SEEPAGE OF ISOTHERMAL AND ADIABATIC FLUIDS THROUGH GRANULAR MEDIA

Ngiangia Alalibo T. *

Orukari Mercy A. **

Abstract

A study of the seepage of isothermal and adiabatic fluids in granular media was carried out. Approximation of the governing equations and its solutions and analysis showed that, in the isothermal case, increase in porosity of the photosphere and increase in its viscosity result in the decrease in pressure of the fluid while increase in permeability of the photosphere increases the pressure of the fluid. For the adiabatic case, increase in porosity of the photosphere, brings about a corresponding increase in the pressure of the fluid which also result in decrease of the granules while increase in permeability and viscosity, decrease the pressure of the fluid through the photosphere.

Keywords; Photosphere, Isothermal, Adiabatic, Granular media, Porosity, Permeability, Viscosity

Introduction

* Department of Physics,University of Port Harcourt,P M B 5323 Choba, Port Harcourt, Nigeria

** Department of Mathematics/Computer Science,Niger Delta University,Wilberforce Island, Bayelsa State,Nigeria

Photosphere is the lowest, densest level of the solar atmosphere. The visible light that reaches the earth from the sun originates in the atmosphere. That light comes from a thin bright shell about three hundred kilometers thick. The photosphere has a temperature of about $5510\,^0C$ [1]. It is a diffuse, tenuous gas with a pressure of about the fraction 0.001 that of earth's atmosphere at sea level. The photosphere is opaque because it contains negative hydrogen ions. This hydrogen ions block, absorb and emit light, all of which prevent light from passing directly through a cloud of hydrogen ions. As a result of the pressure exerted on the photosphere overtime, it is observed that the photosphere breaks into a million tiny bright points, called granules with a strongly textured and varying pattern. Granules are places within the photosphere where hot and bright material reaches the surface. Hot gas rises up, liberating its energy. The study of fluid flow through granular medium is the key to understanding of physical phenomena in astrophysics, atmospheric physics and its sub disciplines. Several studies on compressible fluids are abounded. Seepage velocity of fluid in porous media is one of the parameter in the application of Darcy's law in solving environmental problems; depend on porous media [2-3]. [4] also posits that permeability of porous media especially clay soils decrease with increase in time. Similar studies were carried out by [5] and [6] on the effect of radiation and other parameters on the rate at which the ozone layer is being depleted and came up with the findings that radiation is not a potent factor in the depletion of the ozone layer. [7], considered computational model for speed of efflux in fluids and reported that at a low hydrostatic pressure, the speed of efflux of a viscous fluid is less than that of an inviscid fluid. Thus there is a significant energy loss, if the kinematic viscosity of a fluid is high and this suggest that fluids with a large kinematic viscosity are more likely to support steady flow if subjected to large pressure gradient. Recently, [8] examined the slow seepage of polar fluids through porous media and opined that, the pressure of the fluid is being influenced by the permeability and porosity of the fluid under consideration, also [9] studied the modeling of permeability with porosity and grain size diameter and stated that a strong correlation exist between grain size and hydraulic conductivity and that the value of permeability and hydraulic conductivity can never be zero. Few [10-15] tackled fluid flow through porous medium and made useful findings. Our aim is to examine the effect of pressure of the fluid through the granules and compare with experimental results with a view to validating them or a departure from them. This practice in our view will enrich existing literatures and open the frontiers for further studies.

Formalism

For flow of fluid through porous medium, the smoothed continuity equation and the Darcy's equation respectively are

$$\xi \frac{\partial \rho}{\partial t} = -(\nabla . \rho v_0) \tag{1}$$

$$v_0 = -\frac{\kappa}{\mu}(\nabla p - \rho g) \tag{2}$$

where $\xi, \kappa, \rho, \mu, v_0, p, t, g$ are respectively the porosity, permeability, density of fluid, fluid viscosity, superficial velocity, pressure of fluid, time and acceleration due to gravity.

