

## **QUANTUM SENSORS AND THEIR APPLICATIONS IN PRECISION MEASUREMENTS AND IMAGING**

**Dr. P. Laveena manjulatha**

**Assistant professor of physics**

**MALD gdcgadwal, Jogulambagadwal, palamuru university  
Telangana, India \_ 509125.**

**Email id : laveenamanjulatha@gmail.com**

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### **ABSTRACT**

*In the last 20 years, quantum optical interferometry has seen something of a renaissance as a direct result of the availability of potent sources of entangled photons. The basics of quantum mechanics have been put to the test via the use of optical interferometry, which has also been put to use in the implementation of some of the novel ideas associated with quantum entanglement. These ideas include quantum teleportation, quantum cryptography, quantum lithography, quantum computing logic gates, and quantum metrology. In this article, the authors explore current developments in the use of quantum optical entanglement in quantum metrology, such as in fiber optical gyroscopes and sensors for biological or chemical targets, in an effort to get around the shot-noise limit. In addition to this, we discuss the potential applications of entanglement in imaging technologies such as LIDAR and optical lithography, which might help get around the Rayleigh diffraction limit.*

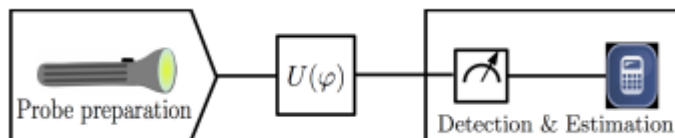
**Keywords**—*Quantum entanglement, Quantum sensors, Imaging*

### **INTRODUCTION**

Sensors are essential to the operation of a wide variety of modern technologies that have an impact on our daily lives, such as autos, the global positioning system (GPS), and mobile phone networks. In addition, they find widespread use in a wide range of commercial settings, such as, but not limited to, production and equipment, petroleum-well mapping, oil refineries, chemical processes, and medical practice. A very sensitive sensor that can pick up on very faint signals is in high demand. The capacity of a sensor is generally determined by either its noise characteristics or its accuracy. Because of this, metrology, which is the study of exact measurements, is a very important aspect of sensor development.

Quantum mechanics, which is a fundamental natural theory, has an impact on the efficiency of information-processing systems. Computing, communication, and encryption are all included in this category. Quantum physics is something that has to be taken into consideration if we want to find out where these technologies will finally reach its pinnacle. As a result of this, researchers have investigated the ways in which quantum physics influences the classical theories of computing, communication, and cryptography that have dominated the field for the better part of the past half century or more. Because of this, new lines of inquiry have been made possible, such as the development of quantum algorithms for quick integer factorization, rapid database search, quantum teleportation, superdense coding, and the distribution of quantum keys. The field of metrology, which focuses on the acquisition, processing, and estimation of information, was also modernized

so that it takes into account the effects of quantum physics. Since it was found that quantum metrology allows for measurements with precisions that surpass the classical limit, this field of study has flourished into an exciting new area of research with the potential for applications in a wide variety of fields, including gravitational wave detection, quantum positioning and clock synchronization, quantum frequency standards, quantum sensing, quantum radar and LIDAR, quantum imaging and lithography, and many more.

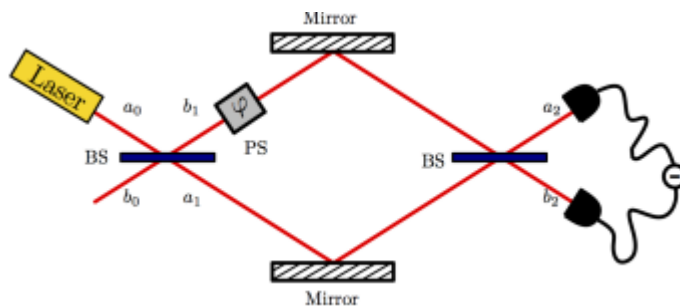


**Fig. 1. A common quantum parameter estimate system is shown. Through a unitary procedure, probes produced in appropriate quantum states are made to develop  $U(\phi)$ , which in our instance is an optical interferometer. The procedure provides the probes with information on the unknown parameter of interest. The probes are then identified at the output, and the measurement results are used to calculate the parameter's unknown value.**

