

STUDY OF PARAMETRIC EFFECT ON THE EFFICIENCY OF THE MATHEMATICAL MODEL

DESCRIBING WASHING OPERATION

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ABSTRACT:

A comprehensive study of the flow of fluid through the packed bed of pulp fibers is attempted by a continuous, dynamic, nonlinear model based on diffusion-dispersion phenomenon. The results obtained from the model are presented in the form of plane and surface plots. Key parameters of the industry are used directly or indirectly in the development of the model. On the basis of the graphs obtained, the theoretical analysis of different parameters is made for exit solute concentration. The model is solved numerically using the technique of orthogonal collocation on finite elements. The effect of Peclet number and Distribution ratio (D_F/R^2) is checked on these parameters.

1. Introduction

Porous particles can be defined as composition of porous particles which comprises of pore space to allow the fluid to flow. The medium may be filled with one or more than one fluid. Sponge, cemented sandstone, karstic limestone, soil, sand, foam rubber, bread etc. are some examples of porous medium. In chemical and process industries, behavior of fluids in

porous medium is studied extensively. Adsorption-desorption, diffusion-dispersion phenomenon are widely studied to analyze the behavior of fluids.

One of such phenomenon is washing of porous solid and semi solid particles. During this process the impurities adsorbed within and on the pores of the particles are detached by the introduction of an external fluid. The mechanics involved is the sum of displacement of the solute by movement of water plug controlled by fluid mechanics, dispersion due to back mixing, diffusion due to concentration gradients and adsorption-desorption due to relative affinity of various solutes towards the particle surface. Time allowed for washable substance to diffuse out of particle pores is also an important factor.

During washing operation, packed bed consists of different zones such as flowing liquor zone, stagnant zone and zone of porous material. Mass transfer through thin layer is represented by film resistance mass transfer coefficient. Dissolved substances in the solute are adsorbed on particle surface to a variable extent depending on the type of particles, solute concentration, consistency and porosity.

The primary objective of washing is to extract the residual liquor that might introduce impurities in the subsequent stages. Simultaneously, the purpose is also to recover the precious dissolved substances. The recovered organic and inorganic substances are used for regeneration.

In the case of real or ideal washing, minimum volume of wash water for complete washing is equal to the volume of liquor initially between the particles. The real washing process is far from the ideal conditions of washing due to axial dispersion, non uniformity of pores, the structure of particles is heterogeneous and longitudinal mixing with diffusion takes place. During washing process only 30% to 86% of the preserved filtrate can be removed by displacement washing whereas ideal washing process needs 100% recovery of residual filtrate (Potůček & Skotnicova, 2002).

A number of complex phenomena are involved in washing of packed bed of porous particles filled with fluid containing soluble material. In case of porous particles the solute is contained both in the pores of the particles and in the lumen. To remove the solute from these particles, first the solutes diffuse from pores to the particle surface and then from surface to interparticle voids. During this unsteady process certain amount of heat is exchanged and associated with transfer of heat and mass in the form of liquid from one medium to another, as long as the driving forces exist. In case of packed beds, the flow pattern causes the dispersion of entering fluid to the original fluid.

The main aim of washing process is to make washing efficient by using minimum amount of wash liquor with maximum removal of solute adsorbed on particle surface. Washing is a crucial operation in post washing and inter-stage bleaching operations to remove and recover chemical liquor, to reduce water consumption and to reuse the fluid (Potůček, 1997).

In this work, an attempt has been made to develop a mathematical model for displacement washing of porous particles. The proposed model is developed on certain basic assumptions which are linked to the model. The summary of these assumptions is given below:

- 1) Temperature of the system is assumed to be constant and the bed is assumed to be macroscopically uniform.
- 2) The displacement washing is controlled by diffusion.
- 3) It is assumed that particles are spherical, homogenous in size and are porous in nature.
- 4) During the process mass transfer coefficient is taken to be uniform
- 5) Axial dispersion coefficient signifies dispersion.
- 6) Langmuir adsorption equilibrium has been followed to relate solute concentration adsorbed on the particle surface and the solute concentration within particle pores.
- 7) Mass transfer within particles occur due to the Fick's law of diffusion and is represented by intraparticle diffusion coefficient D_s
- 8) Flow pattern in the bed is represented by an axial dispersion model

- 9) Bed porosity (ϵ), particle porosity (β) and consistency of particles are assumed to be interrelated to each other.
- 10) Average solute concentration is defined over the bed cross section.

The main aim of this work is to develop a mathematical model which explains the behavior of fluid within the particle phase. Washing behavior of the packed bed of solid particles having cylindrical geometry has been studied through axial dispersion coefficient, molecular diffusion coefficient and intraparticle diffusion coefficient.

In this work, axial dispersion model has been studied which is based on the material balance equation given(Potůček, 1997).

$$\left[\begin{matrix} \text{Entering by} \\ \text{bulk flow} \end{matrix} \right] + \left[\begin{matrix} \text{Entering by} \\ \text{dispersion} \end{matrix} \right] = \left[\begin{matrix} \text{Leaving by} \\ \text{bulk flow} \end{matrix} \right] + \left[\begin{matrix} \text{Leaving by} \\ \text{dispersion} \end{matrix} \right] + \left[\begin{matrix} \text{Accumulation} \\ \text{of solute} \end{matrix} \right]$$

Number of mathematical models have been represented by these equations to study the different physical processes related in the field of fluid dynamics, heat and mass transfer, dispersion – diffusion process, adsorption-desorption process, osmosis etc.

From Brenner (1962) to Arora et. al. (2015) plethora of literature has been contributed to the study of mathematical model based on displacement washing process. Brenner (1962), Sherman (1964) and Potůček *et al.* (2003) has presented a one dimensional axial dispersion model. Pellett (1966), Raghavan & Ruthven (1983), Sridhar (1999), Soos Park (2002), Arora *et al.* (2006) and Kumar *et al.* (2009, 2010) has presented two dimensional model with variable or constant coefficients with different parameters.

According to Arora & Potůček (2009), there are two types of mechanism related to the fluid concentration in packed bed of particles. First one is the transfer rate of material between fluid and particles and is given explicitly by a partial differential equation:

$$\frac{\partial q}{\partial t} = F(n, q) \quad (1)$$

This type of isotherm has been discussed generally for both linear and non-linear isotherms. The second type of rate mechanism is one in which solid phase diffusion into the interior of the bed particles is an important rate step and the rate equation $F(n,q)$ is replaced by an integral equation.

$$q = \frac{2}{R^2} \int_0^R q(r,z,t) r dr \quad (2)$$

Details of this rate mechanism are given in Arora *et al.* (2006).

2. Details of Model

In Model, fiber pores are assumed to be variable. It gives a transient equation for particle phase depending upon time and pore radius of fibers. Adsorption equilibrium, boundary conditions for bulk fluid phase and initial condition will remain same.

