STABILITY CODITIONS OF DISCRETE-TIME PREY-PREDATOR SPECIES WITH SCAVENGER

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ABSTRACT

The discrete- time prey-predator species with scavenger model is proposed. All possible equilibrium points are found. Stability conditions of arising equilibrium points are analyzed with numerical examples. Further, the dynamical behavior of this model is very complicated.

1-INTRODUCTION:

The ecosystem divided into four species due to their eating type, viz. Herbivores, carnivores, top-level carnivores and scavengers [1]. Scavenger is consuming on the decaying meat which keep the ecosystem free from the dead animal bodies [2]. So, the scavenger plays an important role in the food web when they break down the organic material and recycle it into ecosystems as nutrients [3].

Also, a scavenger has no negative effects on the population that it scavengers [4]. On other words, there is no real interaction between the scavenger and the other species. Now a days, there are several mathematical models have been developed to describe the effect of the scavenger in the ecosystem, see [3-6].

Our goal is to involve the scavenger species as third population in the discrete-time prey-predator model which we investigated in [7].The new model become as non-linear discrete-time prey-

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predator species with scavenger such that X_n, Y_n are the prey and predator populations while Z_n is the scavenger one. See the following system:

$$\begin{aligned} X_{n+1} &= X_n + h[aX_n(1-X_n) - bX_nY_n - rZ_n] \\ Y_{n+1} &= Y_n + h[cY_n(1-Y_n) + bX_nY_n - sZ_n] \\ Z_{n+1} &= Z_n + hZ_n[-d + eX_n + fY_n + gZ_n] \end{aligned} \tag{1}$$

Where a, b, c, d, e, f, g, r and s are all positive parameters.

We see that, system (1) includes three subsystems viz. The prey-predator system with absence of scavenger population, the prey- scavenger system with absence of predator population and the predator - scavenger system with absence of prey population.

2- THE EQUILIBRIUM POINTS:

The equilibrium points of the system (1) can be found if it is equal to the vector X^T where $X = (x, y, z)$ .Thus, this implies two sets of the equilibrium points with respect to the scavenger species as follow:

i. With absence of scavenger species:

- $(x, y, z) = (0, 0, 0)$ is the origin equilibrium point.
- $(x, y, z) = (1, 0, 0)$ is the prey equilibrium point
- $(x, y, z) = (0, 1, 0)$ is the predator equilibrium point.
- $(x, y, z)=(\frac{ac-bc}{ac+b^2}, \frac{ac+ab}{ac+b^2}, 0)$ is the prey-predator equilibrium point, where $a > b$.

ii- With the scavenger species

- $(x, y, z)=(\frac{dr-ag}{er-ag}, 0, \frac{a(e-d)}{er-ag})$ is the prey-scavenger equilibrium point, where $e > d$ and $dr > ag$ or($d > e$ and $ag > er$).
- $(x, y, z)=(0, \frac{ds-cg}{fs-cg}, \frac{c(f-d)}{fs-cg})$ is the predator-scavenger equilibrium point, where $f > d$ and $ds > cg$ or($d > f$ and $cg > fs$).
- $(x, y, z)=(x^*, y^*, z^*)$ is the coexisting equilibrium point where

$x^* = \frac{(bs+cr)(af+be-bd)+(ac-bc)(r-bg)}{af(bs+cr)+(ae+b^2)(r-bg)}, y^* = \frac{a(1-x^*)-rz^*}{b}$ and $z^* = \frac{bc-ac+acx^*+b^2x^*}{bs+cr}$.

3- STABILITY AND NUMERICAL ANALYSIS:

A technique for analyzing nonlinear systems qualitatively is the analysis of the behavior of the solutions near equilibrium points using linearization .In general, if we want to study the stability of an equilibrium point in 3 dimensional systems firstly compute the Jacobian matrix and its characteristic equation then find out their roots as follows:

$$J(x,y,z) = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad (2)$$

$$F(\lambda) = \lambda^3 + M_1\lambda^2 + M_2\lambda + M_3 = 0 \quad (3)$$

Where

$M_1 = -(a_{11} + a_{22} + a_{33})$

$M_2 = a_{11}a_{22} + a_{11}a_{33} + a_{22}a_{33} - a_{23}a_{32} - a_{12}a_{21} - a_{13}a_{31}$

$M_3 = a_{11}(a_{23}a_{32} - a_{22}a_{33}) + a_{21}(a_{12}a_{33} - a_{13}a_{32}) + a_{31}(a_{13}a_{22} - a_{12}a_{23})$

Remark: The equilibrium point (x,y,z) is stable if and only if all the absolute values of the roots of the equation (3) are less than one[8].