The study of quantum metrology offers a theoretical basis for analyzing the precision performance of measuring systems that combine quantum-mechanical probes with nonclassical phenomena such as entanglement or squeezing. This may be done by looking at the area of quantum metrology. The theory of quantum parameter estimation is used here. Consider the typical scenario of parameter estimation that is shown in Figure 1. In this scenario, we wish to determine a parameter that is associated with the unitary dynamics that are produced by a physically defined process. Our process involves constructing probes in quantum states that are optimum, continuing the development of those probes throughout, and then measuring those probes using an optimal detection approach at the output. Then, we estimate the value of the unknown physical process parameter by contrasting the states of the probe when it is at the input and when it is at the output. It is reasonable to suppose that the generating Hamiltonian has a linear relationship with the number of probes. When  $N$  classical probes (probes that do not create quantum effects) are deployed, the level of precision that may be achieved is bound by the scale  $1/\sqrt{N}$  the minimum level of gun noise that may be heard. The central limit theorem is an important concept in statistics, and it serves as the basis for this scaling. Quantum entangled probes, on the other hand, have the ability to detect the unknown parameter with an accuracy that may scale as  $1/N$  and go below the shot-noise limit.

The field of optical metrology relies heavily on the method of light interferometry. In the most basic kind of optical interferometer, there are two modes that are employed, and the relative phase between the two modes is unknown. This unknown phase is used by an optical magnetometer to communicate information about the strength of the magnetic field, whilst the light interferometer gravitational wave observation (LIGO) makes use of it to communicate information regarding the existence of a gravitational wave. The configuration of a common optical interferometer is shown in Figure 2, which uses the Mach–Zehnder geometry. A typical interferometer receives its input from a coherent laser source, and detection is carried out by measuring the intensity differences that exist between the two beams that are interfering with one another. If a light with an average number of photons is used, then the  $\bar{n}$  when this method is used, the accuracy of phase estimate is bound by the shot-noise of the  $1/\sqrt{\bar{n}}$  connected with the intensity variations

seen at the output, which may be traced back to the vacuum fluctuations of the quantized electromagnetic field that enter the device via the input port that is not being utilized  $b_0$ .



**Fig. 2 Schematic depicting how coherent light input and intensity difference detection are used in traditional Mach-Zehnder interferometry.**

Phase estimation with sub-shot noise is, nevertheless, possible using quantum optical metrology. To achieve sub-shotnoise accuracy that scales as, Caves demonstrated that the nonclassical squeezed vacuum state, rather than the vacuum state, may be combined in the unused port of the same interferometer  $1/\bar{n}^{2/3}$ ; it is possible to achieve. After then, it was shown that two-mode compressed states might permit phase estimation with an accuracy of  $1/\bar{n}$ . Researchers in the area of quantum optical metrology have postulated and researched states with a restricted number of photons that show quantum entanglement ever since the emergence of single-photon technology. These include, but are not limited to, the Holland-Burnett state and the Berry-Wiseman state, as well as the NOON state, which is a Schrodinger cat-like, maximally mode-entangled state of two modes, where the N photons are in superposition of all N photons being in one mode or the other. [Case in point:] the N photons are in the NOON state. It was discovered that each of these states had the potential to surpass the  $1/N$  limit proposed by Heisenberg. The theoretical discoveries that were achieved earlier made it feasible for several practical demonstrations of sub-shot-noise metrology to be carried out using limited photon number states.

There has been a lot of research done on the different quantum states of light and the different ways of detection. Counting the quantity of photons, establishing the parity of the photons, and the traditional measurement of phase are all instances. The traditional measurement of phase may be reproduced by an adaptive measurement. It has been shown that these measurement devices are capable of achieving the greatest conceivable degrees of accuracy for a wide range of quantum states associated with light.

In recent years, there has been a profusion of study on the effect that photon loss, dephasing noise, and other kinds of decoherence have on the precision of phase estimation in quantum optical metrology. These studies have been conducted both in the United States and abroad. In a few of these situations, both numerical and analytical approaches have been able to zero in on ideal quantum states that are getting close to usable lower bounds on precision.