2.1 Equations for particle phase

The diffusion equation describing the movement of solute within the fiber pores is defined as:

$$D_F \left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} \right) = \frac{\partial w}{\partial t} \quad (3)$$

where w is the local intrapore solute concentration. By definition, w does not distinguish between the solute adsorbed on the fiber surface and the solute within the fiber pores. In order to distinguish between the solute adsorbed on the fiber surface and solute within the fiber pores, it is assumed that the surface diffusion effects are negligible. The transport of solute within the fibers is effectively described by the diffusion equation involving both concentration of solute adsorbed on the fiber surface and the concentration of solute within the fiber pores. The driving force for diffusion is taken to be the intrapore concentration gradient. Hence the intrapore diffusion equation can be written as:

$$D_F \left(\frac{\partial^2 q}{\partial r^2} + \frac{1}{r} \frac{\partial q}{\partial r} \right) = \frac{\partial q}{\partial t} + \frac{(1-\beta)}{\beta} \frac{\partial n}{\partial t} \quad (4)$$

By definition of local intrapore solute concentration Arora et. al., (2006) at radial distance 'r' following relation can be obtained:

$$w = \beta q + (1 - \beta)n \quad (5)$$

For mathematical convenience, it is assumed that local equilibrium prevails in the individual intrafiber pores.

2.2 Initial and boundary conditions

Initially it is assumed that intrapore solute concentration is equal to the bulk fluid concentration.

$$q(r, z, 0) = C_0 \quad r \in (0, R) \text{ and } z \in (0, L) \quad (6)$$

The concentration of solute adsorbed on the fiber surface is equal to the initial concentration of solute adsorbed on the fiber surface.

$$n(r, z, 0) = N_0 \quad r \in (0, R) \text{ and } z \in (0, L) \quad (7)$$

To avoid the problem of maxima or minima of the solute concentration within the fiber pores, the intrapore concentration gradient is assumed to be zero at $r = 0$, i.e.,

$$\frac{\partial q}{\partial r} = 0 \quad \text{at } r = 0 \text{ and } t > 0 \quad (8)$$

External mass transfer resistance exists, i.e., the mass transfer to the surface of the fiber is controlled by film resistance mass transfer coefficient. Therefore, the boundary condition at $r = R$ will be:

$$u \frac{\partial q}{\partial r} \bigg|_{r=R} = \frac{k_f \beta L}{KR^2} (c - q|_{r=R}) \quad \text{at } r = R \text{ and } t > 0 \quad (9)$$

2.3 Mathematical equations for bulk fluid

The transport phenomenon in the porous media having void fraction ε is described by one dimensional axial dispersion model involving the axial dispersion and molecular diffusivity.

The model for bulk fluid is represented by an axial dispersion model involving accumulation term and axial dispersion coefficient as follows.

$$D_L \frac{\partial^2 c}{\partial z^2} = u \frac{\partial c}{\partial z} + \frac{\partial c}{\partial t} + \frac{2(1-\varepsilon)k_f \beta}{KR\varepsilon} (c - q|_{r=R}) \quad (10)$$

After converting the dimensional variables into the dimensionless form, equations (4) to (10) take the following form:

$$\psi \left(\frac{\partial^2 Q}{\partial \eta^2} + \frac{1}{\eta} \frac{\partial Q}{\partial \eta} \right) = \frac{\partial Q}{\partial \tau} + N_1 \frac{(1-\beta)}{\beta} \frac{\partial N}{\partial \tau} \quad (11)$$

$$\frac{\partial N}{\partial \tau} = P_1 [Q(1-N) - k^{*-1}N] \quad (12)$$

$$\frac{\partial C}{\partial \tau} = \frac{1}{Pe} \frac{\partial^2 C}{\partial \xi^2} - \frac{\partial C}{\partial \xi} - 2\theta Bi (C - Q|_{\eta=1}) \quad (13)$$

$$\frac{\partial Q}{\partial \eta} = 0 \quad \text{at } \eta = 0 \text{ and } \tau > 0 \quad (14)$$

$$\frac{\partial Q}{\partial \eta} = Bi(C - Q|_{\eta=1}) \quad \text{at } \eta = 1 \text{ and } \tau > 0 \quad (15)$$

$$C - \frac{1}{Pe} \frac{\partial C}{\partial \xi} = 0 \quad \text{at } \xi = 0 \text{ and } \tau > 0 \quad (16)$$

$$\frac{\partial C}{\partial \xi} = 0 \quad \text{at } \xi = 1 \text{ and } \tau > 0 \quad (17)$$

$$\text{Initially, } Q = N = C = 1 \quad \text{at } \tau = 0 \quad (18)$$

The proposed model is a simplification of the model of Arora et. al., (2006) and is an extension of the model of Arora & Potucek (2009). In this model the effect of swollen radius of fiber pores together with the intrafiber diffusion coefficient is checked. The particle diffusion equation is presented to represent the intrafiber diffusion and the solute adsorbed on the fiber surface. The adsorption isotherm is taken to be Langmuir to link the interfiber and intrapore solute concentrations in a dynamic way.

3. COLLOCATION POINTS

Choice of the collocation points is an important and sensitive part of orthogonal collocation method. In this method, collocation points are taken to be the zeros of orthogonal polynomials. Usually, the zeros of Jacobi polynomial are taken as collocation points. Legendre and Chebyshev polynomials are particular cases of Jacobi polynomial.

The collocation points are obtained by mapping the computational domain of the interval $[-1, 1]$ to $[0, 1]$ with the help of following relationship:

$$\xi_{m+3-j} = \frac{x_j}{2} + \frac{1}{2} \quad (19)$$

where x_j is the j^{th} collocation point in the interval $[-1, 1]$. The $m+2$ interpolation points are chosen to be the extreme values of an $m+1^{\text{th}}$ order shifted Chebyshev polynomial.

$$x_j = \cos \frac{\pi(j-1)}{m+1} ; \quad j = 1, 2, \dots, m+2. \quad (20)$$

The discretization end points are fixed as $\xi_1 = 0$ and $\xi_{m+2} = 1$.

The zeros of Legendre polynomial are calculated from the following recurrence relation:

$$(j-1)P_{j-1}(x) = (2j-3)xP_{j-2}(x) - (j-2)P_{j-3}(x); j=2, \dots, m+1 \quad (21)$$

where $P_0(x) = 1$ and $P_{-1}(x) = 0$. In case of Legendre polynomial, 0 and 1 are taken to be the boundary points.

4. Weight functions

In the method of weighted residuals, the residual so obtained is set orthogonal to the weight function in such a way that $\langle W_j, E \rangle = 0$, where E is the residual obtained after adjusting the approximating function in the given differential equation. In orthogonal collocation method, the residual is set equal to zero at the collocation points.