3-1 proposition: The origin equilibrium point (0,0,0) and prey equilibrium point (1,0,0) are unstable.

3-2 proposition: The predator equilibrium point $(0,1,0)$ is stable if $h < min\{\frac{2}{b-a}, \frac{2}{c}, \frac{2}{d-f}\}$.

Example: In the system (1) we have that the following parameters values $a = 0.12, b = 0.15, c = 0.1, d = 1.51, e = 0.6, f = 1, g = 0.5, r = 0.$ and $s = 0.4$ where initial point $(x_0, y_0, z_0) = (0.5, 0.5, 0.5)$ and$h = 2.33$. Under the above fixed values we see that the predator equilibrium point (0,1,0) is stable. See the phase portrait and time series in the figure (1) bellow.

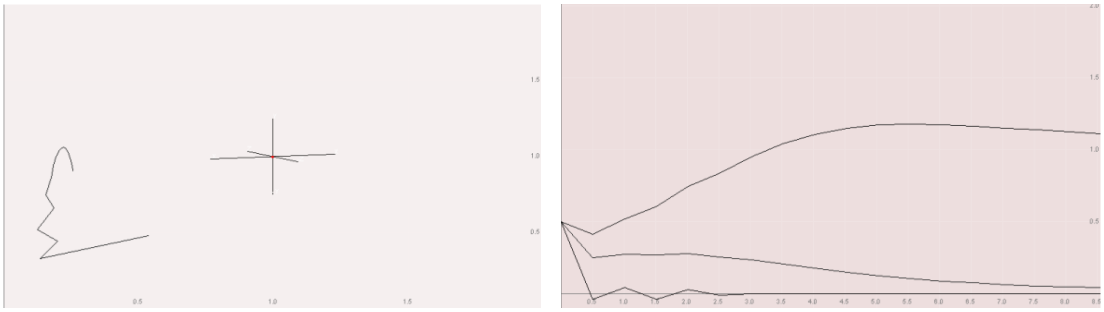

Figure (1): Stability of the predator equilibrium point(0,1,0).

3-3 proposition: Let $a > b$ then we have that the prey-predator equilibrium point $\left(\frac{ac-bc}{ac+b^2}, \frac{ac+ab}{ac+b^2}, 0\right)$ is stable if the following conditions hold:

1. $\left|1 + h\frac{c^2(a-b)+af(a+b)-d(ac+b^2)}{ac+b^2}\right| < 1;$
2. $\left|\frac{-T\pm\sqrt{T^2-4D}}{2}\right| < 1.$

Where

$T = 2 - h\frac{a+c}{ac+b^2}$

and $D = 1 - hac\frac{a+c}{ac+b^2} + h^2\frac{(a-b)(b+c)(ab^2c+a^2c^2)}{(ac+b^2)^2}$.

With the absence of scavenger species, system (1) can be reduced to a sub system as discrete –time prey-predator which we studied before. This sub model exhibit complex dynamic behavior. For more details see [7].

Example: In the system (1) we have that the following parameters values $a = 0.2, b = 0.19, c = 5, d = 1.51, e = 0.6, f = 0.04, g = 0.04, r = 0.6$and $s = 0.6$ where inertial point $(x_0, y_0, z_0) = (0.5, 0.5, 0.5)$ and $h = 0.39$. Under the above fixed values we see that the prey-predator equilibrium point (0.048258,1.001834,0) is stable. See the phase portrait and time series in the figure (2).

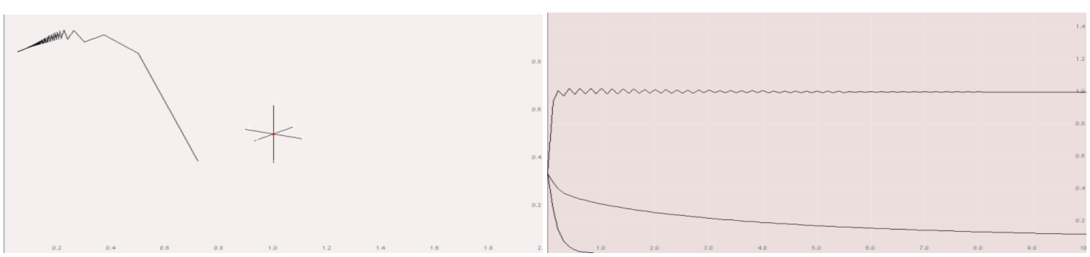

Figure (2): Stability of the prey-predator equilibrium point(0.048258,1.001834,0) .

