
Mathematical study of MHD free convective Casson hybrid nano fluids convective flow through porous medium between two vertical plates with chemical reaction

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Abstract: The numerical investigation of Newtonian heating effect on unsteady MHD free convective flow of radiating and chemically reacting Casson hybrid nanofluid past an infinite oscillating vertical plate embedded in a porous medium is conducted by considering the effects of heat sink and viscous dissipation. The fluid motion is persuaded due to the periodic oscillations of the plate along its length. The effects of several pertinent parameters on the velocity, temperature and concentration are displayed graphically whilst the effects of these parameters on the skin friction, Nusselt and Sherwood numbers are exhibited in tabular format and then discussed in detail. The velocity distribution is enhanced with increasing in permeability parameter, Newtonian heating parameter and thermal Grashof number whereas it is reduced with increasing Hartmann number, heat absorption parameter and Casson fluid parameter. Heat transport participates the significant role during the managing and dispensation of Casson fluids. This is of immense significance in numerous engineering applications, they are, transpiration cooling, termite welding, drag reduction as

well as heat recovery of oils. In addition, this plays the enlargement of bio-convection prototypes produced by the populations of gravitactic microorganism in the porous media.

Key words: MHD, Casson fluid, Porous medium, chemical reaction.

1. Introduction

The recent development in technologies requires the pioneering revolution in the domain of temperature transportation. In spite of the difficulties of non-Newtonian fluids, practical mathematicians and engineering scholars are affiliated in non-Newtonian fluid dynamics since, the flow and heat transportation characteristics of these fluids is an important to plentiful and various systems in bio-technology, pharmacy and chemical engineering, etc. In extensively, non-Newtonian models have non-linear relationship flanked by stress and rate of strain. The mechanical characteristics of non-Newtonian fluids, both thin and/or shear thickness, normal stress differences, and visco-elastic reactions, may not be depicted with the conservation theory; therefore, an innovative and effective prediction is requisite. Casson fluid model became non-Newtonian fluid to the explorers since those of wide-ranging applications in the range of bio-medical and industrial engineering, energy construction, geo-physical fluid mechanics as well as dynamics. Among these abundant non-Newtonian model, Casson fluid model is the foremost significantly rheological model and it was primarily developed with Casson [1]. Casson fluid is the shear thin fluid practiced as one kind of non-Newtonian fluid it displayed yield stress. If the lesser amount of shear stress than the yield stresses is supplied, therefore, the fluids perform like solids, i.e., no flows happened and it gone if apply shear stresses are higher to the yield stresses (Ghosh and Mukhopadhyay [2]). In the shearing thick fluid model, this is presumed to had an unlimited viscosity near vanishes rate of shearing, a yield stresses below which is no flow takes place and a vanish viscosities at an unbounded pace of shearing stresses. Tomatos sauces, Jellies, honey, human blood, soup, biological liquids, etc. were the quantities of recurrent models of Casson fluids. The virtually investigation of Casson fluid through homogeneous and heterogeneous reactions had been explored by Khan et al. [3]. Time reliant MHD Casson fluid liberated flows over a fluctuating perpendicular plate embedded by an absorbent media is discussed by Khalid et al. [4] utilizing Laplace transformations technique. These outcomes shined illumination on decreased speed as well as increased the skin friction on the plate surface by increasing magnetic field parameter. Rauf et al. [5] examined the boundary stratum flows of Casson liquid on the sheet lengthening in two ways depiction to crosswise magnetic domain as well as thermal radiating by combined convective conditions utilizing Range Kutta

Fehlberg method. This is obtained that the systems increased thermally by the higher radiation. Zaib et al. [6] discussed on Casson fluid with consequences of gelatinous dissipation moving on an exponential shrinking sheet. The mathematical systems are evaluated computationally utilizing shooting technique as well as dual resolutions are attained for the velocity along with temperature distribution. This analysis depicted a reduced in systems temperature distributions by an increasing Prandtl quantity where as the system temperature growing up for increased consequence of gelatinous dissipation. Raju et al. [7] discussed the comparison to Casson fluid by the Newtonian fluid past an exponential stretching surface under the effects of thermal radiating, gelatinous dissipation as well as magnetic field.