Combination of Equations (1) and (2) results in

$$\left(\frac{\xi\mu}{\kappa}\right)\frac{\partial \rho}{\partial t} = (\nabla . \rho(\nabla p - \rho g)) \tag{3}$$

We write the equation of state for this study following the argument of [11] as

$$\rho = \rho_0 p^m e^{\beta p} \tag{4}$$

where ρ_0 is the fluid density at unit pressure, m and β are integers.

For Isothermal expansion of fluids, m = 1 , $\beta = 0$ and equations (3) and (4) reduced to

$$\frac{2\xi\mu\rho_0}{\kappa}\frac{\partial \rho}{\partial t} = \nabla^2 \rho^2 \tag{5}$$

With the boundary conditions

$$\rho(0) = 0 \quad \text{and} \quad \rho(-1) = 1 \tag{6}$$

Method of Solution

We approximate ρ^2 by discarding powers of ρ greater than unity using Taylor's series expansion about 0 and rewriting equation (5), we get

$$\frac{\xi\mu\rho_0}{\kappa}\frac{\partial\rho}{\partial t}=\frac{\partial^2\rho}{\partial x^2} \tag{7}$$

To solve equation (7) we assume a solution of the form

$$\rho=\theta(z)e^{-\lambda t} \tag{8}$$

where λ is a constant and the boundary conditions also modified into

$$\theta(0)=0 \text{ and } \theta(-1)=e^{\lambda t} \tag{9}$$

Substitution of equation (8) into equation (7) and simplify we obtain

$$\theta''(x)+\lambda f\theta(x)=0 \tag{10}$$

where $f=\frac{\xi\mu\rho_0}{\kappa}$

Solving equation (10) using method of undetermined coefficients, we get

$$\theta(x)=ACos\sqrt{\lambda f}\,x+BSin\sqrt{\lambda f}\,x \tag{11}$$

where A and B are constants.

Imposing the boundary conditions of equation (9) and applying the transformation of equation (8), we get

$$\rho=-\left(Sin^{-1}\left(\sqrt{\lambda f}\right)\right)Sin\sqrt{\lambda f}\,x \tag{12}$$

Further, using the equation of state as described in equation (4) and considering the case for isothermal expansion of fluids, we write equation (12) as

$$p=-\frac{1}{\rho_0}\left(Sin^{-1}\left(\sqrt{\lambda f}\right)\right)Sin\sqrt{\lambda f}\,x \tag{13}$$

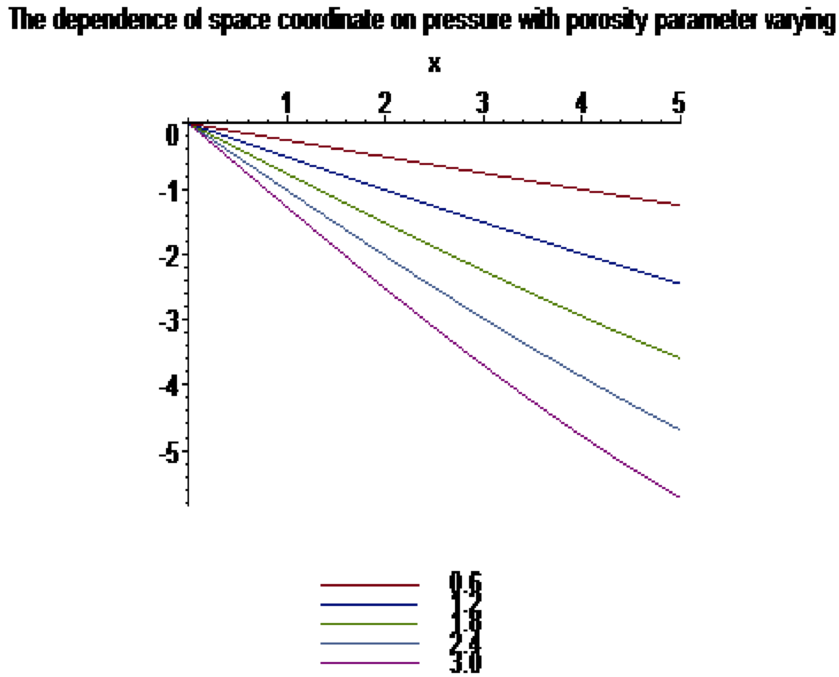

Figure 1: Pressure profile p against boundary layer x for varying porosity term. ξ

