

EFFECTS OF CHEMICAL REACTION AND HEAT  
GENERATION ON MHD FREE CONVECTIVE  
OSCILLATORY COUETTE FLOW THROUGH A  
VARIABLE POROUS MEDIUM

P.Lalitha\*

V. Manjulatha\*\*

S.V.K. Varma\*

V.C.C Raju\*\*\*

**Abstract**

In this paper, the effects of chemical reaction and heat source on free convective oscillatory coquette flow of a viscous incompressible fluid through a variable porous medium bounded by two vertical parallel porous plates are analyzed. The approximate solutions to the velocity, temperature and concentration are obtained by assuming that the free stream velocity oscillates in times about a constant mean and that there is a periodic temperature in the moving plate. The effect of governing parameters on the flow variables are discussed quantitatively with the aid of graphs and tables for the flow field, temperature field, concentration field, skin-friction, Nusselt number and Sherwood number.

**Keywords:** Couette flow, variable porous medium, chemical reaction and heat source.

\* Department of Mathematics, Sri Venkateswara University, Tirupati, Andhra Pradesh, India.

\*\* Department of Mathematics, Noble college, Machilipatnam, Andhra Pradesh, India.

\*\*\* Department of Mathematics, University of Botswana, Gaborone, BOTSWANA.

## 1. Introduction

Couette flow is one of the basic flows in fluid dynamics that refers to the laminar flow of a viscous fluid in the space between two parallel plates, one of which is moving relative to the other. The flow is driven by virtue of viscous drag force acting on the fluid and the applied pressure gradient parallel to the plates. Couette flow is frequently used in physics and engineering to illustrate shear-driven fluid motion. Some important application areas of Couette motion are MHD power generators and pumps, aerodynamic heating, electrostatic precipitation, polymer technology, petroleum industry, purification of crude oil etc.

Free convective flows between two long vertical plates have been studied for many years because of their applications in the fields of nuclear reactors, heat exchangers, cooling appliances in electronic instruments etc. These flows were studied by assuming the plates at two different constant temperatures or temperature of the plates varying linearly along the plates etc.

The growing need for chemical reactions in chemical and hydrometallurgical industries require the study of heat and mass transfer with chemical reaction. The presence of a foreign mass in water or air causes some kind of chemical reaction. This may be present either by itself or as mixtures with air or water. In many chemical engineering processes, a chemical reaction occurs between a foreign mass and the fluid in which the plate is moving. These processes take place in numerous industrial applications, for example, polymer production, manufacturing of ceramics or glassware and food processing. Convection in porous media has applications in geothermal energy storage and flow through filtering devices.

The study of fully developed free convective flow between two parallel plates at constant temperature was initiated by Ostrach [1]. Kelleher and Yang [2] investigated the heat transfer response of laminar free convection boundary layers along vertical heated plates to surface-temperature oscillations. In case of unsteady free convective flows, Soundalgeker [3] analyzed the effects of viscous dissipation on the flow past an infinite vertical porous plate. It was assumed that plate temperature oscillates in such a way that its amplitude is small. Raptis [4] studied the unsteady free convective flow through a porous medium. Raptis and Peridikis [5] further studied the unsteady free convective flow through a highly porous medium bounded by

an infinite porous plate. S.W.Yuan [6, 7] studied the couette flow, which is the flow between two parallel flat plates, one of which is at rest and other moving with a velocity parallel to the fixed plate; and shear stress distribution in a couette flow. Transient free convective flow between two long vertical parallel plates maintained at constant but unequal temperatures was examined by Singhet al.[8]. Das et al [9] have analyzed the transient free convective flow past an infinite vertical plate with periodic temperature variation.

Free convective flow through a porous medium between two vertical parallel plates was discussed by Singh [10]. Chamkha [11] presented a MHD flow of a uniformly stretched vertical permeable surface in the presence of heat generation/absorption and chemical reaction. Pantokratoras [12] considered the fully developed free convective flow between two asymmetrically heated vertical parallel plates for a fluid of varying thermo physical properties. Unsteady free convective oscillatory couette flow through a porous medium with periodic wall temperature was studied by Sharma et al [13].

Ibrahim et al. [14] analyzed the effect of chemical reaction and radiation absorption on the unsteady MHD free convective flow past a semi-infinite vertical permeable moving plate with a heat source and suction. Khare and Jyothi Singh [15] discussed about the MHD flow of a dusty viscous non Newtonian fluid flowing between two parallel inclined plates influenced by gravitational force and within the framework of some physically realistic approximation and suitable boundary conditions, derivations for velocity profile have been obtained by varying the magnetic field and inclination of the plates.

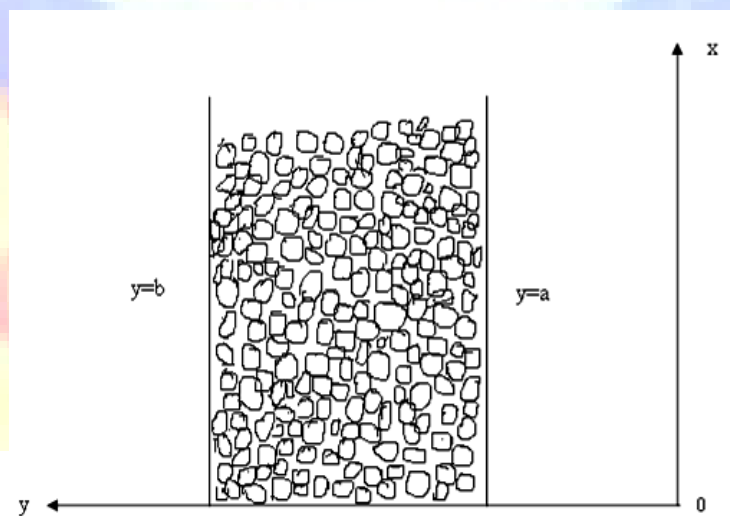
The MHD couette flow of two immiscible fluids in a parallel plate channel in the presence of an applied electric and inclined magnetic field was investigated by Nikodijevic et al. [16]. Raju and Varma [17] studied unsteady MHD free convection oscillatory couette flow through a porous medium. The chemical reaction and combined buoyancy effects of thermal and mass diffusion on MHD convective flow along an infinite vertical porous plate in the presence of Hall current with variable suction and heat generation was discussed by Mastanrao et al. [18].

