FUZZY SOFT MATRIX AND MULTI CRITERIA IN DECISION MAKING BASED ON WEIGHTED T-NORM OPERATORS

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ABSTRACT

The purpose of this paper is to put forward the notion of fuzzy soft matrix theory and some basic results. In this paper , we define fuzzy soft matrices and some new operators on weighted t-norms with properties . Lastly we have given an application in decision making based on different operators of weighted t-norms.

Key words – Soft sets, fuzzy soft matrices , operators of weighted t-norms.

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1. INTRODUCTION

Most of our traditional tools for formal modeling, reasoning, and computing are crisp, deterministic, and precise in character. However, in real life, there are many complicated problems in engineering, economics, environment, social sciences medical sciences etc. that involve data which are not all always crisp, precise and deterministic in character because of various uncertainties typical problems. Such uncertainties are being dealing with the help of the theories, like theory of probability, theory of fuzzy sets, theory of intuitionistic fuzzy sets, theory of interval mathematics and theory of rough sets etc. Molodtsov [1] also described the concept of "Soft Set Theory" having parameterization tools for dealing with uncertainties. Researchers on soft set theory have received much attention in recent years. Maji and Roy [3,4] first introduced soft set into decision making problems. Maji et al.[2] introduced the concept of fuzzy soft sets by combining soft sets and fuzzy sets. Cagman and Enginoglu [5] defined soft matrices which were a matrix representation of the soft sets and constructed a soft max-min decision making method. Cagman and Enginoglu [6] defined fuzzy soft matrices and constructed a decision making problem . Borah et al.[7] extended fuzzy soft matrix theory and its application. Maji and Roy [8] presented a novel method of object from an imprecise multi-observer data to deal with decision making based on fuzzy soft sets. Majumdar and samanta [9] generalized the concept of fuzzy soft sets.In this paper , we have introduced some operators of fuzzy soft matrix on the basis of weighted t-norms . We have also discussed their properties . Finally we have given an application in decision making problem on the basis of weighted t-norms operators .

2. Definition and Preliminaries:

2.1Soft set [1] Let U be an initial universe , P(U) be the power set of U , E be the set of all parameters and A⊆ E. A soft set (f_A,E) on the universe U is defined by the set of order pairs

$$(f_A,E) = \{(e , f_A(e)) : e\in E , f_A(e) \in P(U) \}$$

$$\text{where } f_A : E \to P(U) \text{ such that } f_A(e) = \phi \text{ if } e \notin A.$$

Here f_A is called an approximate function of the soft set (f_A,E). The set f_A(e) is called e-approximate value set or e-approximate set which consists of related objects of the parameter e ∈ E.

Example 1 let U = { u_1 , u_2 , u_3 , u_3} be a set of four balls and E = { white(e_1), red (e_2) ,blue (e_3) } be a set of parameters. If A ={e_1 , e_2} ⊆ E. Let $f_A(e_1)$= { u_1 , u_2 , u_3 } and $f_A(e_2)$= { u_1 , u_2 , u_3 , u_4} , then we write the soft set (f_A,E)= {(e_1,{ u_1 , u_2 , u_3 }), (e_2,{ u_1 , u_2 , u_3 , u_4})} over U which describe the "colour of the balls" which Mr. X is going to buy.

2.2 Fuzzy set [2] A Fuzzy Set (FS) in the universal set U is defined as A= { (x , μ_A(x)) | x ∈ U } where μ_A : U → [0,1] is a mapping called membership function of the fuzzy set A .

Example 2. Consider the example 1, here we can not express with only two real numbers 0 and 1, we can characterized it by a membership function instead of crisp number 0 and 1, which associate with each element a real number in the interval [0,1].Then

(f_A,E) = { $f_A(e_1)$ = { (u_1,.7) ,(u_2,.5) ,(u_3,.4) }, $f_A(e_2)$ = { (u_1,.5) , (u_2,.1) , (u_3,.5) ,(u_4,.2) } } is the fuzzy soft set representing the "colour of the balls" which Mr. X is going to buy.

2.3 Fuzzy Soft Matrices (FSM) [5] Let (f_A, E) be fuzzy soft set over U. Then a subset of U x E is uniquely defined by

$$R_A = \{ (u , e) : e \in A \ , u \in f_A(e) \},$$

which is called relation form of (f_A, E) .