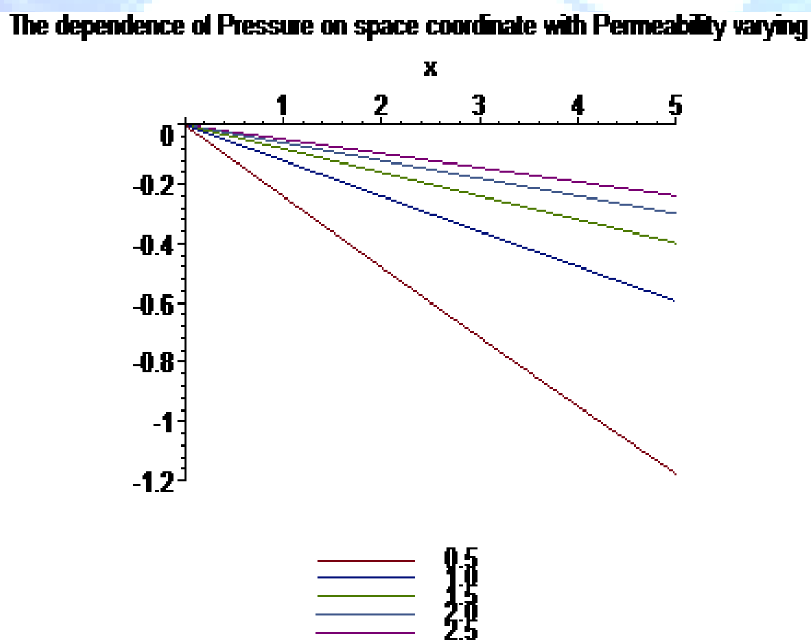

Figure 2: Pressure profile p against boundary layer x for varying permeability term (κ)

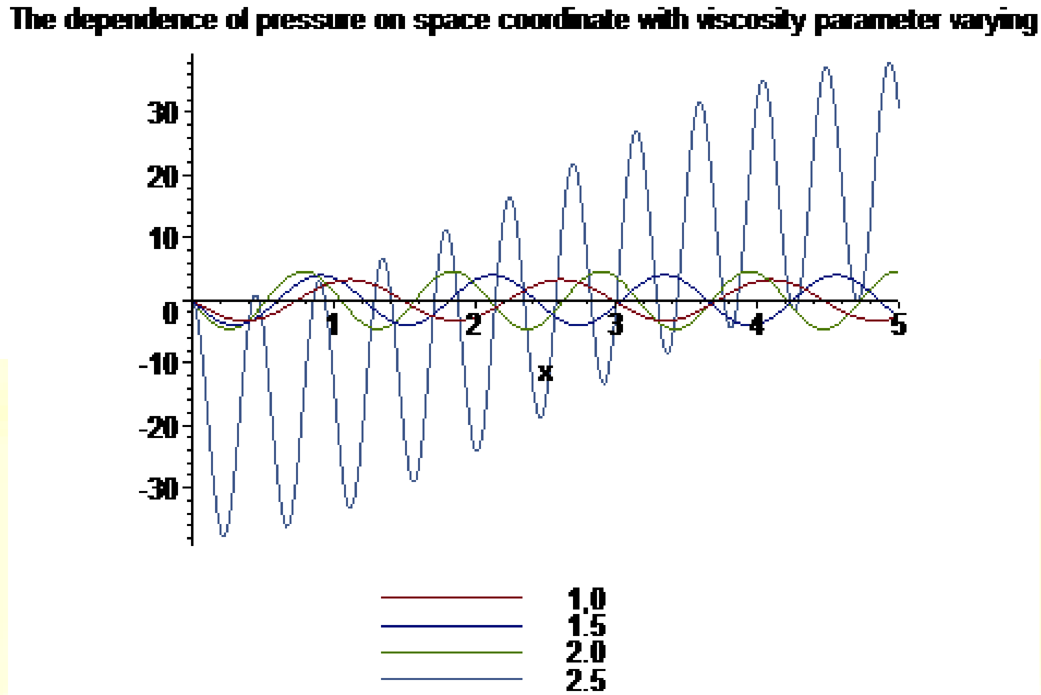

Figure 3: Pressure profile p against boundary layer x for varying viscous term (μ)

