end{aligned}
$$

$$
\begin{aligned}
P_2^{(2)}(\tilde{x}) = \frac{1}{2h^2}\Big\{\Big[&\left(\tilde{x}_1 - \tilde{x}_{13}^{(2)}\right)\left(\tilde{x}_2 - \tilde{x}_{12}^{(2)}\right)\left(\tilde{x}_3 - \tilde{x}_{11}^{(2)}\right)\Big(\tilde{x}_1 \\
&- \tilde{x}_{23}^{(2)})\left(\tilde{x}_2 - \tilde{x}_{22}^{(2)}\right)(\tilde{x}_3 - \tilde{x}_{21}^{(2)})\Big] f\left(\tilde{x}_{01}^{(2)}, \tilde{x}_{02}^{(2)}, \tilde{x}_{03}^{(2)}\right) \\
&- 2\Big[\left(\tilde{x}_1 - \tilde{x}_{03}^{(2)}\right)\left(\tilde{x}_2 - \tilde{x}_{02}^{(2)}\right)(\tilde{x}_3 \\
&- \tilde{x}_{01}^{(2)})\left(\tilde{x}_1 - \tilde{x}_{23}^{(2)}\right)\left(\tilde{x}_2 - \tilde{x}_{22}^{(2)}\right)(\tilde{x}_3 - \tilde{x}_{21}^{(2)})\Big] \\
&\times f\left(\tilde{x}_{11}^{(2)}, \tilde{x}_{12}^{(2)}, \tilde{x}_{13}^{(2)}\right) \\
&+ \Big[\left(\tilde{x}_1 - \tilde{x}_{03}^{(2)}\right)\left(\tilde{x}_2 - \tilde{x}_{02}^{(2)}\right)(\tilde{x}_3 \\
&- \tilde{x}_{01}^{(2)})\left(\tilde{x}_1 - \tilde{x}_{13}^{(2)}\right)\left(\tilde{x}_2 - \tilde{x}_{12}^{(2)}\right)(\tilde{x}_3 - \tilde{x}_{11}^{(2)})\Big] \\
&\times f\left(\tilde{x}_{21}^{(2)}, \tilde{x}_{22}^{(2)}, \tilde{x}_{23}^{(2)}\right)\Big\}
\end{aligned}
$$

$$
\begin{aligned}
P_2^{(2)}(\tilde{x}) = \frac{1}{2h^2}\Big\{\Big[&\left(\tilde{x}_1 - \tilde{x}_{13}^{(2)}\right)\left(\tilde{x}_1 - \tilde{x}_{23}^{(2)}\right) + \left(\tilde{x}_2 - \tilde{x}_{12}^{(2)}\right)\left(\tilde{x}_2 - \tilde{x}_{12}^{(2)}\right) \\
&+ \left(\tilde{x}_3 - \tilde{x}_{11}^{(2)}\right)\left(\tilde{x}_3 - \tilde{x}_{23}^{(2)}\right)\Big] f\left(\tilde{x}_{01}^{(2)}, \tilde{x}_{02}^{(2)}, \tilde{x}_{03}^{(2)}\right) \\
&- 2\Big[\left(\tilde{x}_1 - \tilde{x}_{03}^{(2)}\right)\left(\tilde{x}_1 - \tilde{x}_{23}^{(2)}\right) + \left(\tilde{x}_2 - \tilde{x}_{02}^{(2)}\right)\left(\tilde{x}_2 - \tilde{x}_{22}^{(2)}\right) \\
&+ (\tilde{x}_3 - \tilde{x}_{01}^{(2)})(\tilde{x}_3 - \tilde{x}_{21}^{(2)})\Big] \times f\left(\tilde{x}_{11}^{(2)}, \tilde{x}_{12}^{(2)}, \tilde{x}_{13}^{(2)}\right) \\
&+ \Big[\left(\tilde{x}_1 - \tilde{x}_{03}^{(2)}\right)\left(\tilde{x}_1 - \tilde{x}_{13}^{(2)}\right) + \left(\tilde{x}_2 - \tilde{x}_{02}^{(2)}\right)\left(\tilde{x}_2 - \tilde{x}_{12}^{(2)}\right) \\
&+ (\tilde{x}_3 - \tilde{x}_{01}^{(2)})(\tilde{x}_3 - \tilde{x}_{11}^{(2)})\Big] \times f\left(\tilde{x}_{21}^{(2)}, \tilde{x}_{22}^{(2)}, \tilde{x}_{23}^{(2)}\right)\Big\}
\end{aligned}
$$