Consider the function $f(x)$ as, $f(x) = x^{i-1}$

To reduce the number of calculations, the weight function has been calculated using Lagrangian interpolation polynomial as given by Arora et. al., (2005). The algorithm given by Arora et. al., (2005) is also based on pure Gaussian formula which integrates any polynomial of degree less than or equal to $2m-1$. However, this algorithm is also influenced by the method of Finlayson (1972), but it is simple and convenient.

$$\int_0^1 W(x)y(x,t) dx = \int_0^1 W(x) \sum_{i=1}^{m+2} l_i(x)y_i dx \quad (22)$$

$$= \sum_{i=1}^{m+2} y_i \int_0^1 W(x)l_i(x)dx \quad (23)$$

$$= \sum_{i=1}^{m+2} w_i y_i \quad (24)$$

$$w_i = \int_0^1 W(x)l_i(x)dx \quad (25)$$

It gives the weight functions at the respective collocation points and $\sum_{i=1}^{m+2} w_i = 1$.

5. ORTHOGONAL COLLOCATION ON FINITE ELEMENTS

The orthogonal collocation method does not give fast converging results for large values of the parameters in case of stiff system of boundary value problems as mentioned by Carey and Finlayson (1975), Liu and Bhatia (1999), Arora et al. (2005). To overcome this problem orthogonal collocation method is conjectured with the finite element method to combine the features of both the methods.

Orthogonal collocation on finite elements was first proposed by Patterson and Cresswell (1971). It was further extended by Carey and Finlayson (1975) to solve effectiveness factor problem for large Thiele modulus. Ma and Guiochon (1991) have used orthogonal collocation on finite elements for calculation of chromatographic elution band profiles for different components. Arora et. al., (2009) has checked the asymptotic behavior of parabolic

partial differential equations using finite element collocation method. Liu & Jacobsen (2004) have carried out the bifurcation analysis of heat integrated fixed bed reactor by reducing the order of distributed model using orthogonal collocation on finite elements.

In orthogonal collocation on finite elements, the whole domain is divided into small sub domains, called elements. The orthogonal collocation is applied within each element. In this process it is mandatory that the trial function and its first derivative should be continuous at the nodal points or the boundaries of the elements.

In orthogonal collocation on finite elements, The global variable x varies in the ℓ^{th} element, where $\ell = 1, 2, 3, \dots, ne$. The node points are set at $x_1, x_2, x_3, \dots, x_{ne+1}$.

The boundary points have been placed as $x_1 = 0$ and $x_{ne+1} = 1$. To apply the orthogonal collocation with in ℓ^{th} element, a new variable ξ is introduced in the ℓ^{th} element. The variable ξ is introduced in such a way that as x varies from x_ℓ to $x_{\ell+1}$, ξ varies from 0 to 1

in the ℓ^{th} element such that $\xi = \frac{x - x_\ell}{x_{\ell+1} - x_\ell}$.

Orthogonal collocation is applied on the variable ξ within ℓ^{th} element, $\ell = 1, 2, \dots, ne$. To avoid the problem of double calculation at the node points, the approximating function and its first derivative are taken to be continuous at the node points by using the principle of continuity.

$$y^\ell \Big|_{x_\ell+} = y^{\ell+1} \Big|_{x_{\ell+1}-} \quad (26)$$

$$\frac{dy^\ell}{dx} \Big|_{x_\ell+} = \frac{dy^{\ell+1}}{dx} \Big|_{x_{\ell+1}-} \quad (27)$$

These conditions are also called the continuity conditions. With the help of these conditions, the problem of double calculation arising in the application of orthogonal collocation on finite elements is overcome.

6. CONVERGENCE CRITERION

To check the asymptotic behavior theoretically and rate of convergence of the orthogonal collocation on finite elements, a linear convection diffusion time dependent problem represented by parabolic partial differential equation is considered:

$$\frac{\partial y}{\partial t} = \frac{1}{Pe} \frac{\partial^2 y}{\partial x^2} - a_1(x) \frac{\partial y}{\partial x} - a_2(x)y + f(x), \quad x \in (0,1) \text{ and } t > 0 \quad (28)$$

$$\text{Boundary conditions:} \quad a_3 y + a_4 \frac{\partial y}{\partial x} = b_1; \quad x = 0 \text{ and } t > 0 \quad (29)$$

$$a_5 y + a_6 \frac{\partial y}{\partial x} = b_2; \quad x = 1 \text{ and } t > 0 \quad (30)$$

$$\text{Initial condition:} \quad y = g(x) \quad t = 0 \quad \text{and } x \in (0,1) \quad (31)$$

$a_i(x)$ are positive functions defined on $(0, 1)$, whereas b_1 and b_2 are non negative real numbers. $f(x)$ and $g(x)$ are a continuous functions of x defined on $(0, 1)$. a_3 and a_4 are constants not both zero and a_5 and a_6 are constants not both zero. x is bounded in the open interval $(0,1)$ on the real axis.

To apply the orthogonal collocation on finite elements technique, interval $[0, 1]$ is divided into small $[x_{k-1}, x_k]$ and Let P be the partition $x_1 \leq x_2 \leq \dots \leq x_r$ and the length h of the k^{th} interval $[x_{k-1}, x_k]$ is defined as $h = |x_k - x_{k-1}|$, where $k = 1, 2, 3, \dots, r$.

After applying the collocation, following system of collocation equations is obtained:

$$\frac{\varepsilon}{h^2} \sum_{i=1}^m B_{ji} y_i^\ell - \frac{a_1(x_j)}{h} \sum_{i=1}^m A_{ji} y_i^\ell - \gamma_j \sum_{i=1}^m \delta_{ji} y_i^\ell = g_j; \quad j = 2, \dots, m-1 \text{ and } \ell = 1, 2, \dots, r \quad (32)$$

$$y_n^{\ell-1} = y_1^\ell; \quad \ell = 1, 2, \dots, r \quad (33)$$

$$\frac{1}{h} \sum_{i=1}^m A_{ni} y_i^{\ell-1} = \frac{1}{h} \sum_{i=1}^m A_{ni} y_i^\ell; \quad \ell = 1, 2, \dots, r \quad (34)$$

$$a_3 y_1^1 - \frac{a_4}{h} \sum_{i=1}^m A_{1i} y_i^1 = 0 \quad (35)$$

$$a_5 y_n^r - \frac{a_6}{h} \sum_{i=1}^m A_{ni} y_i^r = 0 \quad (36)$$

After combining the system of equations within each element, following system of differential algebraic equations appear:

$$\mathbf{D} = \mathbf{M} \mathbf{Y} \quad (37)$$

where, $\mathbf{M}$ is the coefficient matrix $[m_{ij}]_{d \times 1}$, where $d = r \times (m-2) + r + 1$, r being the number of elements and m being the number of collocation points. $\mathbf{Y}$ is the matrix of collocation solutions $[y_j]_{d \times 1}$ and $\mathbf{D}$ is the matrix $[g_j]_{d \times 1}$. Entries in matrix $\mathbf{M}$ are defined as:

$$m_{ji} = \begin{cases} a_3 \delta_{1i} - \frac{a_4}{h} A_{1i}; & j=1, i=1, 2, \dots, m \\ \frac{\varepsilon}{h^2} B_{ji} - \frac{a_1(x_j)}{h} A_{ji} - \gamma_j \delta_{ji}; & j=2, \dots, m-1 \text{ and } i=1, 2, \dots, \\ \frac{A_{mi}}{h} - \frac{A_{1i}}{h}; & j=m, i=1, 2, \dots, m \\ a_5 \delta_{1i} - \frac{a_6}{h} A_{mi}; & j=d, i=1, 2, \dots, m \end{cases} \quad (38)$$

After combining all the entries of m_{ji} , matrix $\mathbf{M}$ appears to be a tri-diagonal positive definite matrix.