For the adiabatic expansion of fluids, $\beta = 0$, $m = \dfrac{C_v}{C_p}$, combination of equations (3) and (4) reduced to

$$\frac{(m+1)\xi\mu\rho_0^{\frac{1}{m}}}{\kappa}\frac{\partial\rho}{\partial t} = \nabla^2\rho^{\left(\frac{1+m}{m}\right)} \tag{14}$$

Following the same procedure adopted for the isothermal case, we write equation (14)

$$\frac{(m+1)\xi\mu\rho_0^{\frac{1}{m}}}{\kappa}\frac{\partial\rho}{\partial t} = \frac{(1+m)}{m}\frac{\partial^2\rho}{\partial x^2} \tag{15}$$

Transformation of equation (15) will result into

$$\theta''(x) + \frac{a\lambda}{b}\theta(x) = 0 \tag{16}$$

where $a = \dfrac{(m+1)\xi\mu\rho_0^{\frac{1}{m}}}{\kappa}$. $b = \dfrac{(1+m)}{m}$

The solution of equation (16) is

$$\theta(x) = cCos\sqrt{\frac{a\lambda}{b}}x + dSin\sqrt{\frac{a\lambda}{b}}x \qquad (17)$$

where c and d are constants

Imposition of the boundary conditions and transformation of it using equation (8) is given by

$$\rho = -\left(Sin^{-1}\left(\sqrt{\frac{a\lambda}{b}}\right)\right)Sin\sqrt{\frac{a\lambda}{b}}x \qquad (18)$$

Finally, considering the equation of state for adiabatic expansion of fluids, we get

$$\log p = \frac{1}{m}\log\left[-\frac{1}{\rho_0}\left(Sin^{-1}\sqrt{\frac{a\lambda}{b}}\right)Sin\sqrt{\frac{a\lambda}{b}}x\right] \qquad (19)$$

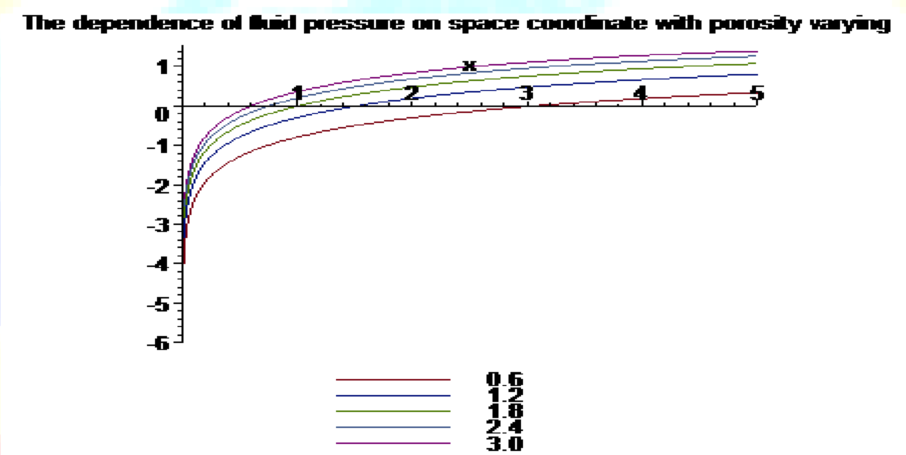

Figure 4: Pressure profile p against boundary layer x for varying porosity term. ξ