The MHD Casson fluid is flowing over a wedge by the influences of binary chemical reaction as well as commencement energy are investigated by Zaib et al. [8]. Through the utilization of adapted Arrhenius functions to signify the establishment energy the modeling are improved and evaluated. The MHD combined convective Casson fluid flow through the consequences of in two stratifications in addition to temperature immersions is discussed by Rehman et al. [9]. The Casson fluid is a shear thinning liquid by the three assumptions: near zero shear rates the viscosities are infinite, no fluid flow survives lower than the yield stresses in addition to, near an unlimited shearing the viscosities are ignorable as elucidated by Reddy et al. [10]. Hamid et al. [11] discussed the stretching sheet enveloped by the MHD Casson liquid manipulated by linear performing thermal radiating. Twofold solutions are evaluated and it is shown that together higher as well as lower profiles of the temperatures are an growing function of radiating. Heat transportation paces are investigated for free convective stream of Casson liquid moving by a moderately heat trapezoidal cavity utilizing Galerkins finite elements technique by Hamid et al. [12]. Das et al. [13] discussed the Newtonian heating effect on unsteady MHD Casson fluid movement past a smooth plate with temperature as well as accumulation transportation. Krishna et al. [14] investigated unsteady radiative MHD flow of Casson hybrid nanofluid over an infinite exponentially accelerated vertical porous surface. Krishna [15] explored the radiative MHD flow of an incompressible viscous electrically conducting non-Newtonian Casson hybrid nanofluid over an exponentially accelerated vertical porous surface. Krishna [16] discussed the analytical study of chemical reaction, Soret, Hall and ion slip effects on MHD flow past an infinite rotating vertical porous plate. Krishna [17] investigated the radiation absorption, chemical reaction; Hall and ion slip impacts on MHD free convection flow past a semi-infinite moving porous surface.

The effects of Hall and ion slip on the radiative MHD rotating flow of viscous incompressible electrically conducting Jeffrey fluid over an infinite vertical flat porous surface by the ramped wall velocity and temperature, and isothermal plate have been explored by Krishna [18]. From those explorations, the conditions at the wall surface are commonly ramped, constant or variable wall temperature. However, there were several problems of physical interest where the heat is transported to the fluid via a bounding surface with a finite heat capacity. In this case, the above thermal boundary conditions fail to work; consequently, the Newtonian heating conditions are highly required. The applications of Newtonian heating are significant in heat exchangers, petroleum industry, turbines, and also in convective flows set up when the bounding surface absorbs heat by solar radiation. Merkin [19] initially established the Newtonian heating conditions in free convective boundary layered flows past an infinite perpendicular surface. Lesnic et al. [20,21] explored the free/forced convective frontier layered flow along the upright as well as parallel surfaces embedded in an absorbent media produced by the Newtonian heating. Chaudhary in addition to Jain [22] explored tremulous natural convective frontier layered flow over an impetuously commenced upright plate by the Newtonian heating. Salleh et al. [23] discussed the free/forced convective boundary layered flow at the forwarded stagnation points of an unbounded plane walls produced with the Newtonian heating. Salleh et al. [24] explored the impacts of Newtonian heating on the frontier layered flows as well as temperature transport past a stretching sheet. Hussanan et al. [25] executed an exact solution of warmth in addition to mass transport flow over an unbounded upright plate in the occurrence of Newtonian heating. Hussanan et al. [26,27] estimated the influences of Newtonian heating on tremulous frontier layered flow with non-Newtonian as well as Newtonian liquids during the spongy media. The tremulous MHD innate convective flow over an impetuously stirring perpendicular plate by the Newtonian heating in a revolving frame has been explored by Seth et al. [28]. More recently, Chand et al. [29] discussed the thermal instability in a low Prandtl number nanofluid layer in a porous medium through planar channel. Ali et al. [30] explored the finite element analysis on the MHD rotating flow of Oldroyd-B nanofluid over a stretching sheet by the effects of Dufour as well as Soret with double diffusion Cattaneo Christov heat flux model. Kareem et al. [31] investigated the mixed convective heat transfer of nanofluids in a lid-driven trapezoidal cavity by the numerical methodology. The MHD mixed convection in an inclined cavity containing adiabatic obstacle and filled with Cu-water nanofluid in the presence of the heat generation and partial slip has been investigated by Ahmed et al. [32]. Ali et al. [33] explored the MHD mixed convection due to a rotating

circular cylinder in a trapezoidal enclosure filled with a nanofluid saturated with a porous media.

2. Mathematical formulation of the problem

"We considered the unsteady MHD free convective oscillating flow of an optically thin incompressible viscous fluid embedded in two parallel porous walls under the influence of an external applied transverse magnetic field with chemical reaction, radiative heat flux, and heat absorption. The physical modeling of the investigation is displayed in Figure I. This is assumed that the fluid has tiny electrical conductivity and the electromagnetic forces created and is too small, so that it is neglected the induced magnetic field.