Interferometry with entangled states of low photon number is the primary focus of our investigation. In particular, we are interested in interferometry based on the NOON states. We highlight new works that reveal NOON state interferometry, which may find use in quantum technologies. This interferometry is performed on tiny numbers of photons N. Both super-sensitivity (reaching the Heisenberg limit of  $1/N$  in phase accuracy) and super-resolution (achieving phase resolution below the Rayleigh diffraction limit) are possible

with the N00N states. The paper will be written in the manner that is outlined below. In this second section, we will discuss some of the underlying ideas behind quantum optical metrology. The investigation of two-mode quantum interferometry may make use of a variety of representations, and the interferometric output statistics can be evaluated using methods derived from the theory of quantum parameter estimation in order to derive the unknown phase. The topic of discussion here is optical quantum metrology. The concept of quantum entanglement, which serves as the major impetus behind quantum enhancement, is presented first, and then a description of the many methods that may be used to generate entanglement for optical metrology is presented thereafter. It is well known that the interferometric method that we discuss here is capable of reaching the Heisenberg limit in phase precision. measured, felt, and photographed by employing technologies related to quantum mechanics. On the basis of the results of a few recent research, this article will explain the benefits that N00N state quantum optical interferometry may provide for the implementation of various technological applications.

## OBJECTIVES

1. To study Quantum Sensors and Their Applications in Precision Measurements and Imaging
2. To study Precision Measurements and Imaging

### A. Classical optical interferometry

Before we go on to discussing quantum interferometry, let us first do a brief examination in purely classical terms of the kind of coherent laser light interferometer that is illustrated in Figure 2. The entering laser beam is cut in half by the first beamsplitter, which is a 50:50 device. After passing through the device, these beams will have their relative phase modified in some way. At the very last beamsplitter, once the two output beams have been combined once again, the difference in average intensity that exists between them may be calculated. A easy calculation using traditional optics demonstrates that one may represent the intensities that are measured at the output ports as a function of the intensity that is measured at the input ports  $I_{a_0}$  and the relative phase  $\varphi$  as

$$\begin{aligned} I_{a_2} &= I_{a_0} \sin^2(\varphi/2), \\ I_{b_2} &= I_{a_0} \cos^2(\varphi/2). \end{aligned} \dots(1)$$

This suggests that there is a variation in intensity between the two output ports  $M(\varphi) \equiv I_{b_2} - I_{a_2} = I_{a_0} \cos \varphi$  fringes of a sinusoidal pattern that are visible whenever the relative phase is changed.

The degree of accuracy with which one is able to infer an unknown relative phase based on the measurement of M, expressed either as the phase error or the minimal phase that may be detected,  $\Delta\phi$ , Applying the following formula for linear error propagation, it is possible to get a reasonable estimate of the answer:

$$\Delta\varphi = \frac{\Delta M}{|dM/d\varphi|} = \frac{\Delta M}{I_{a_0} \sin \varphi}. \dots(2)$$

The above equation suggests that at a local value of phase  $\varphi = \pi/2$ , The precision of phase estimation may be brought down to an infinitesimally small level if the intensity

difference  $M$  is first measured with an accuracy of an infinite and then the input intensity  $I_a$  is raised to an intensity of an arbitrary magnitude. Quantum mechanics, on the other hand, places constraints on the possibility of measuring intensities to an infinite degree of precision (that is, with  $M = 0$ ). This is because counting photons is in and of itself a quantum process, in which the quantity being measured is not an intensity signal that fluctuates constantly but rather a fixed amount of energy. The reason for this is because counting photons is a quantum process. The absorption process is inherently unpredictable due to the fact that the quantized electromagnetic field is affected by fluctuations in the vacuum. The number of photons detected by the coherent laser light follows a Poisson distribution. Because of this, the phase estimate accuracy of classical interferometry is limited to  $\Delta\varphi = 1/\sqrt{\bar{n}}$ , where  $\bar{n}$  is the intensity of the laser beam that is being input..

## B. Quantum optical interferometry

In order to provide a treatment that is completely quantum-based for two-mode optical interferometry, we will now provide a quantized mode of the electromagnetic field and explain the many states that it may take. After that, we will talk about the linear optical transformations that are most important for a pair of independent photonic modes in interferometry, namely the beam splitter transformation and the phase shifter transformation. After this step, Hermitian operators are considered to be measurement observables at the output of the interferometer, and the accuracy of phase estimation is calculated based on the error propagation formula.