The objective of this paper is to investigate the effect of chemical reaction and heat generation on MHD free convective oscillatory Couette flow through a variable porous medium. The governing equations of the flow field are solved for the velocity, temperature, concentration distribution, the rate of heat transfer, the rate of mass transfer, skin friction and the effects of the various flow parameters on the flow field have been studied and the results are presented graphically and discussed quantitatively.

## 2. Formulation of the Problem

The unsteady Couette flow of a viscous incompressible fluid through a variable porous medium bounded between two infinite vertical porous plates, one of which suddenly moves from rest with a free stream velocity that oscillates in time about a constant mean is considered. Further, it is assumed that the temperature and permeability of the moving plate fluctuate in time about a non zero constant mean and a transverse magnetic field  $H_0$  is applied normal to the plate.

We take  $x^*$ -axis along the moving vertical plate in the vertical upward direction and  $y^*$ -axis is taken normal to this plate. The other stationary vertical plate is assumed to be situated  $y^*=b$  at temperature  $T_S^*$ .



Physical model

We consider the free-stream velocity distribution as  $U^*(t^*) = U_0 (1 + \varepsilon e^{i\omega^* t^*})$ . (1)

The governing equations of the flow are

$$\rho \frac{\partial u^*}{\partial t^*} = \frac{-\partial P}{\partial x^*} + \mu \frac{\partial^2 u^*}{\partial y^{*2}} - g\rho^* - \frac{u^*\mu}{K^*} - \sigma_e \mu_e^2 H_0^2 u^*, \quad (2)$$

$$\frac{\partial T^*}{\partial t^*} = \alpha \frac{\partial^2 T^*}{\partial y^{*2}} + \frac{q^*}{\rho c_p} \frac{\partial}{\partial y^*} (T^* - T_s^*), \quad (3)$$

$$\frac{\partial C^*}{\partial t^*} = D \frac{\partial^2 C^*}{\partial y^{*2}} + D_I (C^* - C_s^*), \quad (4)$$

with initial and boundary conditions

$$Y^* = 0: u^* = U_0 (1 + \varepsilon e^{i\omega^* t^*}), T^* = T_n^* + \varepsilon (T_n^* - T_s^*) e^{i\omega^* t^*}, C^* = C_s^* \\ y^* = b: u^* = 0, T^* = T_s^*, C^* = C_s^*. \quad (5)$$

Equation (2) for the stream is reduced to

$$\rho \frac{dU^*}{dt^*} = \frac{-\partial P}{\partial x^*} - g\rho_s^* - \frac{U^*\mu}{K^*} - \sigma_e \mu_e^2 H_0^2 U^*. \quad (6)$$

From equations (2) and (6), we get

$$\rho \frac{\partial u^*}{\partial t^*} = \rho \frac{dU^*}{dt^*} + \mu \frac{\partial^2 u^*}{\partial y^{*2}} + g(\rho_s^* - \rho^*) - \frac{(u^* - U^*)\mu}{K^*} - \sigma_e \mu_e^2 H_0^2 (u^* - U^*). \quad (7)$$

The Boussinesq approximation is

$$g(\rho_s^* - \rho^*) = g\beta\rho^*(T^* - T_s^*) + g\beta^*\rho^*(C^* - C_s^*) \text{ and } K^* = K_0^*/(1 + \varepsilon e^{i\omega^* t^*}). \quad (8)$$

Equation (7) reduces, by using the Boussinesq approximation (8), to

$$\rho \frac{\partial u^*}{\partial t^*} = \rho \frac{dU^*}{dt^*} + \mu \frac{\partial^2 u^*}{\partial y^{*2}} + g\beta\rho^*(T^* - T_s^*) \\ + g\beta^*\rho^*(C^* - C_s^*) - \frac{(1 + \varepsilon e^{i\omega^* t^*})(u^* - U^*)\mu}{K_0^*} - \sigma_e \mu_e^2 H_0^2 (u^* - U^*). \quad (9)$$

where  $U_0$  is the mean constant free-stream velocity,  $\omega^*$  is the frequency of the oscillations and  $t^*$  is the time.  $\beta$  is the volumetric coefficient of thermal expansion,  $\beta^*$  is the volumetric coefficient of chemical expansion,  $\rho_s^*$  is the density of fluid with temperature  $T_s^*$ ,  $\rho^*$  is the density of fluid with temperature  $T^*$ ,  $u^*$  is the velocity,  $U^*$  is the free stream velocity,  $\rho$  is the density,  $\mu$  is the viscosity,  $\nu$  is the Kinematic viscosity,  $P$  is the pressure,  $g$  is the gravity,  $K^*$  is the permeability parameter,  $\alpha$  is the thermal diffusivity,  $T^*$  is the temperature of fluid in the boundary layer,  $T_n^*$  is the temperature of the moving plate,  $T_s^*$  is the temperature of the stationary plate,  $D_I$  is the chemical reaction rate constant,  $C_s^*$  is the concentration at the stationary plate,  $C^*$  is the

concentration of fluid in the boundary layer,  $C_n^*$  is the concentration of moving plate,  $\omega^*$  is the frequency of oscillations and  $H_0$  is the applied magnetic field.