In this work, the theoretical effects of different parameters on exit and average solute concentration profiles have been discussed. To discretize the model, axial domain is divided into 5 elements with 3 interior collocation points in each element. The results obtained from the model are presented in the form of plane and surface plots. Key parameters of the industry are used directly or indirectly in the development of the model.

7.Verification of Model 1 by Literature Data

Verification of the mathematical model has been pursued by using the data from literature. For this purpose the data of Arora *et. al.* (2006b) based on lab scale pilot plant has been used.

They have followed the models for negligible intra fiber diffusion coefficient and constant particle radius.

To validate the model under Indian conditions, the data of Arora *et. al.* (2006b) presented in Table 5.1 has been used. The data was collected from a lab scale pilot plant using Indian wood and non-wood pulp fibers comprising of Eucalyptus and Bamboo.

7.1 Effect of Peclet number (Pe)

Peclet number (Pe) is an important parameter in the study of an axial dispersion model. It is the ratio of advection to dispersion, numerically ranges between perfect mixing($Pe \rightarrow 0$) to perfect displacement($Pe \rightarrow \infty$). Higher the Peclet number, lower will be the axial dispersion coefficient resulting in less back mixing and therefore more leaching of impurities adsorbed on particle surface occur. It will ultimately helps in efficient washing operations. In Figure 1, effect of Peclet number on exit solute concentration is shown for small values of Peclet number. It is observed from this figure, that there is large difference between concentration profile for $Pe=5$ and $Pe=10$. The solution profile for Peclet number, $Pe=5$ converges, but taking more time more than $t=3$, but as the value of Peclet number increases from $Pe=5$ to $Pe=10$, it converges at $t=3$ and for $Pe=15$, it converges more rapidly at $t=2.5$. In Figure 3, the effect of Peclet number is shown for higher values of Peclet number. It is observed from this figure that solution profiles converge most rapidly taking less time for large values of Peclet number. There observed difference in concentration profiles for range of Peclet number $Pe=20$ to 50 , after $Pe=50$ there observed negligible difference in solution profiles. For $Pe=50$ and $Pe=100$ curves overlaps each other after $t=1.5$ and converges at same time i.e. $t=2$. Thus after $Pe=50$, solution profiles are not appreciably affected by increasing value of Peclet number. The reason behind this is that, with the increase in value of Peclet number, axial dispersion coefficient decreases which result in less back mixing, which ultimately decreases the washing time. Also for small value of Peclet number impurities remains attached to the fiber pores due to their less dispersion results in poor washing. From above discussion, it is concluded that value of Peclet number should be selected large for efficient washing in smaller time. Figure 2 and Figure 4, shows the effect of Peclet number on average solute concentration for small and large values of Peclet number, respectively. From Figure 2, it is

clear that for small values of Peclet number, there is more difference observed in solution profiles. In Figure 4, the curves behave same for time less than 1, and after time $t=1$ difference is observed in solution profiles. It is observed that convergence time decreases on increasing the value of Peclet number. Thus efficient washing is achieved for high value of Peclet number as faster breakthrough curves are obtained at higher Pe , indicates rapid flushing of impurities from packed bed which increases efficiency of washing.

7.2 Effect of Distribution Ratio (ψ)

The distribution ratio (ψ) is very important factor in study of washing process. It is defined as ratio of u/L and $\frac{D_F}{R^2}$. Clearly by studying the effect of ψ on solution profiles, the effect of interstitial velocity (u), cake thickness (L), intrafiber diffusion coefficient (D_F) and pore radius of particles (R) can be observed very effectively. Higher value of ψ , results in more diffusion which increases the swelling in pores, which ultimately improves efficiency of washing. From Figure 5, the effect of ψ is observed on exit solute concentration profiles for $Pe=10$, $Bi=0.5$ and $\epsilon=0.986$. It is observed that for higher values of ψ solution profiles converge rapidly. It is due to the reason that interstitial velocity (u) increases for higher values of ψ , which results in downfall in washing time, which increases efficiency of washing. In Figure 6, the effect of ψ can be observed on average solute concentration profiles.

7.3 Effect of bed porosity (ϵ)

Bed porosity (ϵ) is defined as the ratio of void volume to the total volume of the bed. For the fluid flow, small change in porosity leads to a big effective change in pressure drop. Thus bed porosity has very important and sensitive effect on washing process. Pellett (1964) and Arora *et. al.* (2006) discussed that by increase in bed porosity results in more diffusion in pores which increases the efficiency of washing. Figure 7 and Figure 8, depicts the effect of ϵ on solution profiles for $Pe=10$, $Bi=0.5$ and $\psi=0.5$. From Figure 7, it is observed that as the value of porosity increases, the concentration profiles goes on converging to steady state rapidly. Thus higher the value of porosity results in better efficiency of washing as by increase in porosity, solute flows slowly which increases the diffusion of pores which results in better

washing in small time. Figure 8, depicts the effect of ε on average solute concentration profiles and observed same effect as on exit solute concentration.

8 Correlation with Industrial Parameters

A mathematical model is of significance, if it can be implemented in the industry. The model proposed in the present study is also correlated with the efficiency parameters followed in industry. In this section, these efficiency parameters are measured from the model using average solute concentration. The effect of Peclet number(Pe), bed porosity(ε) and distribution ratio(ψ) is checked on these parameters.

8.1 Displacement Ratio

Displacement ratio at any stage is the ratio of the actual possible reduction in the dissolved solids to the maximum possible reduction. Kukreja *et. al.* (1995), Arora *et. al.* (2006), Arora & Potůček (2012) have calculated the displacement ratio using the following formula:

$$DR = \frac{C_0 - c_{av}}{C_0 - C_s} \quad (39)$$

The effect of sorption is ignored in the displacement ratio. Displacement ratio depends on average solute concentration on time and therefore, the displacement ratio is also dependent on time. Breakthrough curves are plotted for displacement ratio with respect to time. As time increases, c_{av} approaches to the concentration of wash liquor and actual reduction in solids approaches to the maximum possible reduction. It signifies that displacement ratio is the maximum when dimensionless average solute concentration is zero. Displacement ratio also depends upon the nature of the substance to be removed. In this case, sodium based solutes are used and therefore, it is calculated for sodium. Figure 9, depicts the effect of different parameters on displacement ratio. It is observed that at higher value of Peclet number, the curves converges to 1 rapidly, than small value of Peclet number. This shows that at higher Peclet number(Pe) solutes displaces rapidly. It is observed that breakthrough curves overlaps each other upto time $t=1$, and after time $t=1$, the difference in curves is observed clearly as for higher value of respective parameters solution profiles converges towards 1 rapidly. Figure

11, explains the behaviour of displacement ratio (DR) with respect to time and average solute concentration via surface plots for different values of parameters. This is observed from these plots that for different value of parameters displacement Ratio and average solute concentration lies between 0 and 1. This approves the applicability of the proposed model for efficient washing.