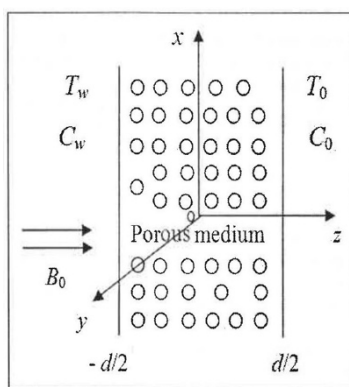


Figure.I. Physical configuration of the problem.

$$\begin{aligned}
 \frac{\partial w}{\partial z} &= 0, \\
 \frac{\partial u}{\partial t} + w \frac{\partial u}{\partial z} &= -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial z^2} - \frac{\sigma B_0^2 u}{\rho} - \frac{\nu}{k} u \\
 &\quad + g\beta(T - T_0) + g\beta^*(C - C_0) \\
 \frac{\partial v}{\partial t} + w \frac{\partial v}{\partial z} &= \nu \frac{\partial^2 v}{\partial z^2} - \frac{\sigma B_0^2 v}{\rho} - \frac{\nu}{k} v \\
 \frac{\partial T}{\partial t} + w \frac{\partial T}{\partial z} &= \frac{k_1}{\rho C_p} \frac{\partial^2 T}{\partial z^2} + \frac{Q_0 T}{\rho C_p} - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial z} \\
 \frac{\partial C}{\partial t} + w \frac{\partial C}{\partial z} &= D \frac{\partial^2 C}{\partial z^2} - \text{Kr}C
 \end{aligned} \tag{1-5}$$

The initial and boundary conditions are

$$\begin{aligned}
 u = 0, v = 0, T = 0, C = 0 &\text{ for all } z \text{ and } t \leq 0 \\
 u = 0, v = 0, T = 0, C = 0 &\text{ at } z = -\frac{d}{2} \\
 u = 0, v = 0, T = T_0 + (T_w - T_0) \cos \omega t, C &= C_0 + (C_w - C_0) \cos \omega t \text{ at } z = \frac{d}{2}
 \end{aligned} \tag{6-9}$$

Combining the equations (2) - (3), let $q = u + iv$, it is achieved as,

$$\frac{\partial q}{\partial t} + w \frac{\partial q}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 q}{\partial z^2} - \left(\frac{\sigma B_0^2}{\rho} + \frac{\nu}{k} \right) q + g\beta (T - T_0) + g\beta^* (C - C_0) \quad (10)$$

Consequently, this is contained that, the fluid is optically thick by comparatively minute density. The radiative heat flux is given by

$$\frac{\partial q_r}{\partial z} = 4\alpha(T - T_0) \quad (11)$$

Where α is the radiating absorption coefficient.

The periodical pressure gradient as,

$$-\frac{1}{\rho} \frac{\partial p}{\partial x} = P \cos \omega t \quad (12)$$

Integrating the equation of continuity (1), it is obtained as, $w' = w_0$

We are introducing the non-dimensional variables as

$$\begin{aligned} x^* &= \frac{x}{d}, z^* = \frac{z}{d}, \xi^* = \frac{\xi}{d}, q^* = \frac{q}{w_0}, \theta = \frac{T - T_0}{T_w - T_0}, \phi \\ &= \frac{C - C_0}{C_w - C_0}, t^* = \omega t, \omega^* = \frac{\omega d^2}{\nu}, p^* = \frac{p}{\rho w_0^2}. \end{aligned} \quad (13)$$

Substituting the non-dimensional variables in the equations (4), (5), and (9), we get

$$\begin{aligned} \omega \frac{\partial q}{\partial t} + \lambda \frac{\partial q}{\partial z} &= -\lambda \frac{\partial p}{\partial x} + \frac{\partial^2 q}{\partial z^2} - \left(M^2 + \frac{1}{K} \right) q \\ &\quad + \text{Gr } \theta + \text{Gm } \phi \\ \omega \frac{\partial \theta}{\partial t} + \lambda \frac{\partial \theta}{\partial z} &= \frac{1}{\text{Pr}} \frac{\partial^2 \theta}{\partial z^2} - \frac{1}{\text{Pr}} (N^2 - Q) \theta \\ \omega \frac{\partial \phi}{\partial t} + \lambda \frac{\partial \phi}{\partial z} &= \frac{1}{\text{Sc}} \frac{\partial^2 \phi}{\partial z^2} - \text{Kc } \phi \end{aligned} \quad (14-15)$$