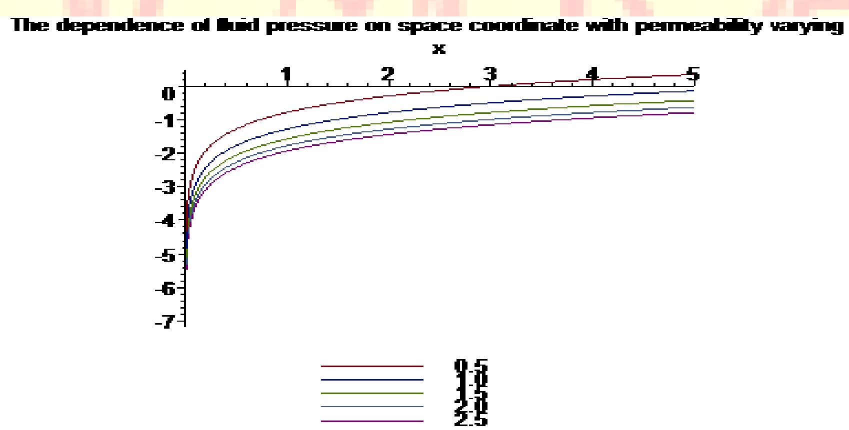

Figure 5: Pressure profile p against boundary layer x for varying permeability term (κ)

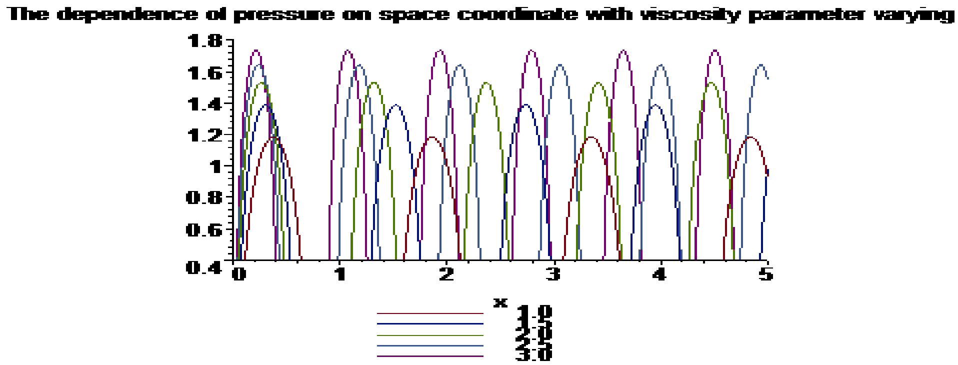

Figure 6: Pressure profile p against boundary layer x for varying viscous term (μ)

Results and discussion

In order to get physical insight and numerical validation of the problem, an approximate value of the ratio of specific heat capacity at constant volume to that at constant pressure $\left(\frac{C_v}{C_p}=2\right)$, constant viscosity of fluid at 20^0C (air $\mu = 1.0N.sm^{-2}$) and decay constant $(\lambda = 0.0035)$ is chosen. The values of other parameters made use of are

$\xi = 0.6,1.2,1.8,2.4,3.0$

$\kappa = 0.5,1.0,1.5,2.0,2.5$

$\rho_0 = 1.29kgm^{-3}$

$\mu = 1.0,1.5,2.0,2.5,3.0$

For the isothermal expansion of fluids, increase in porosity of the photosphere as a result of decrease in the granules brings about a corresponding decrease in the pressure of the fluids as depicted in figure 1. This observation is in agreement with theoretical description of the influence of porosity on the pressure of fluids. Figure 2 shows the increase in permeability of fluid pressure. It is observed that increase in permeability of the photosphere increases the pressure of the fluid. Figure 3 shows the increase in viscosity of isothermal fluids which reduce the pressure and by extension reduce its permeability. For adiabatic expansion of fluids, figure 4, figure 5 and figure 6 showed the influence of porosity, permeability and viscosity respectively on the pressure exerted on the photosphere. If we assume that the negative sign in equation (19) (pressure gradient) is positive, it is shown clearly that from figure 4, that increase in porosity increases the pressure exerted on the photosphere thereby reducing the granules. Similarly, increase in permeability, decreases the pressure of the fluid through the photosphere and finally, increase in viscosity of adiabatic fluid reveals that increase in viscous term result in decreases the pressure and reduce its permeability.

Conclusion

For isothermal and adiabatic expansion of fluids, there is a general trend of fluids pressure being reduced or increased as the porosity and permeability term increases or decreases which is in agreement with earlier studies of [8] and [13]. Also if several gases of varying viscosity are present in the photosphere, the pressure will be affected and flow may be steady.

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