8.2 % efficiency

% efficiency is the key factor to check the efficiency of the equipment. It signifies the percentage of black liquor solids removed during the washing operation. Higher the % efficiency, higher will be the % of solutes removed and therefore, higher will be the efficiency of the equipment. The % efficiency can be calculated using the following formula (Kukreja *et. al.* (1995) and Arora *et. al.* (2006b)):

$$\% \text{efficiency} = \left(1 - \frac{(c_{av} - C_s)(100 - C_{yd})}{(C_0 - C_s)(100 - C_{yi})} \right) 100 \quad (40)$$

Figure 10, depicts the joint effect of Pe, ψ, ε on solution profiles for % efficiency. It has been observed that, maximum efficiency of the equipment is obtained for high value of Peclet number and bed porosity. Figure 12, explains the behaviour of % efficiency with respect to time and average solute concentration via surface plots for different values of parameters. This is observed from these plots that for different value of parameters % efficiency rapidly approaches to 100 and average solute concentration lies between 0 and 1. This approves the applicability of the proposed model for efficient washing.

Table 1: Experimental data of washing cell

Parameter	Unit	Range
Pe	-	7.921-10.651
ε	-	0.6075-0.6314

U	m/s	$(0.1300-0.2071) \times 10^{-3}$
Consistency	%	11.737-12.303
D_L	m^2/s	$(0.611-1.145) \times 10^{-6}$
β	-	0.7246
L	m	30×10^{-3}
R	m	$(10-25) \times 10^{-9}$

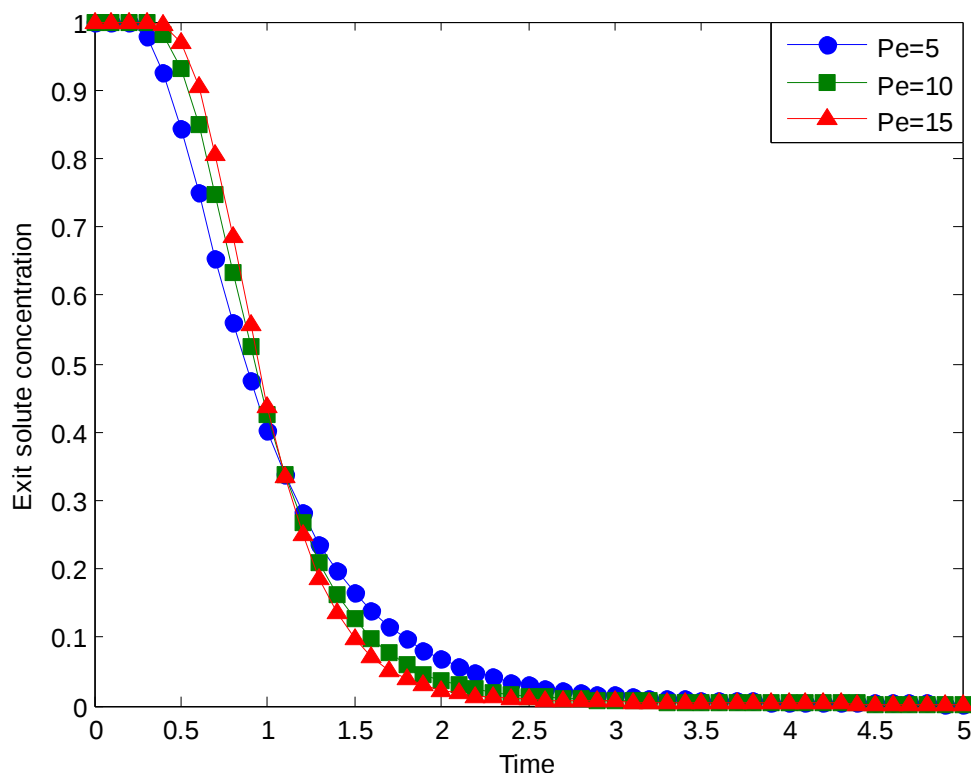


Figure 1:- Behaviour of solution profiles for small values of Peclet number.

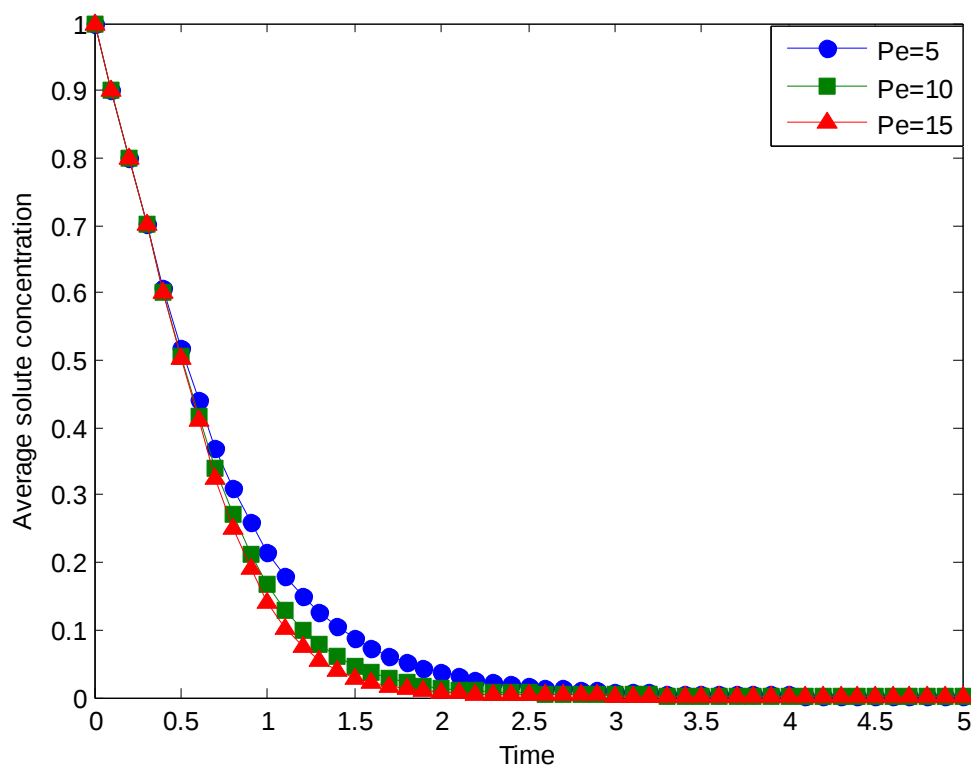


Figure 2:- Behaviour of average solute concentration for small values of Peclet number.