The characteristic function of R_A is written by

μ_{R_A} : U x E → [0 , 1] , where μ_{R_A}(u , e) ∈ [0,1] is the membership value of u∈U for each e ∈U.

If $\mu_{ij} = \mu_{R_A}(u_i,e_j)$, we can define a matrix

$$[\mu_{ij}]_{mXn} = \begin{bmatrix} \mu_{11} & \mu_{12} & \cdots & \mu_{1n} \\ \mu_{21} & \mu_{22} & \cdots & \mu_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \mu_{m1} & \mu_{m2} & \cdots & \mu_{mn} \end{bmatrix}$$

which is called an m x n soft matrix of the soft set (f_A,E) over U.

Therefore we can say that a fuzzy soft set (f_A,E) is uniquely characterized by the matrix $[\mu_{ij}]_{mXn}$ and both concepts are interchangeable.

Example 3. Assume that U = { u_1 , u_2 , u_3 , u_4, u_5, u_6} is a universal set and E = { e_1 , e_2 , e_3, e_4} is a set of all parameters. If A⊆ E = { e_1 , e_2 , e_4} and

$$f_A(e_1) = \{ (u_1 ,.7), (u_2,.4) , (u_3,.6) , (u_4,.1),(u_5,.6), (u_6,.5 \}$$

$$f_A(e_2) = \{ (u_1,.3) , (u_2,.5) , (u_3,.7) , (u_4,.3) ,(u_5,.7), (u_6,.1)\}$$

$$f_A(e_4) = \{ (u_1,.4) , (u_2,.2) , (u_3,.5) , (u_4,.6),(u_5,.7), (u_6,.3) \}$$

Then the fuzzy soft set (f_A,E) is a parameterized family { $f_A(e_1)$, $f_A(e_2)$, $f_A(e_3)$} of all fuzzy sets over U.

Hence the fuzzy soft matrix $[\mu_{ij}]$ can be written as

$$[\mu_{ij}] = \begin{bmatrix} .7 & .3 & 0 & .4 \\ .4 & .5 & 0 & .2 \\ .6 & .7 & 0 & .5 \\ .1 & .3 & 0 & .6 \\ .6 & .7 & 0 & .7 \\ .5 & .1 & 0 & .3 \end{bmatrix}$$

2.4. Zero Fuzzy Soft Matrix [6] Let $[a_{ij}] \in \text{FSM}_{m\,X\,n}$. Then $[a_{ij}]$ is called a Zero Fuzzy Soft Matrix denoted by [0], if a_{ij} = 0 for all i and j.

2.5. Universal Fuzzy Soft Matrix [6] Let $[a_{ij}] \in \text{FSM}_{m\,X\,n}$. Then $[a_{ij}]$ is called a Universal Fuzzy Soft Matrix denoted by [1], if a_{ij} = 1 for all i and j.

2.6. Fuzzy Soft Sub Matrix [6] Let $[a_{ij}]$, $[b_{ij}]$ $\in \text{FSM}_{m\,X\,n}$. Then

$[a_{ij}]$ is said to be a Fuzzy Soft Sub Matrix of $[b_{ij}]$ denoted by $[a_{ij}] \tilde{\subseteq} [b_{ij}]$ if $a_{ij} \le b_{ij}$ for all i and j .

2.7. Union of Fuzzy Soft Matrices [6] Let $[a_{ij}]$, $[b_{ij}]$ $\in \text{FSM}_{m\,X\,n}$. Then

Union of $[a_{ij}]$ and $[b_{ij}]$, denoted by $[a_{ij}] \tilde{\cup} [b_{ij}]$ is defined as

$[a_{ij}] \tilde{\cup} [b_{ij}$ =max $\{a_{ij}, b_{ij}\}$ for all i and j

2.8. Intersection of Fuzzy Soft Matrices [6] Let $[a_{ij}]$, $[b_{ij}]$ $\in \text{FSM}_{m\,X\,n}$. Then

Intersection of $[a_{ij}]$ and $[b_{ij}]$, denoted by $[a_{ij}] \tilde{\cap} [b_{ij}]$ is defined as

$[a_{ij}] \tilde{\cap} [b_{ij}]$ =min $\{a_{ij}, b_{ij}\}$ for all i and j

2.9. Compliment Fuzzy Soft Matrix [6] Let $[a_{ij}]$ $\in \text{FSM}_{m\,X\,n}$. Then Complement of Fuzzy Soft Matrix $[a_{ij}]$, denoted by $[a_{ij}]^0$ is defined as $[a_{ij}]^0 = 1 - a_{ij}$ for all i and j.