The initial and boundary conditions are,

$$u = 0, v = 0, \theta = 0, \phi = 0 \quad \text{for all } z \text{ and } t \leq 0 \quad (16)$$

$$u = 0, v = 0, \theta = 0, \phi = 0, \text{ for } t > 0, \quad \text{at}$$

$$z = -\frac{1}{2}$$

$$u = 0, v = 0, \theta = \cos t, \phi = \cos t, \text{ for } t > 0$$

$$\text{at } z = \frac{1}{2} \quad (17-18)$$

"Where, $M^2 = \frac{\sigma B_0^2 d^2}{\mu}$ Hartmann number, $K = \frac{k}{d^2}$ permeability parameter, $\lambda = \frac{w_0 d}{\nu}$ suction parameter, $\text{Gr} = \frac{g\beta d^2 (T - T_0)}{\nu w_0}$ thermal Grashofs number, $\text{Gm} = \frac{g\beta^* d^2 (C - C_0)}{\nu w_0}$ mass Grashofs

number, $Kc = \frac{\alpha^2 Kr}{\nu}$ chemical reaction parameter, $Sc = \frac{\nu}{D}$ Schmidt number, $Q = \frac{Q_0 \alpha^2}{k_1}$ heat absorption parameter, $Pr = \frac{\mu c_p}{k_1}$ Prandtl number, $N = \frac{2\alpha d}{\sqrt{k_1}}$ radiation parameter." The system of Eqns. (13) to (15) together with initially along with frontier conditions (16)-(18) are resolved by utilizing Laplace Transform technique. The transformed solutions are

$$\begin{aligned}\bar{q} &= a_{25}e^{m_5 s} + a_{24}e^{m_6 s} + a_{13} + a_{14}e^{m_3 s} \\ &+ a_{15}e^{m_4 s} + a_{16}e^{m_1 s} + a_{17}e^{m_1 s} \\ \bar{\theta} &= a_9 \left(\frac{s}{s^2 + 1} \right) e^{m_3 s} + a_{10} \left(\frac{s}{s^2 + 1} \right) e^{m_4 s} \\ \bar{\phi} &= a_{11} \left(\frac{s}{s^2 + 1} \right) e^{m_1 s} + a_{12} \left(\frac{s}{s^2 + 1} \right) e^{m_1 s}\end{aligned}\quad (19-21)$$

"Implement the inverse Laplace transformations for the equations (19), (20) and (21), happens to unwieldy as of the existence of both the multiple poles and branch points. These integrals are willing to the solutions by means of asymptotic approximations. By making use of the convolution theorem for inverse Laplace transform, those equations are estimated by the numerical integration with the helping of MATHEMATICA program codes. It is obtained the velocity, temperature and concentration distributions respectively. For engineering interest, it is obtained that the skinfriction, Nusselts number and Sherwood number near the left side plate as well as right side plate as,"

$$\tau = \left(\frac{\partial q}{\partial z} \right), \quad Nu = \left(\frac{\partial \theta}{\partial z} \right), \text{ and } Sh = \left(\frac{\partial \phi}{\partial z} \right) \quad (22)$$

3. Discussion of the numerical results

The Fig. 1 exhibited the consequences of the Hartmann number on velocity distribution. It is evidently that, the velocity is reducing with enhancing Hartmann number over the plate throughout the fluid medium. It is shown the propensity to refuse to gone together with the flow field throughout the fluid media. As a result, it is attained that, the application of the magnetic strength in the manifestation of porous materials support the lessening outcome on the thicknesses of the momentums frontier layer. Therefore, It is noticed that as the values of M increase velocity profiles gradually decrease. The the peak value drastically decreases with increase in the magnetic field, because the presence of magnetic field in an electrically conducting fluid introduces a force called the Lorentz force, which acts against the flow if the magnetic field is applied in the normal direction, as in the present problem. This type of resisting force slows down the fluid velocity. Figs. 2 and 3 elucidates the effects of Casson parameter α and phase angle ωt on the velocity distribution.

It is observed that an increase in Casson fluid parameter and/or phase angle $\omega\tau$ tend to decrease in the fluid motion. Physically, an increase in α causes to increase of plasticity of the fluid consequently fluid motion decreases. Fig. 4 represent for a variety of values of temperature absorption parameters Q on velocity. This is apparent that liquid velocity as well as the thicknesses of momentum boundary layer decrease on an enhancing heat absorption parameter Q throughout the fluid region. Fig. 5 demonstrated that, the consequences of porosity parameter K on a velocity distribution. It is perceived that, the velocity delivery is enhancing by an increment in permeability parameters K throughout the liquid environment. This was obviously that, in general, proportional values of K developed the velocity distribution and thus an enlargement in the momentum boundary stratum thickness. An increase in the values of K furnishes reduction in the resistance of the porous medium as a result accelerates the fluid motion. Lower the permeability reasons lesser the liquid velocity is scrutinized within the flow medium engaged by the fluid. It is obvious that the presence of porous medium causes higher restriction to the fluid flow which causes the fluid to decelerate. Therefore, with an increase in permeability parameter causes not struggle to the fluid motion and hence velocity increases. From Fig. 6, the velocity distribution and momentous boundary stratum thickness are diminishes by an enhancement in Schmidt number. As the Schmidt number increases, the velocity decreases. This causes the concentration buoyancy effects to decrease yielding a reduction in the fluid velocity. The reduction in the velocity profiles are accompanied by simultaneous reductions in the momentum boundary layer thicken and the analogous to the effect on the thickness of a concentration boundary layer. The velocity field sketches are reduces for an enhancing in chemical reacting parameters K_c . The parameters pertaining to the chemical reaction parameter play the main role on the velocity and concentration field. The dimensionless velocity distribution for different values of chemical reaction parameter, it is seen from the figure that the velocity of the fluid decreases with increase of chemical reaction parameter. Also, it is observed that the velocity of the fluid decreases uniformly near the wall of the plate. Hence, largest values of K_c lead to reduce the diffusion coefficient of mixture chemical species; it reduced velocity and momentous frontier stratum thickness. From Fig. 7, it is noticed that, the concentrations sketches are reduces for an enhancing in chemically reacting parameters K_c . largest values of K_c led to reduced the diffusions coefficients of mixture chemically species, it reduced concentrations as well as frontier stratum thicknesses. Actually, for a harmful case ($K_c > 0$), reliable chemically reacting acquires position through lot of disturbances.