$$
\begin{aligned}
P_2^{(2)}(\tilde{x}) = \frac{1}{2h^2}\Big\{\Big[&\tilde{x}_1^2 - \tilde{x}_1\tilde{x}_{23}^{(2)} - \tilde{x}_1\tilde{x}_{13}^{(2)} + \tilde{x}_{13}^{(2)}\tilde{x}_{23}^{(2)} + \tilde{x}_2^2 - \tilde{x}_2\tilde{x}_{22}^{(2)} \\
&- \tilde{x}_2\tilde{x}_{12}^{(2)} + \tilde{x}_{12}^{(2)}\tilde{x}_{22}^{(2)} + \tilde{x}_3^2 - \tilde{x}_3\tilde{x}_{23}^{(2)} - \tilde{x}_3\tilde{x}_{13}^{(2)} + \tilde{x}_{11}^{(2)}\tilde{x}_{23}^{(2)}\Big] \\
&\times f\left(\tilde{x}_{01}^{(2)}, \tilde{x}_{02}^{(2)}, \tilde{x}_{03}^{(2)}\right) \\
&- 2\Big[\tilde{x}_1^2 - \tilde{x}_1\tilde{x}_{23}^{(2)} - \tilde{x}_1\tilde{x}_{03}^{(2)} + \tilde{x}_{23}^{(2)}\tilde{x}_{13}^{(2)} + \tilde{x}_2^2 - \tilde{x}_2\tilde{x}_{22}^{(2)} \\
&- \tilde{x}_2\tilde{x}_{02}^{(2)} + \tilde{x}_{02}^{(2)}\tilde{x}_{22}^{(2)} + \tilde{x}_3^2 - \tilde{x}_3\tilde{x}_{21}^{(2)} - \tilde{x}_3\tilde{x}_{01}^{(2)} + \tilde{x}_{01}^{(2)}\tilde{x}_{21}^{(2)}\Big] \\
&\times f\left(\tilde{x}_{11}^{(2)}, \tilde{x}_{12}^{(2)}, \tilde{x}_{13}^{(2)}\right) \\
&+ \Big[\tilde{x}_1^2 - \tilde{x}_1\tilde{x}_{03}^{(2)} - \tilde{x}_1\tilde{x}_{13}^{(2)} + \tilde{x}_{03}^{(2)}\tilde{x}_{13}^{(2)} + \tilde{x}_2^2 - \tilde{x}_2\tilde{x}_{12}^{(2)} \\
&- \tilde{x}_2\tilde{x}_{02}^{(2)} + \tilde{x}_{12}^{(2)}\tilde{x}_{02}^{(2)} + \tilde{x}_3^2 - \tilde{x}_3\tilde{x}_{11}^{(2)} - \tilde{x}_3\tilde{x}_{01}^{(2)} + \tilde{x}_{01}^{(2)}\tilde{x}_{11}^{(2)}\Big] \\
&\times f\left(\tilde{x}_{21}^{(2)}, \tilde{x}_{22}^{(2)}, \tilde{x}_{23}^{(2)}\right)\Big\}
\end{aligned}
$$

Combining polynomials in integration

Let us $x_0, x_1, \ldots\ldots\ldots\ldots x_n$ be n+1 There are separate nodes.

Given that $y_0 = f(x_0), y_1 = f(x_1), \ldots\ldots\ldots, y_n = f(x_n)$

To locate a polynomial is the challenge at hand. $P_n(x) \in P_n$, is referred to as an interpolating polynomial, including the fact that $P_n(x_i) = y_i$ (i= 0,1,2………n)

It is theorem 4.1.3.