2.10. Fuzzy Soft Equal Matrices [6] Let $[a_{ij}]$, $[b_{ij}]$ $\in \text{FSM}_{m\,X\,n}$. Then

$[a_{ij}]$ and $[b_{ij}]$ are said to be Fuzzy Soft Equal Matrices , denoted by $[a_{ij}]$ =$[b_{ij}]$ if a_{ij} =b_{ij} for all i and j.

Proposition1. Let $[a_{ij}] \in \text{FSM}_{m\,X\,n}$. Then

i) $[[a_{ij}]^0]^0 = [a_{ij}]$ iv) $[a_{ij}] \tilde{\cap} [a_{ij}] = [a_{ij}]$

ii) $[a_{ij}] \tilde{\subseteq} [a_{ij}]$ v) $a_{ij}] \tilde{\cup} [0] = [a_{ij}]$

iii) $[a_{ij}] \tilde{\cup} [a_{ij}] = [a_{ij}]$ vi) $[a_{ij}] \tilde{\cap} [0] = [0]$

Proposition2. Let $[a_{ij}]$, $[b_{ij}]$, $[c_{ij}]$ $\in$ FSM$_{m \, X \, n}$. Then

i) $[a_{ij}] = [b_{ij}]$ and $[b_{ij}] = [c_{ij}] \Rightarrow [a_{ij}] = [c_{ij}]$

ii) $a_{ij}] \tilde{\subseteq} [b_{ij}]$ and $[b_{ij}] \tilde{\subseteq} [c_{ij}] \Rightarrow [a_{ij}] = [c_{ij}]$

Proposition3. Let $[a_{ij}]$, $[b_{ij}]$, $[c_{ij}]$ $\in$ FSM$_{m \, X \, n}$. Then

i) $[a_{ij}] \tilde{\cup} ([b_{ij}] \tilde{\cap} [c_{ij}]) = ([a_{ij}] \tilde{\cup} [b_{ij}]) \tilde{\cap} ([a_{ij}] \tilde{\cup} [c_{ij}])$

ii) $[a_{ij}] \tilde{\cap} ([b_{ij}] \tilde{\cup} [c_{ij}]) = ([a_{ij}] \tilde{\cap} [b_{ij}]) \tilde{\cup} ([a_{ij}] \tilde{\cap} [c_{ij}])$

2.11. Fuzzy Soft Rectangular Matrix [7] Let $[a_{ij}]$ $\in$ FSM$_{m \, X \, n}$. Then $[a_{ij}]$ is said to be a Fuzzy Soft Rectangular Matrix if m $\neq$ n .

2.12. Fuzzy Soft Square Matrix [7] Let $[a_{ij}]$ $\in$ FSM$_{m \, X \, n}$. Then $[a_{ij}]$ is said to be a Fuzzy Soft Square Matrix if m = n .

2.13. Fuzzy Soft Row Matrix [7] Let $[a_{ij}]$ $\in$ FSM$_{m \, X \, n}$. Then $[a_{ij}]$ is said to be a Fuzzy Soft Row Matrix if m = 1 .

2.14. Fuzzy Soft Column Matrix [7] Let $[a_{ij}]$ $\in$ FSM$_{m \, X \, n}$. Then $[a_{ij}]$ is said to be a Fuzzy Soft Column Matrix if n = 1 .

2.15. Fuzzy Soft Diagonal Matrix [7] Let $[a_{ij}]$ $\in$ FSM$_{m \, X \, n}$. Then $[a_{ij}]$ is said to be a Fuzzy Soft Diagonal Matrix if m = n and $a_{ij} = 0$ for all i $\neq$ j .

2.16. Fuzzy Soft Upper Triangular Matrix [7] Let $[a_{ij}]$ $\in$ FSM$_{m \, X \, n}$. Then $[a_{ij}]$ is said to be a Fuzzy Soft Upper Triangular Matrix if m = n and $a_{ij} = 0$ for all i> j .

2.17. Fuzzy Soft Lower Triangular Matrix [7] Let $[a_{ij}]$ $\in$ FSM$_{m \, X \, n}$. Then $[a_{ij}]$ is said to be a Fuzzy Soft Lower Triangular Matrix if m = n and $a_{ij} = 0$ for all i< j .