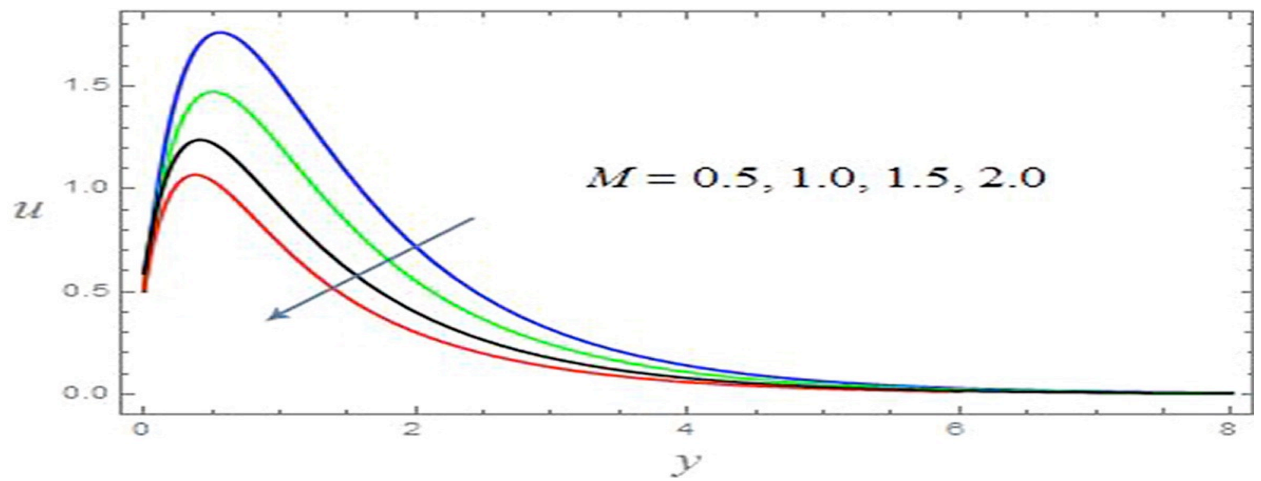


Fig.1. Velocity Profiles against M

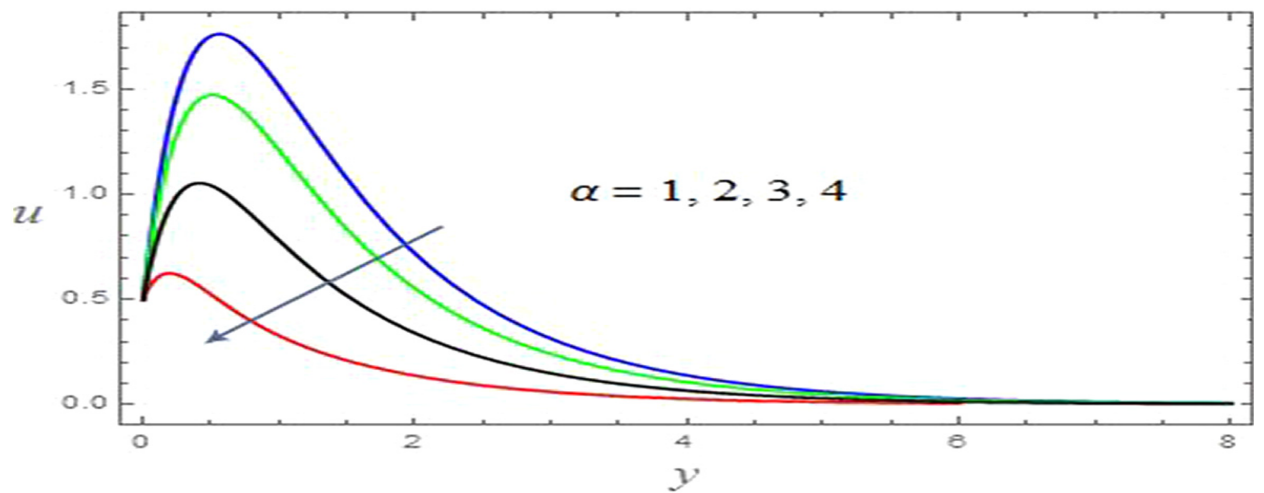


Fig.2. Velocity profiles against α