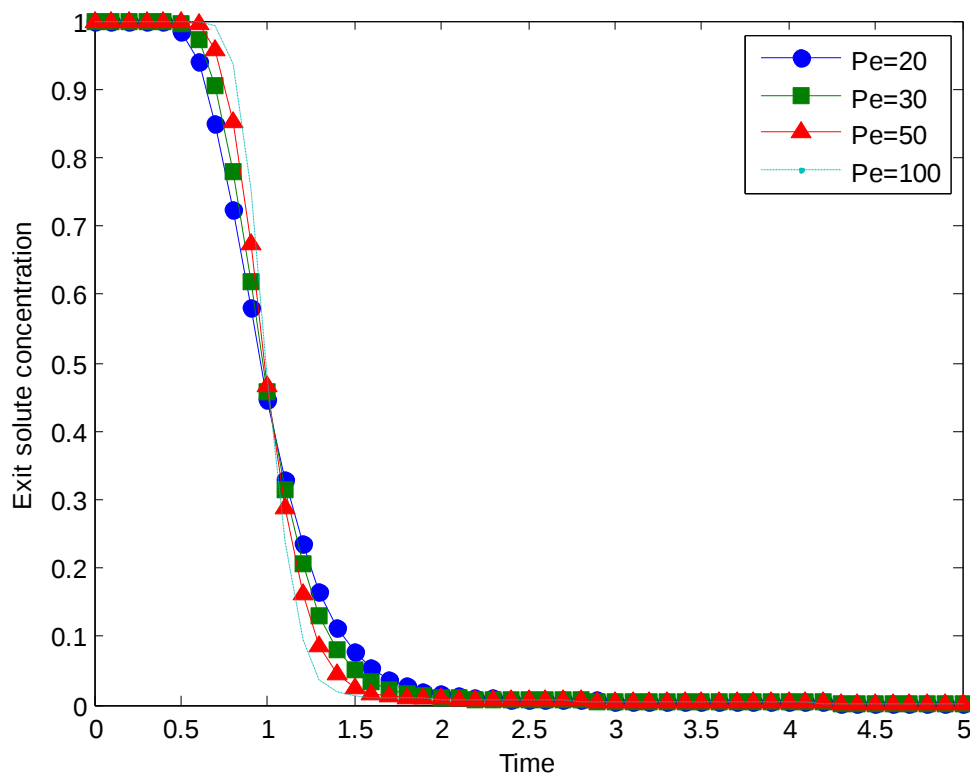


Figure 3:- Behaviour of solution profiles for large values of Peclet number.

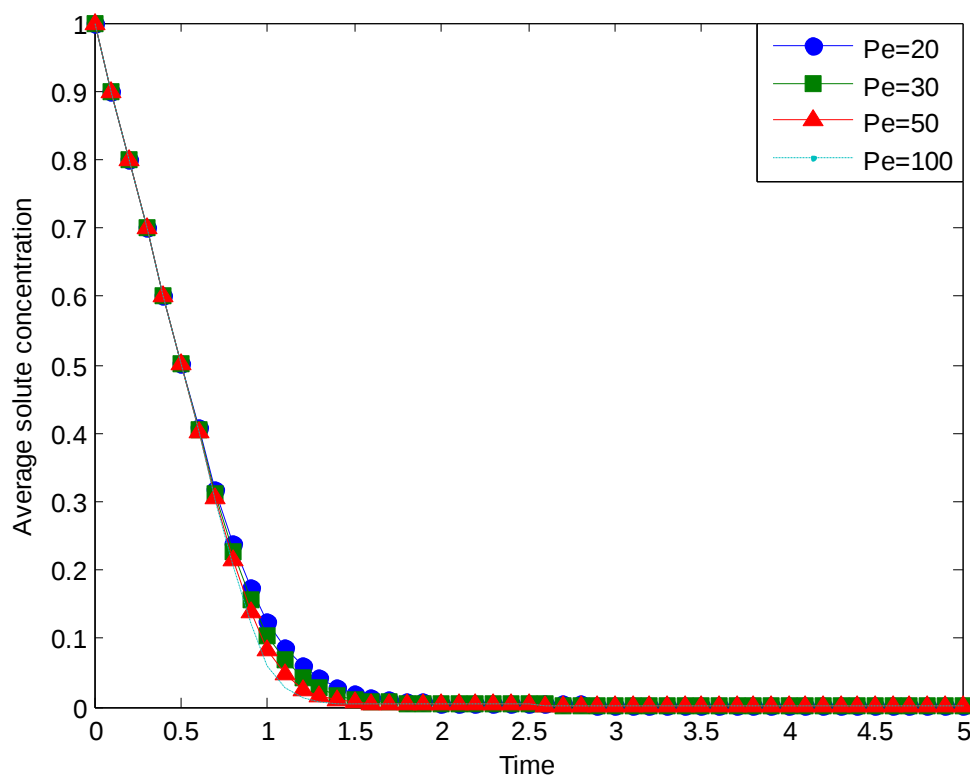


Figure 4:- Behaviour of average solute concentration for large values of Peclet number.

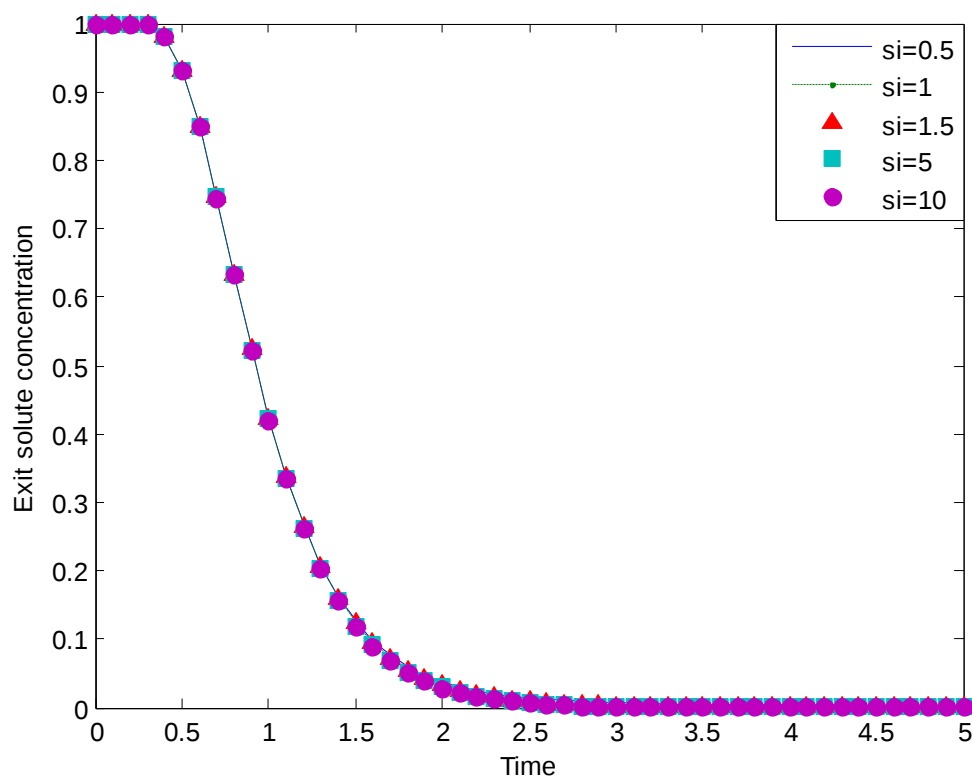


Figure 5:- Behaviour of exit solute concentration for different values of $si(\psi)$.