2.18. Fuzzy Soft Triangular Matrix [7] Let $[a_{ij}]$ $\in$ FSM$_{m \, X \, n}$. Then $[a_{ij}]$ is said to be a Fuzzy Soft Triangular Matrix if is either fuzzy soft lower or fuzzy soft upper triangular matrix for all i and j .

2.19. t-Norm [10]: Let T : [0,1] x [0,1] be a function satisfying the following axioms:

i) T(a , 1) = a , ∀ a ∈ [0,1] (Identity)

ii) T(a, b) = T(b ,a) , ∀ a ,b ∈ [0,1] (Commutativity)

iii) if $b_1 \leq b_2$, then T(a, b_1) ≤ T(a, b_2) , ∀ a , b_1 , b_2 ∈ [0,1] (Monotonicty)

iv) T (a, T(b,c)) = T(T (a,b) , c) , ∀ a ,b ∈ [0,1] (Associativity)

Then T is called t-norm.

A t-norm is said to be continuous if T is continuous function in [0,1].

An example of continuous t- Norm is a b .

N.B. :The functions used for intersection of fuzzy sets are called t-norms.

2.20. t-Conorm[10]: Let S : [0,1] x [0,1] be a function satisfying the following axioms:

i) S(a , 0) = a , ∀ a ∈ [0,1] (Identity)

ii) S(a,b) = S(b,a) , ∀ a ,b ∈ [0,1] (Commutativity)

iii) if $b_1 \leq b_2$, then S(a, b_1) ≤ S(a, b_2) , ∀ a , b_1 , b_2 ∈ [0,1] (Monotonicity)

iv) S (a, S(b,c)) = S(S (a,b) , c) ∀ a ,b ∈ [0,1] (Associativity)

Then S is called t-conorm.

A t-conorm is said to be continuous if S is continuous function in [0,1].

N.B. :The functions used for union of fuzzy sets are called t-conorms.

An example of continuous t- Conorm is a + b− a .b .

2.21. Union of Fuzzy Soft Matrices on t-norm: Let $[a_{ij}]$, $[b_{ij}]$ ∈FSM$_{m\,X\,n}$.Then

Union of Fuzzy Soft Matrices $[a_{ij}]$ and $[b_{ij}]$ on t-norm is defined by

$[a_{ij}] \tilde{\cup} [b_{ij}] = [a_{ij} + b_{ij} - a_{ij} \,.\, b_{ij}]$ for all i and j.

2.22. Intersection of Fuzzy Soft Matrices on t-norm: Let$[a_{ij}]$, $[b_{ij}]$ ∈FSM$_{m\,X\,n}$.Then

Intersection of Fuzzy Soft Matrices $[a_{ij}]$ and $[b_{ij}]$ on t-norm is defined by

$[a_{ij}] \tilde{\cap} [b_{ij}] = [a_{ij} \,.\, b_{ij}]$ for all i and j.

Proposition4. Let $[a_{ij}]$, $[b_{ij}]$, $[c_{ij}]$ ∈ FSM$_{m\,X\,n}$. Then

i) $[a_{ij}] \tilde{\cup} [0] = [a_{ij}]$ iii) $[a_{ij}] \tilde{\cup} [b_{ij}] = [b_{ij}] \tilde{\cup} [a_{ij}]$

ii) $[a_{ij}] \tilde{\cup} [1] = [1]$ iv) $([a_{ij}] \tilde{\cup} [b_{ij}]) \tilde{\cup} [c_{ij}] = [a_{ij}] \tilde{\cup} ([b_{ij}] \tilde{\cup} [c_{ij}])$

Proposition5. Let $[a_{ij}]$, $[b_{ij}]$, $[c_{ij}]$ $\in$ FSM$_{m\,X\,n}$. Then

i) $[a_{ij}] \tilde{\cap} [0] = [a_{ij}]$ iii) $[a_{ij}] \tilde{\cap} [b_{ij}] = [b_{ij}] \tilde{\cap} [a_{ij}]$

ii) $[a_{ij}] \tilde{\cap} [1] = [a_{ij}]$ iv) $([a_{ij}] \tilde{\cap} [b_{ij}]) \tilde{\cap} [c_{ij}] = [a_{ij}] \tilde{\cap} ([b_{ij}] \tilde{\cap} [c_{ij}])$