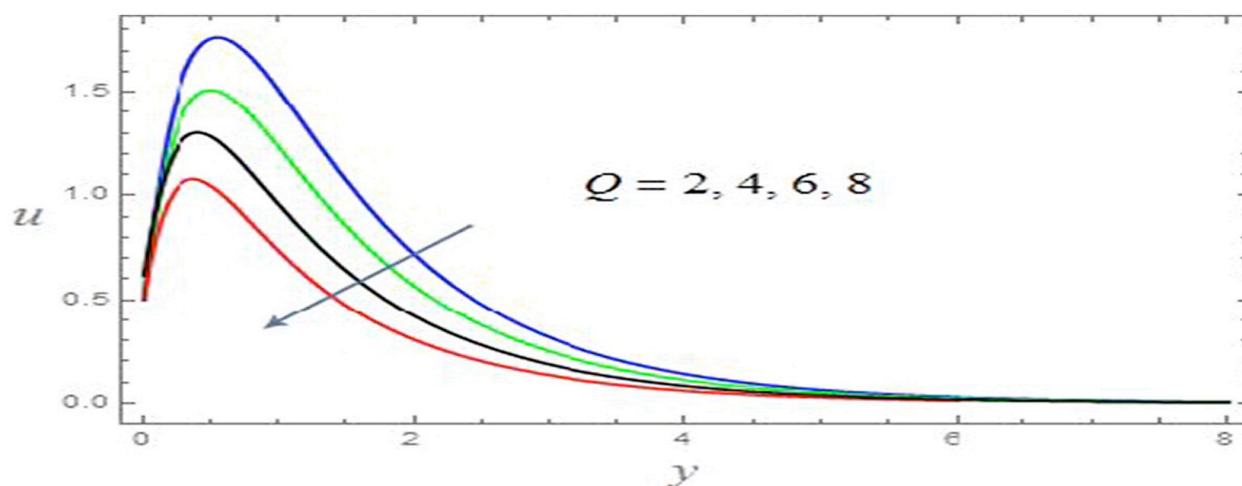


Fig.3.Velocity profiles against Q

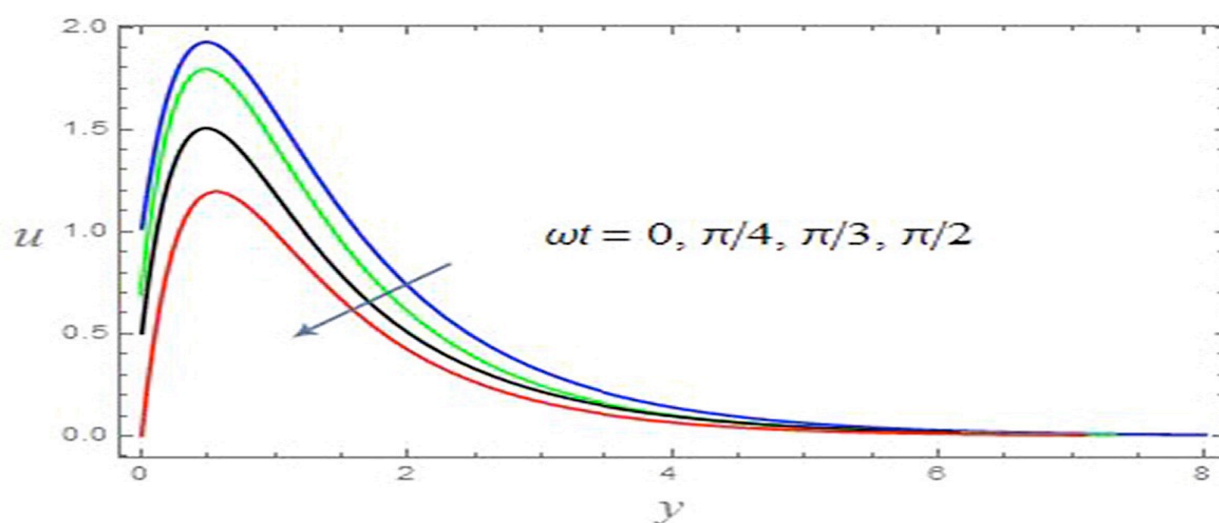


Fig. .4. Velocity profiles against ωt .