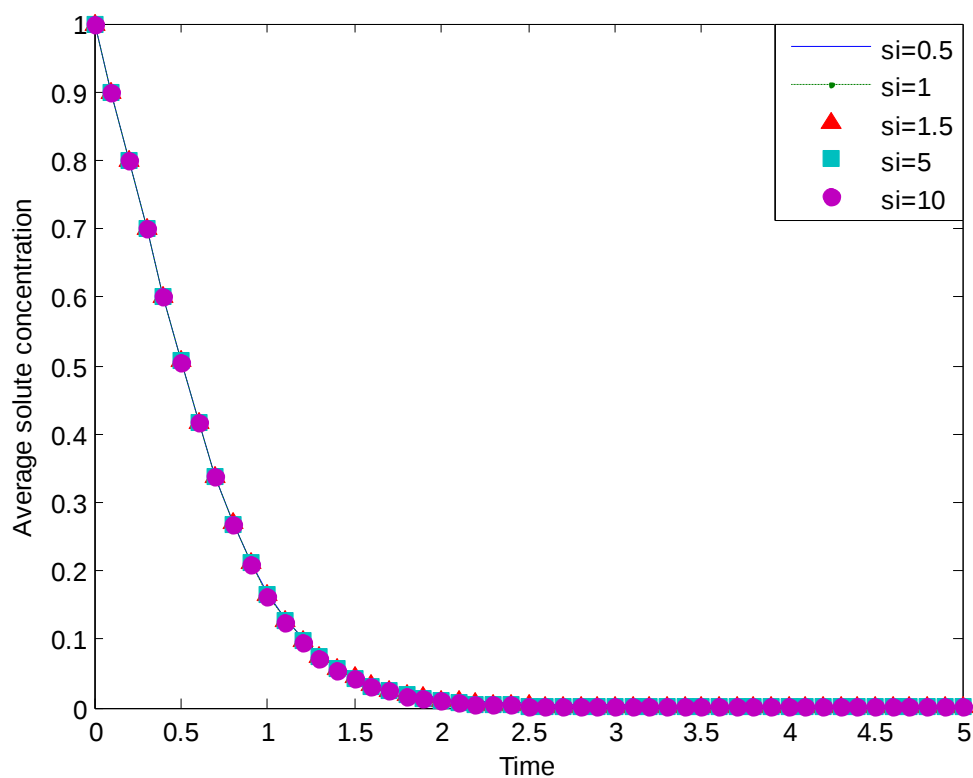


Figure 6:- Behaviour of average solute concentration for different values of $si(\psi)$.

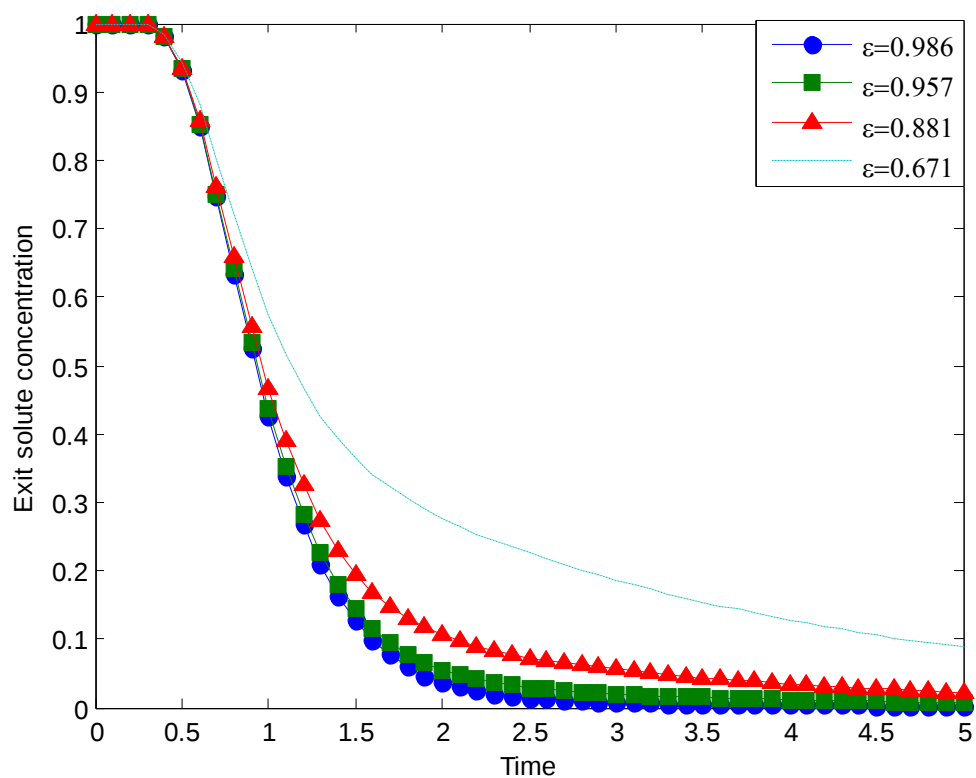


Figure 7:- Behaviour of exit solute concentration for different values of ϵ .

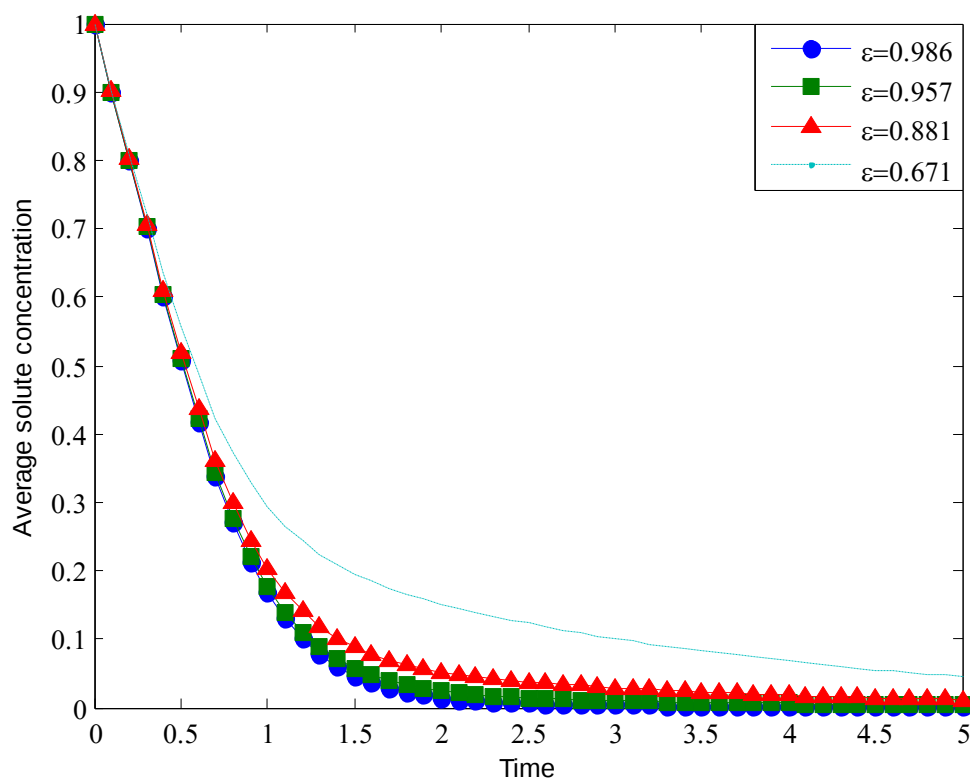


Figure 8:- Behaviour of average solute concentration for different values of ϵ .