Proposition6. Let $[a_{ij}]$, $[b_{ij}]$ $\in$ FSM$_{m\,X\,n}$. Then

De Morgan's type results are true :

i) $([a_{ij}] \tilde{\cup} [b_{ij}])^0 = [a_{ij}]^0 \tilde{\cap} [b_{ij}]^0$

ii) $([a_{ij}] \tilde{\cap} [b_{ij}])^0 = [a_{ij}]^0 \tilde{\cup} [b_{ij}]^0$

Proof: for all i and j ,

i) $([a_{ij}] \tilde{\cup} [b_{ij}])^0 = [a_{ij} + b_{ij} - a_{ij}.b_{ij}]^0$

$= [1 - (a_{ij} + b_{ij} - a_{ij}.b_{ij})]$

$= [1 - a_{ij} - b_{ij} + a_{ij}.b_{ij})]$

$= [(1 - a_{ij})(1 - b_{ij})]$

$= [1 - a_{ij}] \tilde{\cap} [1 - b_{ij}]$

$= [a_{ij}]^0 \tilde{\cap} [b_{ij}]^0$ □

ii) Similar proof for ii).

2.23. Scalar Multiplication of Fuzzy Soft Matrix : Let $[a_{ij}]$ $\in$ FSM$_{m\,X\,n}$. Then

Scalar Multiplication of Fuzzy Soft Matrix $[a_{ij}]$ by a scalar k denoted by k $[a_{ij}]$ is defined as k $[a_{ij}]$ = [ka_{ij}] , $0 \leq k \leq 1$.

Proposition7. Let $[a_{ij}]$ $\in$ FSM$_{m\,X\,n}$ and s and t are two scalars such that $0 \leq s, t \leq 1$. Then

i) $s(t[a_{ij}]) = (st)[a_{ij}]$

ii) $s \leq t \Rightarrow s[a_{ij}] \tilde{\subseteq} t[a_{ij}]$

iii) $[a_{ij}] \tilde{\subseteq} [b_{ij}] \Rightarrow s[a_{ij}] \tilde{\subseteq} s[b_{ij}]$

2.24. Three Important Operators of t- Norms[11] **:**

i) **Minimum Operator :** $T_M(\mu_1, \mu_2, \ldots, \mu_n) = \min(\mu_1, \mu_2, \ldots, \mu_n)$

ii) **Product Operator :** $T_P(\mu_1, \mu_2, \ldots, \mu_n) = \prod_{i=1}^{n} \mu_i$

iii) **Operator Lukasiewicz t-norm (Bounded t-norm) :**
$T_L(\mu_1, \mu_2, \ldots, \mu_n) = \max(\sum_{i=1}^{n} \mu_i - n + 1, 0)$

Example4. Let $[a_{ij}]$, $[b_{ij}]$ $\in$ FSM$_{3\,X3}$ where

$$[a_{ij}] = \begin{bmatrix} .3 & .2 & .6 \\ .1 & .7 & .5 \\ .3 & .4 & .6 \end{bmatrix} \text{ and } [b_{ij}] = \begin{bmatrix} .5 & .7 & .6 \\ .5 & .6 & .3 \\ .3 & .4 & .3 \end{bmatrix}. \text{ Then}$$

$$T_M([a_{ij}], [b_{ij}]) = \begin{bmatrix} .3 & .2 & .6 \\ .1 & .6 & .3 \\ .3 & .4 & .3 \end{bmatrix}$$

$$T_P([a_{ij}], [b_{ij}]) = \begin{bmatrix} .15 & .14 & .36 \\ .5 & .42 & .15 \\ .09 & .16 & .18 \end{bmatrix}$$

$$T_L([a_{ij}], [b_{ij}]) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

2.25.Arithmetic Mean (A.M.) of Fuzzy Soft Matrix : Let $\tilde{A} = [a_{ij}] \in$ FSM$_{m\,X\,n}$. Then Arithmetic Mean of Fuzzy Soft Matrix of membership value denoted by $\tilde{A}_{AM}$ is defined as $\tilde{A}_{AM} = \frac{\sum_{j=1}^{n} \mu_{ij}^{\tilde{A}}}{n}$.