Introducing the following non- dimensional quantities

$$Y = y^*/b, u = u^*/U_0, U = U^*/U_0, t = \omega^*t^*, \omega = \omega^*b^2/\nu, \theta = \frac{T^* - T_s^*}{T_n^* - T_s^*}$$

$$C = \frac{c^* - c_s^*}{c_n^* - c_s^*}, Re = \frac{U_0 b}{\nu}, Gr = \frac{g\beta b^2(T_n^* - T_s^*)}{\nu U_0}, Gc = \frac{g\beta^* b^2(C_n^* - c_s^*)}{\nu U_0}, M^2 = \frac{\sigma_e \mu_e^2 H_0^2 b^2}{\mu}$$

$$Q^* = \frac{QK}{b}, Kr = \frac{\nu D_1}{U_0^2}, K = K_0^*/b^2, \nu = \mu/\rho^*$$

the dimensionless equations (9), (3) and (4) become

$$\omega \frac{\partial u}{\partial t} = \omega \frac{\partial U}{\partial t} + \frac{\partial^2 u}{\partial y^2} + Gr \theta + Gc C - \frac{(u-U)(1+\epsilon e^{it})}{K} - M^2 (u - U), \quad (10)$$

$$\omega Pr \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} + Q \frac{\partial \theta}{\partial y}, \quad (11)$$

$$\omega Sc \frac{\partial C}{\partial t} = \frac{\partial^2 C}{\partial y^2} + Sc Kr Re^2 C, \quad (12)$$

with corresponding boundary conditions

$$\begin{aligned} y = 0: & \quad u = 1 + \epsilon e^{it}, \theta = 1 + \epsilon e^{it}, C = 1 + \epsilon e^{it} \\ y = 1: & \quad u = 0, \theta = 0, C = 0 \end{aligned} \quad (13)$$

where  $Gr$  is the thermal Grashof number,  $Gc$  is the modified Grashof number,  $Pr(= \nu/\alpha)$  is the Prandtl number,  $Sc(= \nu/D)$  is the Schmidt number,  $M$  is the magnetic parameter,  $Q$  is the heat source parameter,  $Kr$  is the chemical reaction rate constant and  $Re$  is the Reynolds number.

### 3. Solution of the problem

Since the amplitudes of the free stream velocity and temperature variation  $\epsilon$  ( $\ll 1$ ) are very small, we now assume the solution in the following form:

$$u(y, t) = u_0(y) + \epsilon u_1(y)e^{it}, \theta(y, t) = \theta_0(y) + \epsilon \theta_1(y)e^{it}, C(y, t) = C_0 + \epsilon C_1(y)e^{it} \quad (14)$$

and for stream velocity  $U = 1 + \epsilon e^{it}$ .

Substituting equation (14) in to equations (10), (11) and (12), we get

$$u_0'' - \frac{u_0}{K} - M^2 u_0 = -Gr \theta_0 - Gc C_0 - \frac{1}{K} - M^2, \quad (15)$$

$$\theta_0'' + Q \theta_0' = 0, \quad (16)$$

$$u_1'' - (i\omega + \frac{1}{K}) u_1 - M^2 u_1 = - Gr \theta_1 - Gc C_1 - i\omega - \frac{2}{K} + \frac{u_0}{K} - M^2, \quad (17)$$

$$\theta_1'' - i\omega Pr \theta_1 + Q \theta_1' = 0 \quad (18)$$

$$C_0'' + Sc Kr Re^2 C_0 = 0, \quad (19)$$

$$C_1'' - i\omega Sc C_1 + Sc Kr Re^2 C_1 = 0, \quad (20)$$

with the corresponding boundary conditions

$$y = 0: u_0 = 1, u_1 = 1, \theta_0 = 1, \theta_1 = 1, C_0 = 1, C_1 = 1,$$

$$y = 1: u_0 = 0, u_1 = 0, \theta_0 = 0, \theta_1 = 0, C_0 = 0, C_1 = 0. \quad (21)$$

Solving the equations (15) to (20) subject to the boundary conditions (21), we get

$$\theta_0 = \frac{e^{-Q} - ye^{-Qy}}{e^{-Q}} \quad (22)$$

$$C_0 = \frac{(1-y)e^{h(1-y)} + (1-y)e^{-h(1-y)}}{e^h + e^{-h}} \quad (23)$$

$$u_0 = A_2 e^{\sqrt{\frac{1}{K} + M^2} y} + B_2 e^{-\sqrt{\frac{1}{K} + M^2} y} + \frac{Gr(1-y)}{M_1^2} + \frac{Gc(1-y)}{M_1^2} + 1 \quad (24)$$

$$\theta_1 = A e^{\frac{-Q + \sqrt{Q^2 + 4(1+i)^2 \lambda^2}}{2} y} + B e^{\frac{-Q - \sqrt{Q^2 + 4(1+i)^2 \lambda^2}}{2} y} \quad (25)$$

$$C_1 = A_1 e^{\sqrt{(1+i)^2 \lambda_1^2 + h^2} y} + B_1 e^{-\sqrt{(1+i)^2 \lambda_1^2 + h^2} y} \quad \text{where } h = i\sqrt{ScKr Re} \quad (26)$$

$$u_1 = A_3 e^{(a_2 + ia_3)y} + B_3 e^{-(a_2 + ia_3)y} - AA_4 e^{\lambda(1+i)y} - BA_4 e^{-\lambda(1+i)y} - A_1 A_5 e^{\lambda_1(1+i)y} - B_1 A_5 e^{-\lambda_1(1+i)y} + A_2 S_1 e^{a_1 y} + B_2 S_1 e^{-a_1 y} - (1-y)S_2 - (1-y)S_3 + 1 \quad (27)$$

where the constants are given in appendix.

