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ON αG*–PRECLOSED SETS IN TOPOLOGICAL SPACES

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Abstract:

The aim of this paper is to introduce the new class of closed sets called αg*-pre-closed sets in topological spaces and some of their properties are investigated. The class of αg*-preclosed sets lies between the class of preclosed sets and the class of g*p-closed sets.

Keywords:g-closed sets;gp-open sets;αg*p-closed sets;

1. Introduction:

Levine [9] introduced and studied the generalized closed (briefly g-closed) sets in 1970. Maki et al [12,13, 14] and Veera Kumar [20] introduced and studied the concepts of generalized α-closed (gα-closed) sets and α-generalized closed (αg-closed) sets, gp-closed sets and αgr-closed sets respectively in topological spaces.

In this paper, we define and study the properties of αg*-preclosed (briefly αg*p-closed) sets in topological spaces which is properly placed between the class of preclosed sets and the class of g*p-closed sets.

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2. Preliminaries:

Throughout this paper, the space (X, τ) (or simply X) always means a topological space on which no separation axioms are assumed unless explicitly stated. For a subset A of a space X, cl(A), int(A) and A^c denote the closure of A, the interior of A and the complement of A in X respectively.

Definition 2.1: A subset A of a topological space (X,τ) is called a

(i) regular open set [17] if A = int(cl(A)) and a regular closed set if A = cl(int(A)).

(ii) α-open set [15] if A $\subseteq$ int(cl(int(A))) and a α-closed set [12] if cl(int(cl(A))) $\subseteq$ A.

(iii) pre-open set [10] if A $\subseteq$ int(cl((A)) and a pre-closed set if cl(int((A)) $\subseteq$ A.

(iv) semi-open set [8] if A $\subseteq$ cl(int(A)) and a semi-closed set [3] if int(cl(A) $\subseteq$ A.

(v) semi-preopen set [2] (=β-open set[1]) if A $\subseteq$ cl(int(cl(A))) and semi-preclosed [2](=β-closed set [1]) if int(cl(int(A))) $\subseteq$ A.

The α-closure of A is the smallest α-closed set containing A. and this is denoted by αcl(A). Similarly the semi-closure (resp. pre-closure and semi-pre-closure) of a set A in a topological space (X, τ) is the intersection of all semi-closed (resp. pre-closed and semi-pre-closed) sets containing A and is denoted by scl(A) (resp. pcl(A) and spcl(A)).

Definition 2.2: A subset A of a topological space (X,τ) is called a

(i) generalized closed (briefly g-closed) [9] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X,τ).

(ii) semi-generalized closed (briefly sg-closed) [4] if scl(A) $\subseteq$ U whenever A$\subseteq$U and U is semi-open in (X,τ).

(iii) generalized-semi closed (briefly gs-closed) [3] if scl(A) $\subseteq$ U whenever A$\subseteq$U and U is open in (X,τ).

(iv) α-generalized closed (briefly αg-closed) [13] if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X,τ).

(v) generalized pre-closed (briefly gp-closed) [14] if pcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X,τ).

(vi) generalized semi-preclosed (briefly gsp-closed) [6] if spcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X,τ).

(vii) generalized pre regular closed (briefly gpr-closed) [7] if pcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is regular- open in (X,τ).

(viii) α-generalized regular-closed (briefly αgr-closed) set [20] if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is regular-open in (X,τ).

(ix) g*p-closed [19] if pcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is g-open in (X,τ).

(x) $g^{\#}\alpha$-closed [16] if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is g-open in (X,τ).

3. αg*-preclosed sets in topological spaces:

In this section we introduce αg*p-closed sets in topological spaces and study some of their properties.

Definition 3.1: A subset A of a topological space (X,τ) is said to be αg*-pre closed (briefly αg*p-closed) set if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is gp-open in (X,τ).

The family of all αg*p-closed sets in a topological space (X,τ) is denoted by αg*pC(X,τ).

Theorem 3.2: Every closed set is αg*p-closed set but not conversely.

Proof: Let A be a closed set in (X, τ). Note that αcl(A) $\subseteq$ cl(A) is always true and cl(A) = A as A is closed. So if A $\subseteq$ G where G is gp-open set in (X, τ). Then αcl(A) $\subseteq$ G. Hence A is αg*p-closed set in (X, τ).

Example 3.3: Let X = {a, b, c} and τ = {X, ϕ, {a}}. Then the subsets {b} and {c} are αg*p-closed sets but not closed sets in (X, τ).

Theorem 3.4: Every αg*p-closed set is g*p-closed set.

Proof: Let A be a αg*p-closed set in (X, τ). Let A $\subseteq$ U, where U is g-open and so it is gp-open set. Then αcl(A) $\subseteq$ U. Note that pcl(A) $\subseteq$ αcl(A) is always true. Therefore pcl(A) $\subseteq$ U. Hence A is g*p-closed set in (X, τ).