1. It is possible for electromagnetic fields to exist in quantized modes, and the operators that are responsible for their production and destruction can completely describe the states that result from this possibility  $\hat{a}$  and  $\hat{a}^\dagger$ , which satisfy the commutation relation  $[\hat{a}, \hat{a}^\dagger] = 1$ . They are characterized by the effect that they have on the various stages of the mode  $|n\rangle$ —also called Fock states, given by:

$$\hat{a}|n\rangle = \sqrt{n}|n-1\rangle, \hat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle. \dots(3)$$

The action of an appropriate function of the mode creation and annihilation operators on a pure state of the single-mode field (a vector in Hilbert space) may be used to define the vacuum state. This action can be thought of as the vacuum state  $|0\rangle$ . For example, a Fock state  $|n\rangle$  can be expressed as  $\frac{\hat{a}^\dagger^n}{\sqrt{n!}}|0\rangle$ , where  $|0\rangle$ , where  $|0\rangle$  is referred to as the vacuum condition. The coherent state may be expressed in the following way:

$$|\alpha\rangle = \hat{D}(\alpha)|0\rangle, \dots\dots\dots 4$$

where  $\hat{D}(\alpha) = \exp(\alpha\hat{a}^\dagger - \alpha^*\hat{a})$  is what's known as the displacement operator, and it acts as a complex number that indicates the coherent amplitude of the state. Full bases may be constructed using the Fock states as well as the Coherent states (the coherent states really provide an over-complete basis). As a result, it is possible to utilize these states to represent any pure state of the quantum field operating in a single mode. Mixed states, which are ensembles of pure states, may also be expressed in terms of these states as a density operator, just as any other state of the single-mode field can. This is because mixed states are ensembles of pure states. The term "density operator" refers to the positive semi-definite tracing one operators. The positive semi-definiteness criteria stipulates that the eigenvalues of the state must be non-negative real numbers. This is done to guarantee that



the eigenvalues may be correctly interpreted as probabilities. In addition, the trace one condition ensures that the aggregate of these probabilities is equal to one, so transforming the state into a normalized version of itself. The following density operator provides a description of the most generic condition that can be applied to a single-mode field when using the Fock basis.

$$\hat{\rho} = \sum_{n,n'} p_{n,n'} |n\rangle\langle n'|, \text{Tr}(\rho) = 1, \rho \geq 0. \dots\dots\dots 5$$

An alternative way to represent a quantized mode is in terms of quasi-probability distributions in the phase space of eigenvalues  $x$  and  $p$  of the mode's quadrature operators  $\hat{x}$  and  $\hat{p}$ . The mode's creation and destruction operators are used to define these operators, as  $\hat{x} = \hat{a}^\dagger + \hat{a}$  and  $\hat{p} = i(\hat{a}^\dagger - \hat{a})$ , to put it another way. It is possible to derive the Wigner distribution of a single-mode state by using the density operator's form, which may be found in equation (5) as

$$W(\alpha) = \frac{1}{2\pi^2} \int d^2\tilde{\alpha} \text{Tr} \{ \hat{\rho} \hat{D}(\tilde{\alpha}) \} e^{-\tilde{\alpha}\alpha^* - \tilde{\alpha}^*\alpha}, \dots\dots\dots 6$$

where  $\tilde{\alpha} = \tilde{x} + i\tilde{p}$  and  $\alpha = x + ip$ .

2. The Mach-Zehnder interferometer's quantum states as well as its dynamic behavior: We correlate mode generation and mode annihilation operations with each of the two modes in the quantum description of the Mach-Zehnder interferometer (MZI). Within this region, we refer to them as  $\hat{a}_i, \hat{a}_i^\dagger$  and  $\hat{b}_i, \hat{b}_i^\dagger, i \in \{0, 1, 2\}$ , where the different values of  $i$  indicate the various modes that may be found at the input, the interior, and the output of the interferometer. It is possible for a MZI to have two different modes: either spatial modes or polarization modes.