### Skin friction

The skin-friction coefficient at the surfaces  $y=0$  and  $y=1$  is given by

$$\tau = - \left( \frac{\partial u}{\partial y} \right)_{y=0}$$

$$\begin{aligned} \tau_{y=0} = & -A_2\sqrt{\frac{1}{K} + M^2} + B_2\sqrt{\frac{1}{K} + M^2} + \frac{Gr}{\frac{1}{k} + M^2} + \frac{Gc}{\frac{1}{k} + M^2} - \varepsilon e^{it} ( A_3(a_2 + ia_3) \\ & - B_3(a_2 + ia_3) - AA_4\lambda(1 + i) + BA_4\lambda(1 + i) - A_1A_5\lambda_1(1 + i) \\ & + B_1A_5\lambda_1(1 + i) + A_2S_1a_1 - B_2S_1a_1 - S_2 + S_3) \end{aligned} \quad (28)$$

$$\tau = -\left(\frac{\partial u}{\partial y}\right)_{y=1}$$

$$\begin{aligned} \tau_{y=1} = & -A_2\sqrt{\frac{1}{K} + M^2}e^{\sqrt{\frac{1}{K} + M^2}} + B_2\sqrt{\frac{1}{K} + M^2}e^{-\sqrt{\frac{1}{K} + M^2}} + \frac{Gr}{\frac{1}{k} + M^2} + \frac{Gc}{\frac{1}{k} + M^2} \\ & - \varepsilon e^{it} (A_3(a_2 + ia_3)e^{(a_2+ia_3)} - B_3(a_2 + ia_3)e^{-(a_2+ia_3)} - AA_4\lambda(1 + \\ & i)e^{\lambda(1+i)} + BA_4\lambda(1 + i)e^{-\lambda(1+i)} - A_1A_5\lambda_1(1 + i)e^{\lambda_1(1+i)} + B_1A_5\lambda_1(1 + \\ & i)e^{-\lambda_1(1+i)} + A_2S_1e^{a_1}a_1 - B_2S_1e^{-a_1}a_1 - S_2 + S_3) \end{aligned} \quad (29)$$

### Rate of heat transfer

Heat Transfer Coefficient at the surfaces  $y = 0$  and  $y = 1$  in terms of Nusselt number is given by

$$\begin{aligned} Nu = & -\left(\frac{\partial \theta}{\partial y}\right)_{y=0} \\ Nu_{y=0} = & \frac{1}{e^{-Q}} - \varepsilon e^{it} \left[ A \left( \frac{-Q + \sqrt{Q^2 + 4(1+i)^2 \lambda^2}}{2} \right) + B \left( \frac{-Q - \sqrt{Q^2 + 4(1+i)^2 \lambda^2}}{2} \right) \right] \end{aligned} \quad (30)$$

$$\begin{aligned} Nu = & -\left(\frac{\partial \theta}{\partial y}\right)_{y=1} \\ Nu_{y=1} = & \left( \frac{e^{-Q} - Qe^{-Q}}{e^{-Q}} \right) - \varepsilon e^{it} \left( A \left( \frac{-Q + \sqrt{Q^2 + 4(1+i)^2 \lambda^2}}{2} \right) e^{\left( \frac{-Q + \sqrt{Q^2 + 4(1+i)^2 \lambda^2}}{2} \right)} + \right. \\ & \left. B \left( \frac{-Q - \sqrt{Q^2 + 4(1+i)^2 \lambda^2}}{2} \right) e^{\left( \frac{-Q - \sqrt{Q^2 + 4(1+i)^2 \lambda^2}}{2} \right)} \right) \end{aligned} \quad (31)$$



### Rate of Mass Transfer

Mass Transfer Coefficient at the surfaces  $y=0$  and  $y=1$  in terms of Sherwood number is given by

$$Sh = -\left(\frac{\partial c}{\partial y}\right)_{y=0}$$

$$Sh_{y=0} = 1 - \varepsilon e^{it} (A_1 \sqrt{(1+i)^2 \lambda_1^2 + h^2} - B_1 \sqrt{(1+i)^2 \lambda_1^2 + h^2}) \quad (32)$$

$$Sh = -\left(\frac{\partial c}{\partial y}\right)_{y=1}$$

$$Sh_{y=1} = -\varepsilon e^{it} \left( A_1 \sqrt{(1+i)^2 \lambda_1^2 + h^2} e^{\sqrt{(1+i)^2 \lambda_1^2 + h^2}} - \sqrt{(1+i)^2 \lambda_1^2 + h^2} e^{-\sqrt{(1+i)^2 \lambda_1^2 + h^2}} \right) \quad (33)$$

### 4. Results and Discussion

Numerical calculations are carried out for different values of dimensionless parameters and a representative set of results are reported graphically in Figures 1-11 and tables 1-2. These results are obtained to illustrate the influence of the chemical reaction parameter  $Kr$ , the Reynolds number  $Re$ , the heat source parameter  $Q$ , the magnetic field parameter  $M$ , permeability parameter  $k$ , thermal Grashof number  $Gr$ , modified Grashof number  $Gc$ , frequency of the oscillation  $\omega$ , Schmidt number  $Sc$  and Prandtl number  $Pr$  on the concentration, temperature and the velocity profiles.

The effect of chemical reaction and Reynolds number on concentration field  $C_1$  is shown in figure 1. It is observed that the species concentration decreases as the chemical reaction parameter or the Reynolds number increases. Figure 2 displays the effect of heat source parameter on transient temperature ( $\theta_1$ ) field. It is observed that the temperature decreases with the increase of the heat source parameter.