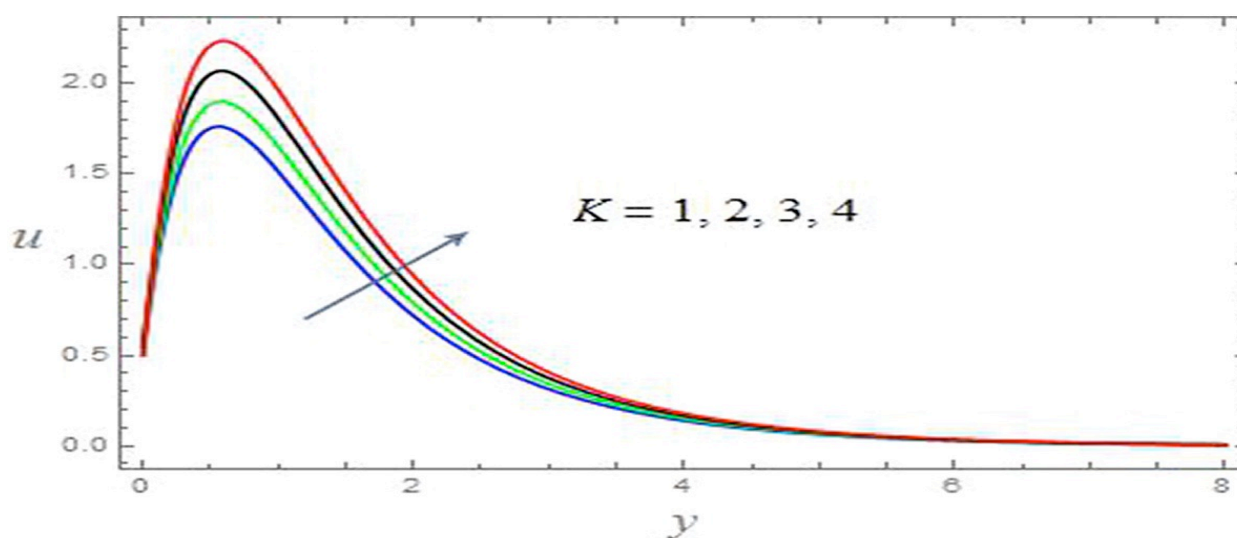


Fig.5 .Velocity profiles against K

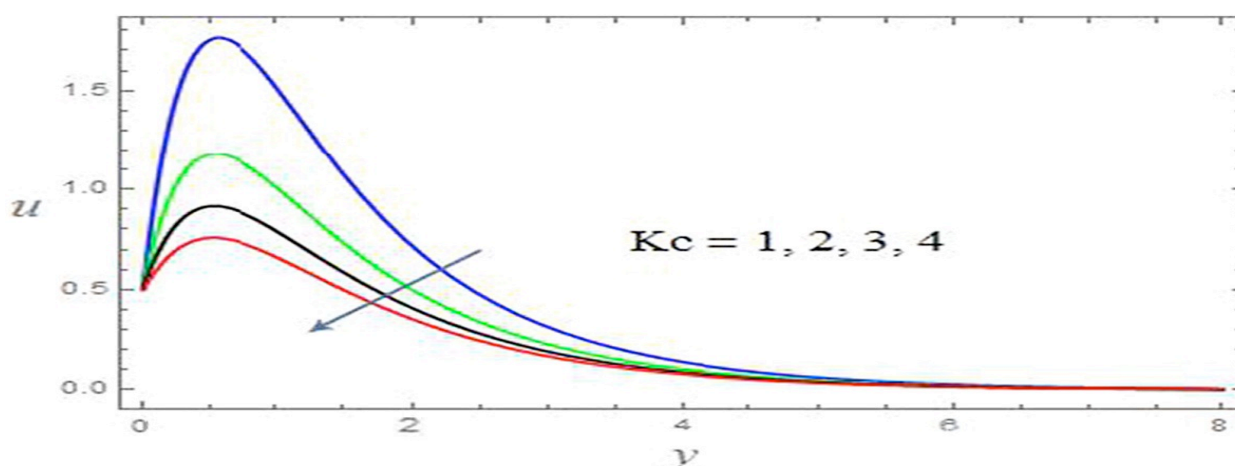


Fig.6. Velocity profiles against K_c

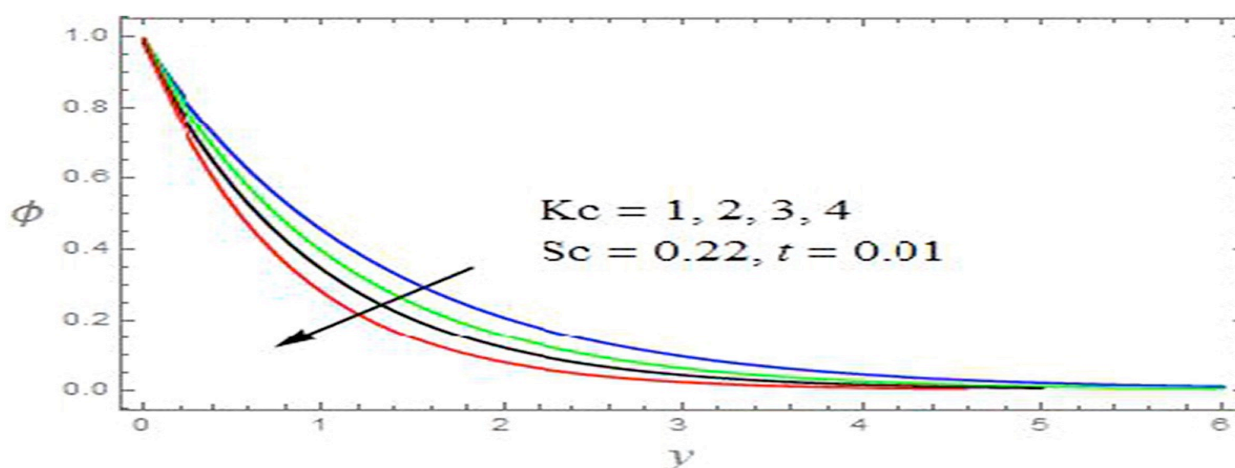


Fig.7. Concentration profiles against K_c

It is considered the numerical study of Newtonian heating effect on unsteady free convection MHD flow of radiating and chemically reacting Casson hybrid nanofluid past an oscillating vertical porous plate embedded in a porous medium. It was conducted by the effects of heat sink and viscous dissipation. Water and ethylene glycol mixture has been taken as a base Casson fluid. First order consistent chemical reaction and heat absorption are also regarded. Silver and Titania nano-particles are disseminated in base fluid water and ethylene glycol combination to be formed hybrid nanofluid. The precise solutions of the leading equations of the stream domain were accomplished by utilizing of the Finite difference procedure. The fluid flow is presided by the indimensional parametrics named as, viz. Hartmanns quantity M , penetrability parameters K , Casson fluid parameter α , Newtonian heating parameters γ , heat Grashofs quantity Gr , mass Grashofs quantity Gm , temperature absorption parameter Q , radiation parameter Nr , chemically reacting parameters

Kr, Schmidt quantity Sc, as well as phase angle ωt . The velocity profiles are executed a distinguishing highest quantity at the surface of the plate and then diminish appropriately on escalating boundary layer coordinate y to reach free stream conditions. Also the