Let $x_0, x_1, \ldots\ldots\ldots . x_n$ be $n + 1$ distinct nodes and let $\tilde{y}_i = (y_i{}^l, y_i{}^c, y_i{}^r)$ $i = 0,1,2 \ldots\ldots n$ be $n + 1$triangular fuzzy numbers. There exists at least one fuzzy polynomial such that $\tilde{P}_2(x_i) = \tilde{y}_i$ for $i = 0,1,2 \ldots\ldots n$

The evidence

In order to demonstrate the presence of anything, let us use a constructive method that offers an expression for

$\tilde{P}_2(x)$. For arbitrary y_i, $i = 0,1,2 \ldots\ldots . . n$

If we assume that $P_2(x, y_0, y_1, \dots \dots \dots . . y_n)$ is a polynomial that may be interpolated in it

$$P_2(x_i, y_0, y_1, \dots \dots \dots . . y_n) = y_{i,} \quad (i = 0,1,2 \dots \dots \dots n)$$

such that

Define the term

$$\tilde{P}_2(x) = \left(P_2{}^{(1)}(x), P_2{}^{(2)}(x), P_2{}^{(3)}(x)\right)$$

In what

$$P_2{}^{(1)}(x) = \inf_{\substack{\forall y_i \in \tilde{y}_i \\ i = 0,1,\dots \dots . n}} P_n(x, y_0, y_1, \dots \dots \dots . y_n)$$

$$P_2{}^{(3)}(x) = \sup_{\substack{\forall y_i \in \tilde{y}_i \\ i = 0,1,\dots \dots . n}} P_n(x, y_0, y_1, \dots \dots \dots . y_n)$$

$$P_2^{(2)}(x) = f'(\tilde{x}) = \frac{\partial P_2^{(2)}(\tilde{x})}{\partial(\tilde{x}_2)}$$

$$= \frac{1}{2h^2}\Big[(2\tilde{x}_2 - \tilde{x}_{22} - \tilde{x}_{12}) \times f\left(\tilde{x}_{01}^{(2)}, \tilde{x}_{02}^{(2)}, \tilde{x}_{03}^{(2)}\right) -$$

$$2(2\tilde{x}_2 - \tilde{x}_{22} - \tilde{x}_{02}) \times f\left(\tilde{x}_{11}^{(2)}, \tilde{x}_{12}^{(2)}, \tilde{x}_{13}^{(2)}\right) +$$

$$(2\tilde{x}_2 - \tilde{x}_{12} - \tilde{x}_{02}) \times f\left(\tilde{x}_{21}^{(2)}, \tilde{x}_{22}^{(2)}, \tilde{x}_{23}^{(2)}\right)\Big]$$

As a result,

$$\tilde{P}_2(x_i) = \left(P_2{}^{(1)}(x), P_2{}^{(2)}(x), P_2{}^{(3)}(x)\right)$$

$$= (y_i^1, y_i^2, y_i^3)$$

$$= \tilde{y}_i$$

Consequently, the existence of the interpolating polynomial of the triangular fuzzy number may be acknowledged.

Example

Given the values of Б that are listed below $`f(x) = In\,x,$ a rough estimate of the worth of Б $f'(2.0)$ By employing the Quadratic interpolation and the ς$f'(2.0)$ acquired in the following manner

i	x_i	f_i
0	(1.8,2.0,2.2)	(0.58778,0.69314,0.78845)
1	(2.0,2.2,2.4)	(0.69314,0.78845,0.87546)
2	(2.2,2.4,2.6)	(0.69314,0.78845,0.87546)

CONCLUSION

"Fuzzy Numerical Differentiation and Integration" is a compilation of a considerable body of work that describes various contributions that will undoubtedly increase the area of the examination of numerical differentiation and integration in fuzzy situations. The title of this compilation is "Fuzzy Numerical Differentiation and Integration." At the end of each chapter, a floodgate of chances for more study is presented, each of which is distinct from the others. We have developed the fuzzy trapezoidal rule, the fuzzy Simpson's 1/3 rule, and the fuzzy Simpson's 3/8 rule as part of our attempts to accomplish fuzzy integration of fuzzy functions. These rules help us realize our goal of achieving fuzzy integration. There are a number of different interpolation algorithms that have been developed. Some of them are the Fuzzy Hermite interpolation formula, the Fuzzy Divided Difference interpolation method, the Fuzzy Stirling interpolation formula, the Fuzzy Quadratic interpolation formula, and the Fuzzy Gauss Laguerre interpolation formula.

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