For different values of magnetic parameter, the velocity profile ( $u_0$ ) is plotted in figure 3. It is noticed that the velocity increases as the magnetic parameter increases. Figure 4 is the graphical representation of the influence of magnetic parameter on velocity profiles ( $u_1$ ). It is observed that the velocity decreases with increasing magnetic parameter near the plate and reverse phenomenon is observed for  $y \geq 0.1$ . Figure 5 shows the effect of Reynolds number on velocity profiles ( $u_1$ ). It is observed that for  $y \leq 0.4$ , velocity decreases as the Reynolds number ( $Re$ ) increases and for  $y \geq 0.6$ , velocity increases as the Reynolds number increases. The phenomena change in the region  $0.4 < y < 0.6$ .

Figure 6 shows the effect of thermal Grashof number and modified Grashof number on velocity profile ( $u_1$ ). It is observed that the velocity decreases with decreasing thermal Grashof

number and modified Grashof number near the plate and reverse phenomenon is observed for  $y \geq 0.2$ . The effect of permeability parameter and frequency of oscillations, on velocity profiles ( $u_1$ ) is plotted in figure 7. It is observed that velocity increases as the frequency of oscillations increases whereas it decreases as permeability parameter increases.

Figure 8 shows the effect of chemical reaction parameter on the Sherwood number at plate  $y=0$ . It is observed that Sherwood number decreases with increasing chemical reaction parameter. Figure 9 is the graphical representation of the effect of chemical reaction parameter on the Sherwood number at plate  $y=1$ . It is observed that Sherwood number decreases as chemical reaction parameter increases. Figure 10 shows the effect of Reynolds number on the Sherwood number at plate  $y=0$ . It is observed that Sherwood number decreases as the Reynolds number increases. Figure 11 shows the effect of Reynolds number on the Sherwood number at plate  $y=1$ . It is noticed that Sherwood number decreases as the Reynolds number increases.

Table 1 shows the variations in the rate of heat transfer at plate  $y=0$  and at plate  $y=1$  for different values of heat source parameters. It is observed that Nusselt number increases as the heat source parameter increases at plate  $y=0$  while it decreases at plate  $y=1$ .

For different values of thermal Grashof number, modified Grashof number and Prandtl number, the variations in skin friction at plate  $y=0$  and plate  $y=1$  are shown in Table 2. It is observed that the skin friction increases as thermal Grashof number or modified Grashof number increases at both the plates  $y=0$  and  $y=1$ . Skin Friction increases as Prandtl number increases at plate  $y=0$  where as it decreases at plate  $y=1$ .

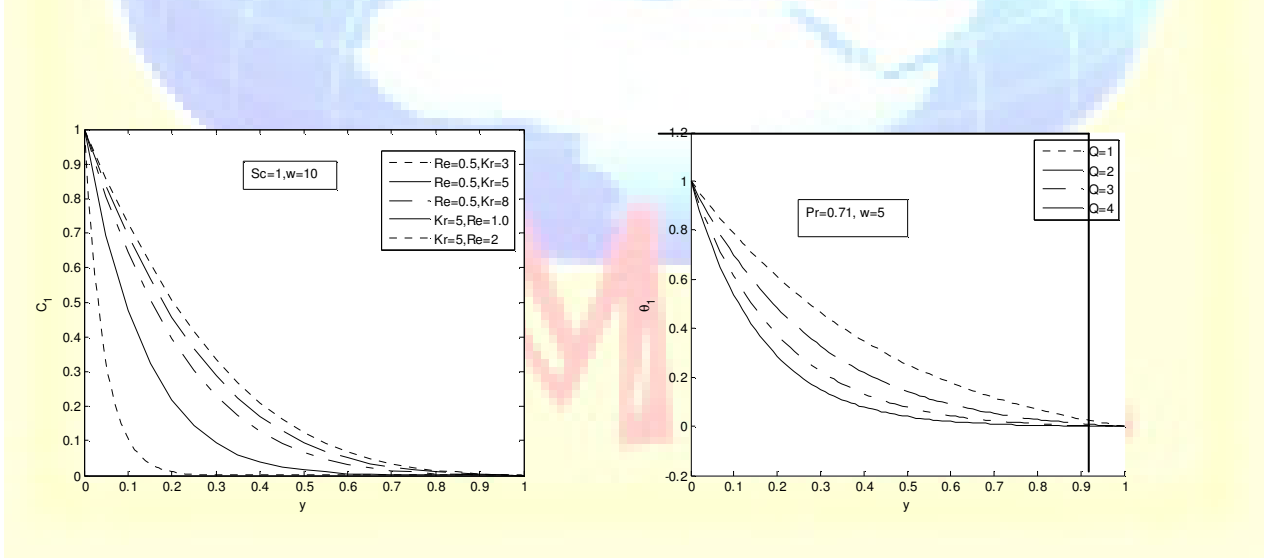


Fig 1: The effects of chemical reaction and Reynolds number on concentration field ( $C_1$ )

Fig 2: The effect of heat source parameter ( $Q$ ) on transient temperature ( $\theta_1$ )