3-4 proposition: Let $e > d$ and $dr > ag$ then we have that the prey-scavenger equilibrium point $(\frac{dr-ag}{er-ag}, 0, \frac{a(e-d)}{er-ag})$ is stable if the following conditions hold:

1- $\left|1 + h\frac{c(er-ag)+b(dr-ag)-as(e-d)}{er-ag}\right| < 1;$

2- $\left|\frac{-T\pm\sqrt{T^2-4D}}{2}\right| < 1.$

Where

$$T = 2 + ha\frac{g(e-d)-(d-ag)}{er-ag}$$

and $D = 1 - ha\frac{g(e-d)-(dr-ag)+h(e-d)(dr-ag)}{er-ag}$.

Example: In the system (1) we have that the following parameters values $a = 25, b = 5, c = 0.3, d = 5, e = 4, f = 7, g = 2, r = 1$ and $s = 1$ where inertial point $(x_0, y_0, z_0) = (0.5, 0.5, 0.5)$ and $h = 1.2$. Under the above fixed values we see that the prey-scavenger equilibrium point (0.978261,0,0.543478) is stable. See the phase portrait and time series in the figure (3).

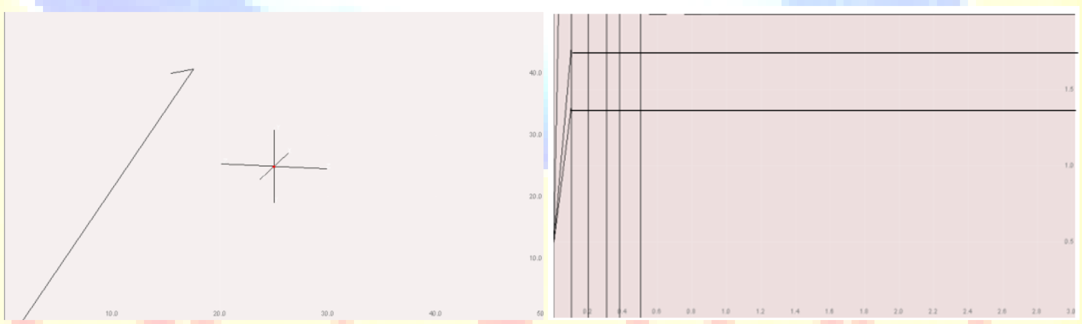

Figure (3): Stability of the prey-scavenger equilibrium point(0.978261,0,0.543478) .

3-5 proposition: Let $f > d$ and $ds > cg$ then we have that the predator-scavenger equilibrium point $(0, \frac{ds-cg}{fs-cg}, \frac{c(f-d)}{fs-cg})$ is stable if the following conditions hold:

1- $\left|1 + h\frac{a(fs-cg)-b(ds-cg)-cr(f-d)}{fs-cg}\right| < 1;$

2- $\left|\frac{-T\pm\sqrt{T^2-4D}}{2}\right| < 1.$

Where

$T = 2 + hc\frac{g(f-d)-(ds-cg)}{ds-cg}$

and $D = 1 + hc\frac{g(f-d)-(ds-cg)+h(f-d)(ds-cg)}{ds-cg}$.

Example: In the system (1) we have that the following parameters values $a = 2, b = 5, c = 0.3, d = 5, e = 4, f = 7, g = 2, r = 1$and $s = 1$ where inertial point $(x_0, y_0, z_0) = (0.5, 0.5, 0.5)$ and $h = 0.5$. Under the above fixed values we see that the predator-scavenger equilibrium point (0,0.6875,0.069375) is stable. See the phase portrait and time series in the figure (4).

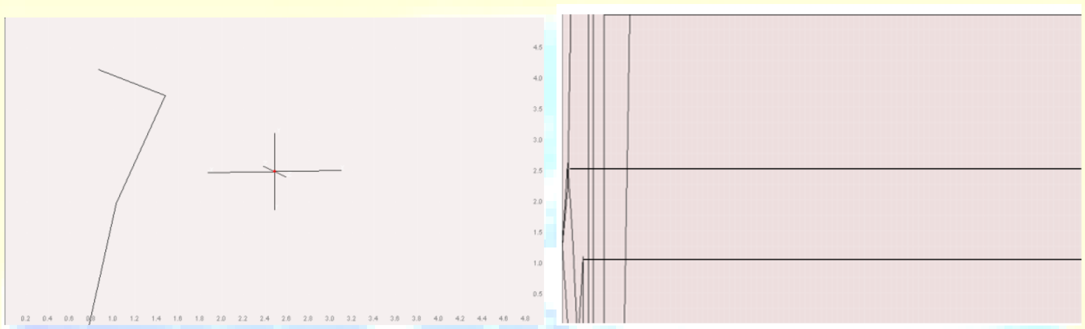

Figure (4): Stability of the predator-scavenger equilibrium point(0,0.6875,0.069375) .