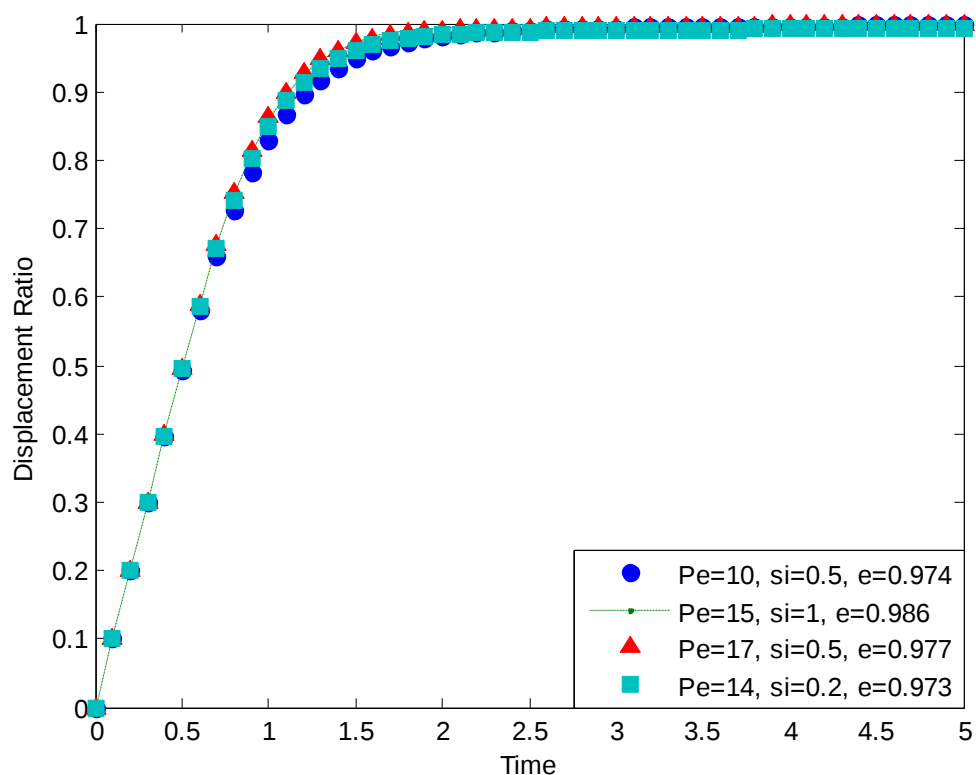


Figure 9:- Behaviour of displacement ratio.

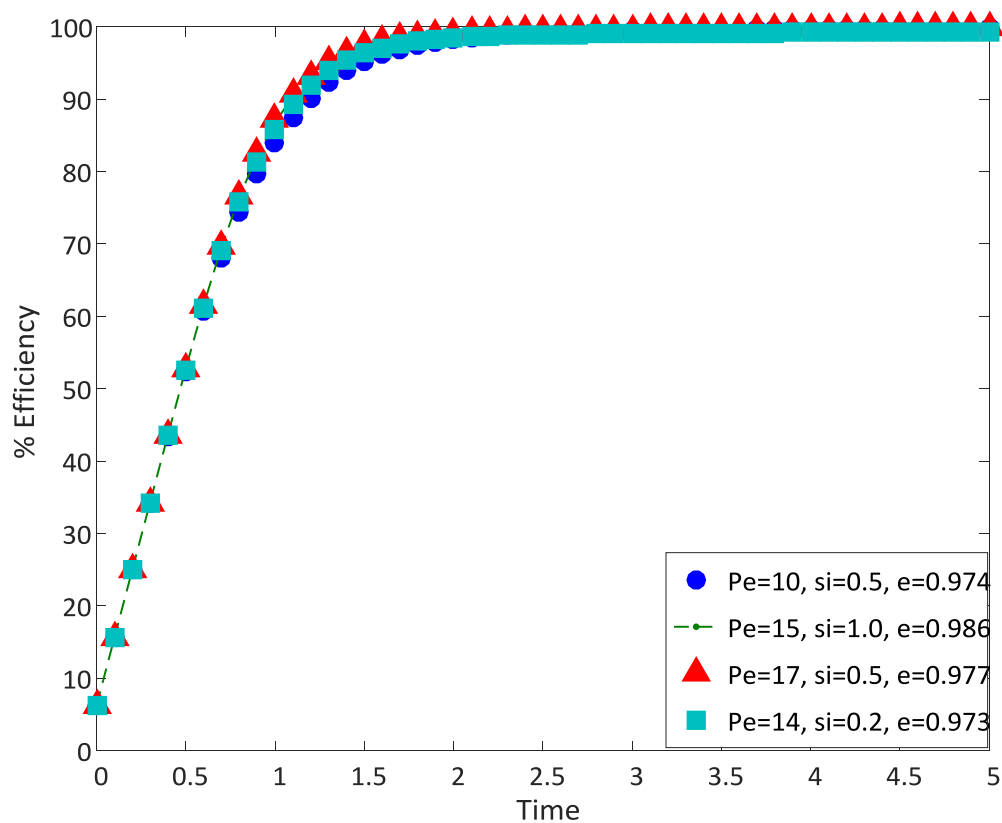


Figure 10:- Behaviour of % efficiency.

Figure 11:- Surface profiles for displacement ratio for different parameters.

Figure 12:- Surface profiles for % efficiency for different parameters.

9. CONCLUSION

Two-dimensional axial dispersion model describing washing zone of rotary vacuum washer has been solved using orthogonal collocation on finite elements. The effect of various parameters such as *Bed Porosity* (ϵ), *Interstitial velocity* (u) and *axial dispersion coefficient* (D_L) has been discussed. It is concluded that not only Peclet number which effects the convergence of solution profiles but other parameters has a vital effect on washing process. It is concluded that the smooth and efficient washing takes place for small values of u and D_L and for large values of bed porosity (ϵ), Peclet number (Pe) and Distribution ratio (ψ). Efficiency of the model and numerical technique has been checked via several 2D and 3D plots for different values of parameters.

This is observed from these plots that for different value of parameters % efficiency rapidly approaches to 100, where average solute concentration and displacement ratio lies between 0 and 1. This approves the applicability of the proposed model for efficient washing.

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