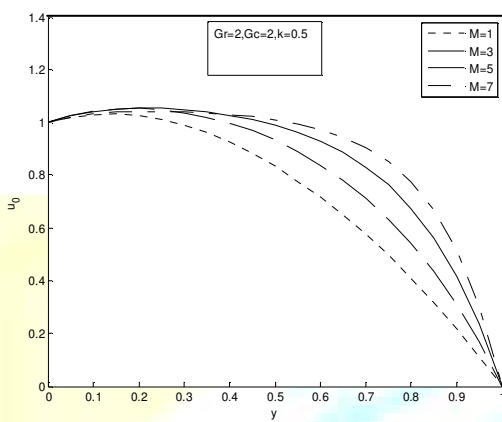


Fig 3: The effect of magnetic parameter ( $M$ ) on fluid velocity  $u_0$

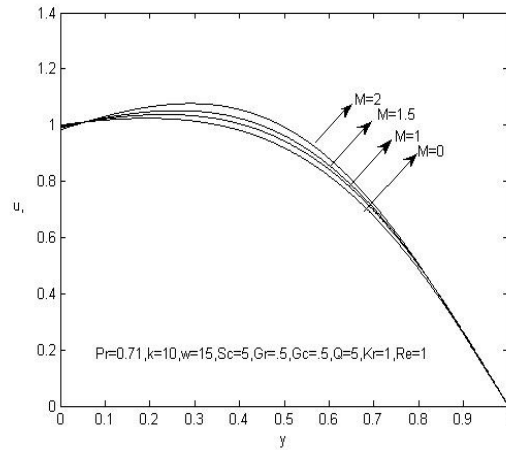


Fig 4: The effect of magnetic parameter ( $M$ ) on fluid velocity  $u_1$

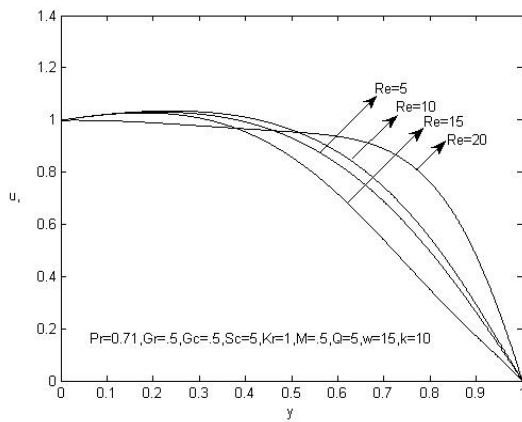


Fig 5: The effect of Reynolds number ( $Re$ ) fluid velocity  $u_1$

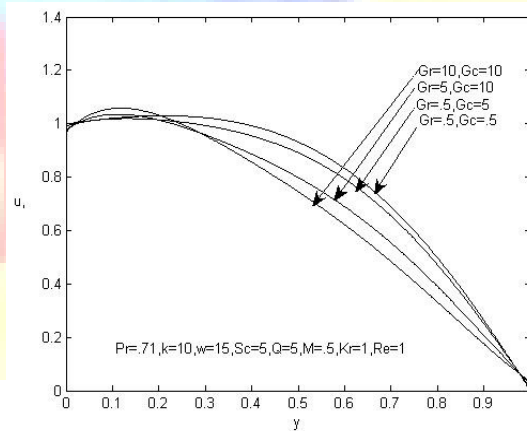


Fig 6: The effects of Grashof number ( $Gr$ ) and modified Grashof number ( $Gc$ ) on fluid velocity  $u_1$

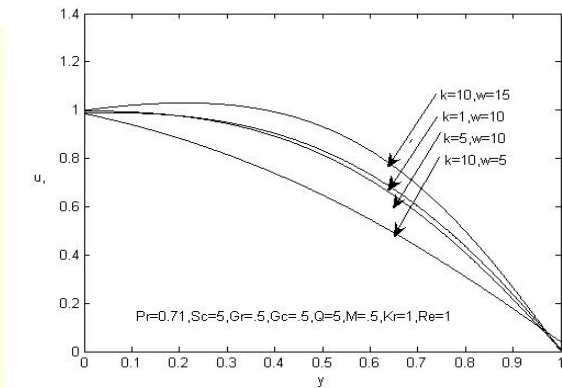


Fig 7: The effects of permeability parameter ( $k$ ) and frequency of collisions ( $w$ ) on fluid velocity  $u_1$

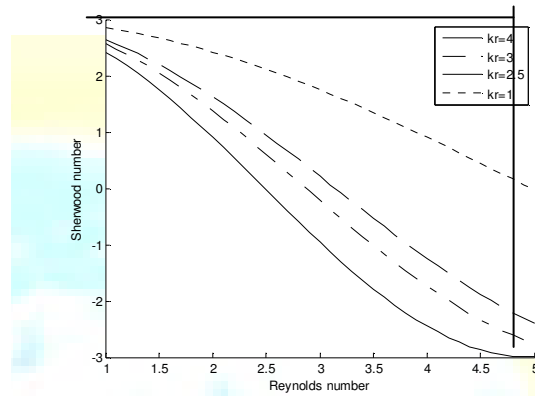


Fig 8: The effect of chemical reaction parameter ( $Kr$ ) on Sherwood number ( $Sh$ ) at plate  $y=0$

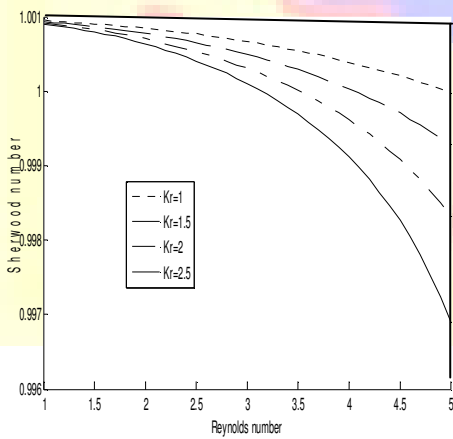


Fig 9: The effect of chemical reaction parameter ( $Kr$ ) on Sherwood number at plate  $y=1$