The converse of the theorem need not be true as seen from the following example.

Example 3.5: Let X = {a, b, c} and τ = {X, ϕ, {a}, {b}, {a, b}}. Then the subsets {a} and {b} are g*p-closed sets but not αg*p-closed sets in (X, τ).

Theorem 3.6: Every αg*p-closed set is αgr-closed set.

Proof: Let A be a αg*p -closed set in (X,τ). Let G be a regular-open set and so it is gp-open set such that A $\subseteq$ G. As A is αg*p-closed we have αcl(A) $\subseteq$ G. Therefore αcl(A) $\subseteq$ G. Hence A is αgr-closed set in (X, τ).

The converse of the theorem need not be true as seen from the following example.

Example 3.7: In Example 3.3, the subset {a} is αgr-closed set but not a αg*s-closed set in (X, τ).

Theorem 3.8: Every α-closed set is αg*p-closed set.

Proof: Proof is follows from the definitions.

The converse of the theorem need not be true as seen from the following example.

Example 3.9: Let X={a, b, c} and τ = {X, ϕ, {a}, {b, c}}. Then the subset {a, c} is αg*p-closed set but not an α-closed set in (X, τ).

Theorem 3.10: Every αg*p-closed set is gp-closed set but not conversely.

Proof: Let A be a αg*p-closed set in (X, τ). Let A $\subseteq$ U, where U is open and so it is gp-open set. Then αcl(A) $\subseteq$ U. Note that pcl(A) $\subseteq$ αcl(A) is always true. Therefore pcl(A) $\subseteq$ U. Hence A is gp-closed set in (X, τ).

Example 3.11: In Example 3.3, the subsets {a, c} are gp-closed sets but not αg*p-closed sets in (X, τ).

Theorem 3.12: Every αg*p-closed set is gs-closed set but not conversely.

Proof: Let A be a αg*p-closed set in (X, τ). Let A $\subseteq$ U, where U is open and so it is gp-open set. Then αcl(A) $\subseteq$ U. Note that scl(A) $\subseteq$ αcl(A) is always true. Therefore scl(A) $\subseteq$ U. Hence A is gs-closed set in (X, τ).

Example 3.13: In Example 3.3, the subsets {a, c} are gs-closed sets but not αg*p-closed sets in (X, τ).

Theorem 3.12: Every αg*p-closed set is gpr-closed set but not conversely.

Proof: Since every gp-closed set is gpr-closed and by Theorem 3.10, Therefore every αg*p-closed set is gpr-closed in (X, τ).

Example 3.13: In Example 3.3, the subsets {a, c} are gpr-closed sets but not αg*p-closed sets in (X, τ).

Theorem 3.14: Every αg*p-closed set is $g^{\#}\alpha$-closed set but not conversely.

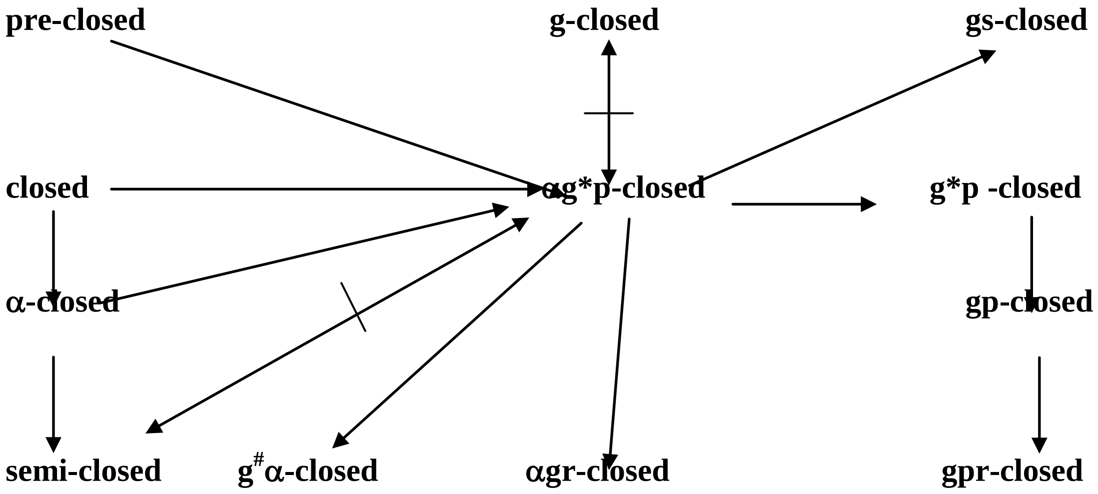
Proof: Let A be a αg*p-closed set in (X, τ). Let G be a g-open set and so it is gp-open set such that A ⊆ G. Then αcl(A) ⊆ G. Therefore αcl(A) ⊆ G. Hence A is $g^{\#}\alpha$ -closed set in (X, τ).