2.26. Three Important t- Norm Operators With Weights :

i) **Minimum Operator With Weight :** $T_M^w(\mu_1; w_1, \mu_2; w_2, \ldots, \mu_n; w_n) = \min\{1 - w_1(1-\mu_1), 1 - w_2(1-\mu_2), \ldots, 1 - w_n(1-\mu_n)\}$

ii) **Product Operator With Weight :** $T_P^w(\mu_1; w_1, \mu_2; w_2, \ldots, \mu_n; w_n) = \prod_{i=1}^{n}\{1 - w_i(1-\mu_i)\}$

iii) **Operator of Lukasiewicz t- norm With Weight (Bounded t-norm With Weight) :**
$T_L^w(\mu_1; w_1, \mu_2; w_2, \ldots, \mu_n; w_n) = \max(\sum_{i=1}^{n} 1 - w_i(1-\mu_i) - n + 1, 0)$

Proposition8. Let $[a_{ij}] \in$ FSM$_{m\,X\,n}$ and w_1 be the weight . Then

i) $[a_{ij}]\ \tilde{\cap}_{T_M^w}\ [a_{ij}] = [a_{ij}]$ iv) $[a_{ij}]\ \tilde{\cap}_{T_M^w}\ [1] = [a_{ij}]$

ii) $[a_{ij}]\ \tilde{\cap}_{T_P^w}\ [a_{ij}] = [a_{ij}]$ v) $[a_{ij}]\ \tilde{\cap}_{T_P^w}\ [1] = [a_{ij}]$

iii) $[a_{ij}] \widetilde{\cap}_{T_L^w} [a_{ij}] = [a_{ij}]$ vi) $[a_{ij}] \widetilde{\cap}_{T_L^w} [1] = [a_{ij}]$

vii) $[a_{ij}] \widetilde{\cap}_{T_M^w} [0] = [0]$ viii) $[a_{ij}] \widetilde{\cap}_{T_P^w} [0] = [0]$

ix) $[a_{ij}] \widetilde{\cap}_{T_L^w} [0] = [0]$

Proof :

i) $[a_{ij}] \widetilde{\cap}_{T_M^w} [a_{ij}] = [\min\{1 - w_1(1 - a_{ij}), 1 - w_1(1 - a_{ij})\}]$

$= [1 - w_1(1 - a_{ij})]$

$= [a_{ij}]$ □

Similar proofs for others.

Proposition9. Let $[a_{ij}], [b_{ij}] \in FSM_{m\,X\,n}$ and w_1, w_2 be the weights . Then

i) $[a_{ij}] \widetilde{\cap}_{T_M^w} [b_{ij}] = [b_{ij}] \widetilde{\cap}_{T_M^w} [a_{ij}]$

ii) $[a_{ij}] \widetilde{\cap}_{T_P^w} [b_{ij}] = [b_{ij}] \widetilde{\cap}_{T_P^w} [a_{ij}]$

iii) $[a_{ij}] \widetilde{\cap}_{T_L^w} [b_{ij}] = [b_{ij}] \widetilde{\cap}_{T_L^w} [a_{ij}]$

Proof : Straight forward from definition .

Proposition10. Let $[a_{ij}], [b_{ij}], [c_{ij}] \in FSM_{m\,X\,n}$ and w_1, w_2, w_3 be their respective weights . Then

i) $([a_{ij}] \widetilde{\cap}_{T_M^w} [b_{ij}]) \widetilde{\cap}_{T_M^w} [c_{ij}] = [a_{ij}] \widetilde{\cap}_{T_M^w} ([b_{ij}] \widetilde{\cap}_{T_M^w} [c_{ij}])$

ii) $([a_{ij}] \widetilde{\cap}_{T_P^w} [b_{ij}]) \widetilde{\cap}_{T_P^w} [c_{ij}] = [a_{ij}] \widetilde{\cap}_{T_P^w} ([b_{ij}] \widetilde{\cap}_{T_P^w} [c_{ij}])$

iii) $([a_{ij}] \widetilde{\cap}_{T_L^w} [b_{ij}]) \widetilde{\cap}_{T_L^w} [c_{ij}] = [a_{ij}] \widetilde{\cap}_{T_L^w} ([b_{ij}] \widetilde{\cap}_{T_L^w} [c_{ij}])$

Proof :

$([a_{ij}] \widetilde{\cap}_{T_M^w} [b_{ij}]) \widetilde{\cap}_{T_M^w} [c_{ij}]$

$= \min\{1 - w_1(1 - a_{ij}), 1 - w_2(1 - b_{ij})\} \widetilde{\cap}_{T_M^w} \{1 - w_3(1 - c_{ij})\}$

$= \min\{1 - w_1(1 - a_{ij}), 1 - w_2(1 - b_{ij}), 1 - w_3(1 - c_{ij})\}$

$= \{1 - w_1(1 - a_{ij})\} \widetilde{\cap}_{T_M^w} \min\{1 - w_2(1 - b_{ij}), 1 - w_3(1 - c_{ij})\}$

$= [a_{ij}] \widetilde{\cap}_{T_M^w} ([b_{ij}] \widetilde{\cap}_{T_M^w} [c_{ij}])$ □

Similar proof for others.