Take into consideration the transmission of the input quantum states of the two modes across the many different linear optical components that the MZI has. According to the so-called Heisenberg picture, propagation is shown as a scattering matrix that acts as a transform on the mode operators.  $M_i$ :

$$\begin{bmatrix} \hat{a}_0 \\ \hat{b}_0 \end{bmatrix} = \hat{M}_i^{-1} \begin{bmatrix} \hat{a}_1 \\ \hat{b}_1 \end{bmatrix} \dots\dots\dots 7$$

It can be shown that the scattering matrices that correspond to a phase shifter and a beam splitter with a ratio of 50:50 are provided by

$$\hat{M}_{BS} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}, \hat{M}_\varphi = \begin{bmatrix} 1 & 0 \\ 0 & e^{-i\varphi} \end{bmatrix} \dots\dots\dots 8$$

respectively. (Note that this form for  $\hat{M}_{BS}$  holds for beamsplitters that are constructed as a single dielectric layer, in which case the reflected and the transmitted beams gather a relative phase of  $\pi/2$ .) Therefore, the two-mode quantum state that is produced at the output of a MZI in the Fock basis may be derived by exchanging the mode operators that are present in the input state with those that are present in the output state. In this case, the overall scattering matrix is represented as follows:  $\hat{M}_{MZI} = \hat{M}_{BS}\hat{M}_\varphi\hat{M}_{BS}$  and is found to be:

$$\hat{M}_{MZI} = ie^{-i\frac{\pi}{2}} \begin{bmatrix} \sin \frac{\theta}{2} & \cos \frac{\theta}{2} \\ \cos \frac{\theta}{2} & -\sin \frac{\theta}{2} \end{bmatrix} \dots\dots 9$$

The propagation of the Mach-Zehnder interferometer may be similarly characterized in terms of phase space quasi-probability distributions such as the Wigner distribution function. This is done by linking the initial complex variables in the Wigner function to their final expressions as follows:

$$W_{out}(\alpha_1, \beta_1) = W_{in}[\alpha_0(\alpha_1, \beta_1), \beta_0(\alpha_1, \beta_1)] \dots\dots 10$$

Similar to how the link between the complex variables is expressed in terms of the scattering matrices, which are two by two in size  $\hat{M}$  :

$$\begin{bmatrix} \alpha_0 \\ \beta_0 \end{bmatrix} = \hat{M}^{-1} \begin{bmatrix} \alpha_1 \\ \beta_1 \end{bmatrix}, \dots\dots 11$$

$\alpha_0, \beta_0, \alpha_1,$  and  $\beta_1$  being the complex amplitudes of the field in the modes  $\hat{a}_0, \hat{b}_0, \hat{a}_1,$  and  $\hat{b}_1$ , respectively. When dealing with Gaussian states, which are states that have a Gaussian Wigner distribution, the approach that is based on phase space probability distributions is very simple to use and powerful. This is because Gaussian operations also have this distribution. The coherent state, the state of a compressed vacuum, and the thermal state are three excellent examples each. This is due to the fact that the mean and covariances of Gaussian states of any number of independent photonics modes may be transmitted using methods that are based on the algebra of the symplectic group, and also due to the fact that a Gaussian distribution can be totally defined by its first and second moments.

The Schwinger model is a different kind of framework that may be used to describe quantum states and the dynamics of those states in a multi-zone interference (MZI) environment. The model is predicated on an intriguing relationship between the algebra of the angular momentum and the algebra of the mode operators for two distinct photonic modes. This connection is at the heart of the model's foundation.

Some of the functions that are performed by the mode operators of a pair of two photonic modes that are fully distinct from one another are as follows:  $\hat{a}_1, \hat{a}_1^\dagger, \hat{b}_1,$  and  $\hat{b}_1^\dagger$

$$\begin{aligned} \hat{J}_x &= \frac{1}{2}(\hat{a}_1^\dagger \hat{b}_1 + \hat{b}_1^\dagger \hat{a}_1), \quad \hat{J}_y = \frac{1}{2i}(\hat{a}_1^\dagger \hat{b}_1 - \hat{b}_1^\dagger \hat{a}_1), \\ \hat{J}_z &= \frac{1}{2}(\hat{a}_1^\dagger \hat{a}_1 - \hat{b}_1^\dagger \hat{b}_1). \end{aligned} \dots\dots 12$$

(The modes after the first beamsplitter are represented by the "1" in the mode zone index shown in Fig. 2. This is important to keep in mind when we talk about the practical applications of the  $\hat{J}^q$  operators shortly.)