Example 3.15: : Let X={a, b, c} and τ = {X, ϕ, {a}, {a, b}}. Then the subset {a, c} is αg*p-closed set but not an g#α-closed set in (X, τ).

Theorem 3.16: If A and B are αg*p-closed sets, then A∪B is αg*p-closed set in (X, τ).

Proof: If A∪B ⊆ G and G is gp-open. Then A ⊆ G and B ⊆ G. Since A and B are αg*p-closed sets, αcl(A) ⊆ G and αcl(B) ⊆ G and hence G ⊇ αcl(A) ∪ αcl(B) = αcl(A∪ B). Thus A∪B is αg*p-closed set in (X, τ).

Remark 3.17: From the above results we have the following diagram.



Where $A \rightarrow B$ ($A \leftrightarrow B$) represents A implies B but not conversely (A and B are independent).

Theorem 3.18: If A is both open and g-closed, then A is αg^*p-closed.

Proof: A is open and g-closed. Let U be a gp-open set containing A. $A \subseteq A$, an open set. And A is g-closed. Therefore $cl(A) \subseteq A$, $plc(A) \subseteq cl(A) \subseteq A \subseteq U$. Hence $pcl(A) \subseteq U$. Thus every gp-open set U containing A contains pcl(A). Therefore A is αg^*p-closed.

Theorem 3.18: If A is an αg^*p-closed set in (X, τ) if and only if $\alpha cl(A) - A$ contains no non-empty gp-closed set in (X, τ).

Proof: Let F be a gp-closed set contained in $\alpha cl(A) - A$. Then $A^c \subseteq F^c$ and F^c is a gp-open set of (X, τ). Since A is αg^*p-closed set, $\alpha cl(A) \subseteq F^c$. This implies $F \subseteq X - \alpha cl(A)$. Then $F \subseteq (X - \alpha cl(A)) \cap (\alpha cl(A) - A)$. $F \subseteq (X - \alpha cl(A)) \cap \alpha cl(A) = \phi$. Therefore $F = \phi$.

Conversely, suppose that $\alpha cl(A) - A$ contain no non-empty gp-closed set in (X, τ). Let G be a gp-open such that $A \subseteq G$. If $\alpha cl(A) \not\subset G$, then $\alpha cl(A) \cap G^c$ is a non-empty gp-closed set of $\alpha cl(A) - A$. which is a contradiction. Therefore $\alpha cl(A) \subseteq G$ and hence A is an αg^*p-closed set in (X, τ).

Theorem 3.19: If a subset A of a topological space (X, τ) is αg^*p-closed such that $A \subseteq B \subseteq \alpha cl(A)$, then B is also αg^*p-closed.

Proof: Let U be a gp-open set in X such that $B \subseteq U$, then $A \subseteq U$. Since A is αg^*p-closed, $\alpha cl(A) \subseteq U$. By hypothesis, $B \subseteq \alpha cl(A)$ and hence $\alpha cl(B) \subseteq \alpha cl(\alpha cl(A)) = \alpha cl(A) \subseteq U$. Consequently, $\alpha cl(B) \subseteq U$. Therefore B is also αg^*p-closed set in (X, τ).

Theorem 3.20: If A is gp-open and αg^*p-closed set, then A is α-closed set.

Proof: Let $A \subseteq A$, where A is gp-open. Then $\alpha cl(A) \subseteq A$ as A is αg^*p-closed in (X, τ). But $A \subseteq \alpha cl(A)$ is always true. Therefore $A = \alpha cl(A)$. Hence A is α-closed set in (X, τ).

Theorem 3.21: Let $A \subseteq Y \subseteq X$ and suppose that A is αg^*p-closed set in X, then A is αg^*p-closed relative to Y.

Proof: Given that $A \subseteq Y \subseteq X$ and A is αg^*p-closed set in (X, τ). To prove that A is αg^*p-closed relative to Y. Let $A \subseteq Y \cap G$, where G is open and so gp-open in (X, τ). Since A is an αg^*p-

closed set in X, A $\subseteq$ G which implies that αcl(A) $\subseteq$ G. That is Y$\cap$ αcl(A) $\subseteq$ Y$\cap$ G. where Y$\cap$ αcl(A) is the α-closure of A of Y. Thus A is αg*p-closed relative to Y.

We introduce the following

Definition 3.22: A subset A of topological space (X, τ) is called αg*- pre open (briefly αg*p-open) set if its complement A^c is αg*p-closed.

Theorem 3.23: A subset A of a topological space X is αg*p-open if and only if F $\subseteq$ αint(A) whenever F $\subseteq$ A and F is gs-closed.

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