3. FUZZY SOFT MATRICES IN DECISION MAKING BASED ON WEIGHTED T–NORMS

In this section , we put forward fuzzy soft matrices in decision making by using different operators of weighted t- norms.

Input: Fuzzy soft set of m objects , each of which has n parameters.

Output: An optimum result .

ALGORITHM

Step- 1: Choose the set of parameters.

Step -2: Construct the fuzzy soft matrix for the set of parameters.

Step- 3: Compute the arithmetic mean of membership value of fuzzy soft matrix as A_{AM} of different weighted t-norm operators as A_{AM}^w .

Step-4: Choose the object with highest membership value.

Example 5. Suppose the management of a company established an annual university undergraduate scholarship to support high school students with excellent performance in science (Mathematics, Physics , Chemistry) . Suppose s_1, s_2, s_3, s_4, s_5 be five best students of different universities apply for the scholarship such that U = { s_1, s_2, s_3, s_4, s_5} and E = { e_1 (Mathematics) , e_2(Physics) , e_3 (Chemistry) } be the set of parameters . Suppose three Officers Mr. A , Mr. B and Mr. C of that company decide that preference will be given on Mathematics .So .8 , .1 , .1 are given weights on Mathematics, Physics , Chemistry respectively and the following fuzzy soft matrices are constructed on the basis of the parameters as follows :

$$A = \begin{bmatrix} .8 & .9 & .8 \\ .7 & .8 & .4 \\ .9 & .6 & .7 \\ .5 & .3 & .5 \\ .9 & .6 & .7 \end{bmatrix}, B = \begin{bmatrix} .8 & .7 & .6 \\ .9 & .4 & .5 \\ .8 & .8 & .7 \\ .6 & .4 & .7 \\ .7 & .5 & .7 \end{bmatrix} \text{ and } C = \begin{bmatrix} .9 & .5 & .9 \\ .6 & .5 & .6 \\ .5 & .6 & .8 \\ .7 & .8 & .6 \\ .9 & .6 & .8 \end{bmatrix}$$

$$T_M^w = \begin{bmatrix} .84 & .95 & .96 \\ .68 & .94 & .94 \\ .6 & .96 & .97 \\ .6 & .93 & .95 \\ .76 & .95 & .97 \end{bmatrix} \text{ and } T_{M(AM)}^w = \begin{bmatrix} .92 \\ .85 \\ .84 \\ .82 \\ .89 \end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots(3.1)$$

$$T_P^w = \begin{bmatrix} .649152 & .912285 & .931392 \\ .475456 & .87514 & .85728 \\ .46368 & .903168 & .922082 \\ .31008 & .856716 & .88464 \\ .778688 & .87552 & .922082 \end{bmatrix}$$ and

$$T_{P(AM)}^w) = \begin{bmatrix} .830943 \\ .735959 \\ .772521 \\ .683812 \\ .858763 \end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(3.2)$$

$$T_L^w = \begin{bmatrix} .2 & .1 & .2 \\ 0 & 0 & 0 \\ .2 & .3 & .1 \\ 0 & 0 & 0 \\ .2 & 0 & .2 \end{bmatrix} \text{ and } T_{L(AM)}^w = \begin{bmatrix} .167 \\ 0 \\ .2 \\ 0 \\ .133 \end{bmatrix} \ldots\ldots\ldots\ldots\ldots..(3.3)$$

From the above result (3.1) ,it is obvious that s_1 student ; from (3.2) , $s_5$student and from (3.3) , s_3 student will be selected for the scholarship for their highest membership scores.

Conclusion

In this paper ,we proposed fuzzy soft matrices and defined different types of fuzzy soft matrices . We have given some definitions on t-norm operators with weight and their properties . Some of the properties have been proved. Finally , we extend our approach on weighted t-norm operators in application of decision making problems. It is obvious from the results that decisions are different for different methods on same application. This method can also be applied on other decision making problems with uncertain parameters.

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