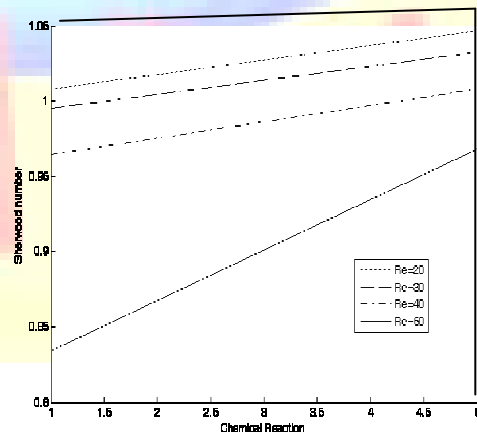
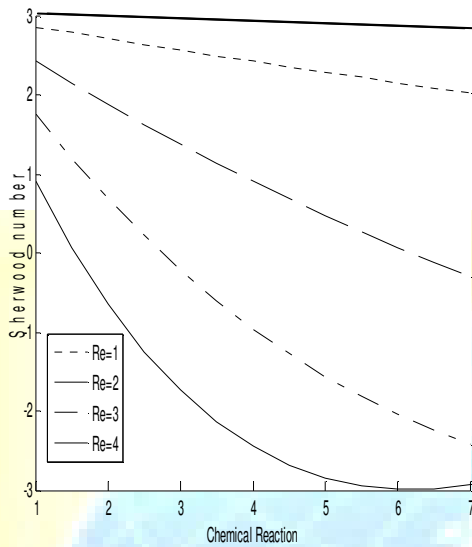


Fig 10: The effect of Reynolds number ( $Re$ ) on Sherwood number ( $Sh$ ) at plate  $y=0$ .

**Table1: Variations in rate of heat transfer when  $t=0.1, \omega = 10, \varepsilon = 0.01$**



$Pr$	$Q$	$Nu_{y=0}$	$Nu_{y=1}$
0.71	1	2.7394	0.0026
0.71	2	7.4165	-0.9985
0.71	3	20.1202	-1.9992
0.71	4	54.6409	-2.9996

**Fig 11: The effect of Reynolds number ( $Re$ ) on Sherwood number ( $Sh$ ) at plate  $y=1$**

**Table 2: Variations in skin friction when**

$k = 1, M = 3, Q = 1, Kr = 1, Re = 10, Sc = 1, \omega = 10, t = 1.0$  and  $\varepsilon = 0.01$

$Gr$	$Gc$	$Pr$	$\tau_{y=0}$	$\tau_{y=1}$
5	10	0.71	1.7159	4.5504
10	10	0.71	2.2224	4.9810
15	10	0.71	2.7291	5.4116
10	5	0.71	1.7219	4.5569
10	15	0.71	2.7229	5.4052
10	10	1	2.2282	4.9849
10	10	2	2.3333	4.9644
10	10	5	3.1041	4.7856

## 5. Conclusion

An increase in the Reynolds number or chemical reaction rate constant leads to a decrease in the concentration and Sherwood number. The temperature decreases for increasing values of heat source parameters. An increase in the magnetic parameter leads to a decrease in the velocity. The rate of heat transfer increases with an increase in the heat source parameter at  $y=0$  whereas it decreases at  $y=1$ . The skin friction increases with an increase in the thermal Grashof number or modified Grashof number at both the plates  $y=0$  and  $y=1$ .

## 6. References

- [1] Ostrach S, Laminar natural-convection flow and heat transfer of fluids with and without heat sources in channels with constant wall temperatures. Technical Report 2863, NASA, USA, 1952.
- [2] M.D.Kelleher and K.T.Yang, Heat transfer response of laminar free convection boundary layer along vertical heated plate to surface temperature oscillation, ZAMP 19, pp.31-44, 1968.
- [3] V.M.Soundalgar viscous Dissipation effects on unsteady free convective flow past an infinite vertical porous plate with constant suction, Int.J.Heat Mass Transfer, 15, pp.1253-1261, 1972.
- [4] A.Raptis Unsteady free convection flow through a porous medium, Int.J.Engin.Sci.21, pp.345-348, 1983.
- [5] A.Raptis and C.P.Pericikis, oscillatory flow through a porous medium by the presence of free convective flow, Int.J.Engin.Sci.23, pp.51-55, 1985.
- [6] S.W.Yuan, Couette flow between two parallel flat plates, University of Texas, Prentice Hall of India Private Limited, New Delhi, 1988.
- [7] S.W.Yuan, Shearing stress distribution in a Couette flow, University of Texas, Prentice Hall of India Private Limited, New Delhi, 1988.
- [8] Singh AK, Gholami HR, and Soundalgekar VM, Transient free convection flow between two vertical parallel plates. Wärme and Stoffübertragung, Heat and Mass Transfer, 31, pp. 329-331, 1996.
- [9] U.N.Das, R.K.Deka and V.M.Soundalgekar, Transient free convection flow past an infinite vertical plate with periodic temperature variation, J.Heat Transfer(ASME)121, pp.1091-1094, 1999.
- [10] A.K.Singh MHD free convective flow through a porous medium between two vertical parallel plates, Ind.J.Pure and Appl.Phys.40, pp.709-713, 2002.
- [11] A. J. Chamkha, MHD flow of a uniformly stretched vertical permeable surface in the presence of heat generation /absorption and chemical reaction, Int. Commun. Heat

- Mass Trans., 30,pp.413-422, 2003.
- [12] Pantokratoras A, Fully developed laminar free convection with variable asymmetrically with large temperature differences. ASME Journal of Heat Transfer, 128, pp. 405-408, 2006.
- [13] P.K.Sharma, B.K.Sharma and R.C.Chaudhary, Unsteady free convection oscillatory Couette flow through a porous medium with periodic wall temperature, Tamkang Journal of mathematics volume 38, Number 1, 93-102 Spring 2007.
- [14] F. S. Ibrahim, A. M. Elaiw and A. A. Bakr, Effect of the chemical reaction and Radiation absorption on the unsteady MHD free convection flow past a semi-infinite vertical permeable moving plate with heat source and suction, Commun. Non-linear Sci. Numer. Simul., 13, pp. 1056-1066, 2008.
- [15] R.K.Khare and Jyothi Singh, MHD flow of a dusty viscous non Newtonian fluid flowing between two parallel inclined plates influenced by gravitational force, Journal of International Academy of Physical Sciences Vol. 15, pp. 7-15, 2011.
- [16] D.Nikodijevic, D.Milenkovic and Z.Stamenkovic, MHD Couette flow of two immiscible fluids in a channel in the Presence of an applied electric and inclined magnetic field. Heat Mass Transfer, 47: pp. 1525-1535, 2011.
- [17] Raju M.C, Varma S.V.K, " Unsteady MHD free convection oscillatory Couette flow through a porous medium with periodic wall temperature", i-manager's Journal on Future Engineering and Technology, Vol.6(4), pp.7-12, 2011.
- [18] S. Masthanrao, K. S. Balamurugan, S. V. K. Varma and V. C. C. Raju., Chemical Reaction and Hall Effects on MHD Convective Flow along an Infinite Vertical Porous Plate with Variable Suction and Heat Absorption, International Journal of Applications and Applied Mathematics, Vol. 8, Issue 1, pp. 268 – 288, 2013.