values of the velocity components are higher for Ag-WEG than that of Ag-TiO₂/WEG. So that, it is considered velocity, temperature and concentration profiles for Ag-WEG, since similar nature of both profiles of Ag-WEG than that of Ag- TiO₂/WEG. It is also observed that, the velocity is slowly downs. The skin frictions, Nusselt quantity in addition to Sherwood number are also evaluated as well as exhibited in the Tables.1-3.

Table.1 . Sherwood number (Sh).

Kc	Sc	T	Sh
1.0	0.22	0.2	2.091078546
2.0			2.7603859
3.0			3.540385598
	3.0		2.88348548
	7.0		3.672252589
		0.4	1.990388001
		0.6	1.897639855

Table .2 Nusselt number (Nu)

Nr	Q	Ec	Γ	t	Nu
1	2	0.01	0.02	0.02	2.921379661
2					4.006754789
3					5.026890002
	4				2.437145699
	6				2.005855479
		0.02			3.1920963
		0.03			3.4514287
			0.4		3.02645479
			0.6		3.121359662
				0.3	3.133754459
				0.4	3.333487413

Table.3. Skin friction

M	α	Q	ωt	K	γ	Gr	Gm	Nr	Kc	Sc	Cf
0.5	1	2	$\pi/4$	1	0.2	5	3	1	1	0.22	2.978469221
1.0											2.95596489
1.5											2.935945887
	2										2.434954479
	3										1.945746699
		4									2.653330022
		6									2.318700145
			$\pi/3$								2.971599666
			$\pi/2$								2.95739666
				2							3.283665441
				3							3.575714523
					0.4						2.390284475
					0.6						0.879579885
						10					3.425845223
						15					4.309178899
							6				3.419574459
							9				3.946358989
								2			3.213254567
								3			3.40929478
									2		1.922557997
									3		2.76068879
										0.30	2.89355779
										0.60	2.72624779

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