It is possible to demonstrate that they are compliant with the SU(2) algebra of angular momentum operators, more specifically  $[\hat{J}_q, \hat{J}_r] = i\hbar \epsilon_{q,r,s} \hat{J}_s$ , where  $\epsilon$  is the antisymmetric tensor and where  $q, r, s \in \{x, y, z\}$ . On the basis of this connection, a two-mode N-photon pure state is uniquely mapped onto a pure state in the spin-N/2 subspace of the angular momentum Hilbert space, which may also be written as spin-N/2 subspace of angular momentum Hilbert space.

$$|n_a, n_b\rangle \rightarrow |j = \frac{n_a + n_b}{2}, m = \frac{n_a - n_b}{2}\rangle. \dots 13$$

The Schwinger representation may be understood in terms of the propagation of the quantized single-mode field. This transformation can be considered as an SU(2) group transformation that is induced by the angular momentum operators  $\hat{J}_x$ ,  $\hat{J}_y$  and  $\hat{J}_z$ . As an example, the beamsplitter transformation of the number eight may be expressed as:

$$\begin{bmatrix} \hat{a}_0 \\ \hat{b}_0 \end{bmatrix} = U_{BS}^\dagger \begin{bmatrix} \hat{a}_1 \\ \hat{b}_1 \end{bmatrix} U_{BS}, \dots 14$$

where  $U_{BS} = \exp(i\pi/2\hat{J}_x)$ , In a similar fashion, the change that was brought about by the phase shifter that was housed inside the interferometer may be described as

$$\begin{bmatrix} \hat{a}_1 \\ \hat{b}_1 \end{bmatrix} \rightarrow U_\varphi^\dagger \begin{bmatrix} \hat{a}_1 \\ \hat{b}_1 \end{bmatrix} U_\varphi, \dots 15$$

The total unitary transformation that corresponds to the MZI may be written as where is the SU(2) algebra of the angular momentum operators and B is the Baker-Hausdorff lemma. This gives the formula for the MZI  $\hat{U}_{MZI} = \exp(-i\varphi\hat{J}_y)$ . Operationally, for any given two-mode state, the operator  $\hat{J}_z$  monitors the difference in the number of photons that are produced by the two modes included inside the interferometer  $\propto \hat{a}_1^\dagger\hat{a}_1 - \hat{b}_1^\dagger\hat{b}_1$ . In a similar vein, by using the SU(2) commutation relations, it is possible to demonstrate that the operators  $\hat{J}_x$  and  $\hat{J}_y$  monitor the variations in the amount of photons at the input (which is)  $\propto \hat{a}_0^\dagger\hat{a}_0 - \hat{b}_0^\dagger\hat{b}_0$  and the output (which is  $\propto \hat{a}_2^\dagger\hat{a}_2 - \hat{b}_2^\dagger\hat{b}_2$ ), respectively.

## CONCLUSION

We offered a synopsis of the subject of quantum optical metrology in this work, with a particular emphasis on NOON-based quantum technologies that have been shown to be effective in the laboratory. The concepts of quantum squeezing, quantum entanglement, and quantum interferometry were presented, together with the principles that lie underneath them. We devised an interferometric method that combines coherent and compressed vacuum light on a beam splitter in order to achieve our goal of obtaining Nphoton components that are in a condition that is effectively denoted as NOON. research that use NOON states formed by this method (or other ways under particular circumstances) were subsequently presented. These research exploit NOON states for the purpose of using them in technological applications such as quantum-enhanced biosensing, imaging, and spatial resolution lithography, amongst others. In addition to that, we looked into a great deal of additional possibilities. This review does not even come close to covering the vast amount of information that may be obtained about quantum optical metrology. Please visit for a comprehensive discussion on the topic in question. This article does not address the Bayesian methodology of quantum metrology, which is a method of quantum metrology in which the unknown parameter is meant to be distributed according to an unknown probability distribution.



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