## 7. Appendix

$$\lambda = \sqrt{\frac{\omega p_r}{2}}, \lambda_1 = \sqrt{\frac{\omega S_c}{2}}, A = -\frac{e^{\frac{(-Q-\sqrt{Q^2+4(1+i)^2\lambda^2})}{2}}}{\frac{e^{\frac{(-Q+\sqrt{Q^2+4(1+i)^2\lambda^2})}{2}}}{2} - e^{\frac{(-Q-\sqrt{Q^2+4(1+i)^2\lambda^2})}{2}}}, B = \frac{e^{\frac{-Q+\sqrt{Q^2+4(1+i)^2\lambda^2}}{2}}}{\frac{e^{\frac{-Q+\sqrt{Q^2+4(1+i)^2\lambda^2}}{2}}}{2} - e^{\frac{-Q-\sqrt{Q^2+4(1+i)^2\lambda^2}}{2}}},$$

$$A_1 = -\frac{e^{-\sqrt{(1+i)^2\lambda_1^2+h^2}}}{e^{\sqrt{(1+i)^2\lambda_1^2+h^2}} - e^{-\sqrt{(1+i)^2\lambda_1^2+h^2}}}, B_1 = \frac{e^{\sqrt{(1+i)^2\lambda_1^2+h^2}}}{e^{\sqrt{(1+i)^2\lambda_1^2+h^2}} - e^{-\sqrt{(1+i)^2\lambda_1^2+h^2}}},$$

$$A_2 = -\frac{(1 - \frac{Gr e^{-\sqrt{\frac{1}{k}+M^2}}}{\frac{1}{k}+M^2} - \frac{Gc e^{-\sqrt{\frac{1}{k}+M^2}}}{\frac{1}{k}+M^2})}{\frac{e^{\sqrt{\frac{1}{k}+M^2}}}{\frac{1}{k}+M^2} - e^{-\sqrt{\frac{1}{k}+M^2}}}, B_2 = \frac{(1 - \frac{Gr e^{-\sqrt{\frac{1}{k}+M^2}}}{\frac{1}{k}+M^2} - \frac{Gc e^{-\sqrt{\frac{1}{k}+M^2}}}{\frac{1}{k}+M^2})}{\frac{e^{\sqrt{\frac{1}{k}+M^2}}}{\frac{1}{k}+M^2} - e^{-\sqrt{\frac{1}{k}+M^2}}}, a_1 = \sqrt{\frac{1}{k}+M^2},$$

$$a_2 = \frac{1}{\sqrt{2}} \left[ \sqrt{\frac{1}{K^2} + \omega^2 - M^4 - \frac{2\omega M^2}{i}} + \frac{1}{K} \right]^{\frac{1}{2}}, a_3 = \frac{1}{\sqrt{2}} \left[ \sqrt{\frac{1}{K^2} + \omega^2 - M^4 - \frac{2\omega M^2}{i}} - \frac{1}{K} \right]^{\frac{1}{2}},$$

$$A_4 = \frac{Gr}{\lambda^2(1+i)^2 - (i\omega + \frac{1}{k} + M^2)}, A_5 = \frac{Gc}{\lambda_1^2(1+i)^2 - (i\omega + \frac{1}{k} + M^2)},$$

$$S_1 = \frac{i}{K\omega + \frac{KM^2}{i}}, S_2 = \frac{Gr}{i\omega + \frac{1}{k} + M^2}, S_3 = \frac{Gc}{i\omega + \frac{1}{k} + M^2},$$

$$A_3 = \{ AA_4(e^{-(a_2+ia_3)} - e^{\lambda(1+i)}) + BA_4(e^{-(a_2+ia_3)} - e^{-\lambda(1+i)}) + B_1A_5(e^{-(a_2+ia_3)} - e^{-\lambda_1(1+i)}) + A_1A_5(e^{-(a_2+ia_3)} - e^{\lambda_1(1+i)}) + A_2S_1(e^{a_1} - e^{-(a_2+ia_3)}) + B_2S_1(e^{-a_1} - e^{-(a_2+ia_3)}) +$$

$$(S_2+S_3)e^{-(a_2+ia_3)} + 1 \} / e^{-(a_2+ia_3)} - e^{(a_2+ia_3)},$$

$$B_3 = - \{ AA_4(e^{(a_2+ia_3)} - e^{\lambda(1+i)}) + B_1A_5(e^{(a_2+ia_3)} - e^{-\lambda_1(1+i)}) +$$

$$A_1A_5(e^{(a_2+ia_3)} - e^{\lambda_1(1+i)}) + A_2S_1(e^{a_1} - e^{(a_2+ia_3)}) + B_2S_1(e^{-a_1} - e^{(a_2+ia_3)}) +$$

$$(S_2+S_3)e^{(a_2+ia_3)} + 1 \} / e^{-(a_2+ia_3)} - e^{(a_2+ia_3)}.$$