3-6 proposition: The coexisting equilibrium point (x^*, y^*, z^*) is locally asymptotically stable if all the following Routh-Hurwitz conditions [3] hold:

1. $3 + M_1 - M_2 - 2M_3 > 0;$
2. $1 - M_2 + M_3(M_1 - M_3) > 0;$
3. $1 - M_1 + M_2 - M_3 > 0.$

Where

$M_1 = -3 - h[(b + e - 2a)x^* + (f - b - 2c)y^* + (2g - r - s)z^* + a + c - d]$

$$\begin{aligned} M_2 = &\{1 + h[a(1 - 2x^*) - by^* - rz^*]\}\{1 + h[c(1 - 2y^*) + bx^* - sz^*]\} \\ &+ \{1 + h[a(1 - 2x^*) - by^* - rz^*]\}\{1 + h(-d + ex^* + fy^* + 2gz^*]\} \\ &+ \{1 + h[c(1 - 2y^*) + bx^* - sz^*]\}\{1 + h(-d + ex^* + fy^* + 2gz^*]\} \\ &+ h^2b^2x^*y^* + h^2erx^*z^* + h^2fsy^*z^* \end{aligned}$$

$$M_3 = \{1 + h[a(1-2x^*) - by^* - rz^*]\}\{-h^2 fsy^*z^* - \{\{1 + h[c(1-2y^*) + bx^* - sz^*]\}\{1 + h(-d + ex^* + fy^* + 2gz^*]\}\} + \{h^2b^2x^*y^*\{1 + h(-d + ex^* + fy^* + 2gz^*]\} + h^3 bfrx^*y^*z^*\} + (hez^*)\{(-hbx^*)\{1 + h[c(1-2y^*) + bx^* - sz^*]\} - h^2 bsx^*y^*\}$$

Example: In the system (1) we have that the following parameters values $a = 150, b = 20, c = 0.3, d = 15, e = 4, f = 2.5, g = 2, r = 10$ and $s = 10$ where inertial point $(x_0, y_0, z_0) = (0.5, 0.5, 0.5)$ and $h = 0.01$. Under the above fixed values we see that the coexisting equilibrium point (0.491403,3.371929,0.885095) is stable. See the bellow phase portrait and time series in the figure (5).

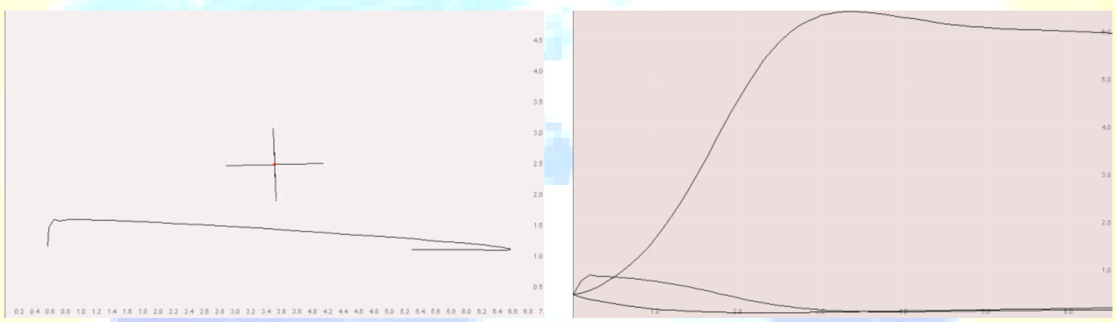

Figure (5): Stability of the coexisting equilibrium point(0.491403,3.371929,0.885095)

4-CONCLUSION:

In our model, there are seven equilibrium points. Each prey and predator population has logistic growth rate with absence of others. Scavenger works as top predator. Also, the subsystems prey-scavenger and predator-scavenger can be used as future work models.

The model of discrete-time prey-predator with scavenger makes an interesting system in mathematical modeling as well as ecosystems science. Also, scavenger species need to shed more lights on it due to its importance. Finally, the phaser program is used to analysis our model numerically.

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