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A note on Square Sum Prime Labeling

Sunoj B S*
Mathew Varkey T K**

Abstract (10pt)

Square sum prime labeling of a graph is the labeling of the vertices with {0,1,2-------,p-1} and the edges with sum of the squares of the labels of the incident vertices. The greatest common incidence number of a vertex (***gcin***) of degree greater than one is defined as the greatest common divisor of the labels of the incident edges. If the ***gcin*** of each vertex of degree greater than one is one, then the graph admits square sum prime labeling. Here we identify some graphs for square sum prime labeling.

Keywords:

Graph labeling;
Square sum;
Greatest common incidence number;
Prime labeling.

Author correspondence:

Sunoj B S,
Assistant Professor, Department of Mathematics
Government Polytechnic College, Attingal, Kerala, India

1. Introduction (10pt)

All graphs in this paper are simple, finite, connected and undirected. The symbol V(G) and E(G) denotes the vertex set and edge set of a graph G. The graph whose cardinality of the vertex set is called the order of G, denoted by p and the cardinality of the edge set is called the size of the graph G, denoted by q. A graph with p vertices and q edges is called a (p,q)- graph.

A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [2],[3] and [4] . Some basic concepts are taken from [1] and [2]. The square sum labeling was defined by V Ajitha, S Arumugan and K A Germina in [8]. In [5], we introduced the concept of square sum prime labeling and proved that some cycle related graphs admit this kind of labeling. In [6] and [7], we proved that some path related graphs and some tree graphs admit square sum prime labeling. In this paper we proved that jewel graph, splitting graph of star, jelly fish graph, double graph of star, modified gear graph and some planar graphs admit square sum prime labeling.

* Assistant Professor, Department of Mathematics, Government Polytechnic College, Attingal, Kerala, India
** Assistant Professor, Department of Mathematics, T K M College of Engineering, Kollam, Kerala, India

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Definition: 1.1 Let G be a graph with p vertices and q edges. The greatest common incidence number (***gcin***) of a vertex of degree greater than or equal to 2, is the greatest common divisor (gcd) of the labels of the incident edges.

2.Main Results

Definition 2.1 Let G = (V, E) be a graph with p vertices and q edges. Define a bijection f : V(G) → {0,1,2,3,----------------,p-1} by $f(v_i) = i-1$, for every i from 1 to p and define a 1-1 mapping f^*_{sqsp} : E(G) → set of natural numbers N by $f^*_{sqsp}(uv) = \{f(u)\}^2 + \{f(v)\}^2$. The induced function f^*_{sqsp} is said to be a square sum prime labeling, if the greatest common incidence number of each vertex of degree greater than one is 1.

Definition 2.2 A graph which admits square sum prime labeling is called a square sum prime graph.

Theorem 2.1 Jewel graph J_n admits square sum prime labeling.

Proof: Let G = J_n and let a, b, x, y, v_1, v_2,----------------,v_n are the vertices of G

Here |V(G)| = n+4 and |E(G)| = 2n+5

Define a function f : V → {0,1,2,3,----------------,n+3 } by

$f(v_i) = i+3$, i = 1,2,------,n

f(a) = 0, f(b) = 1, f(x) = 2, f(y) = 3

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{sqsp} is defined as follows

$f^*_{sqsp}(ay)$ = 9

$f^*_{sqsp}(by)$ = 10

$f^*_{sqsp}(ax)$ = 4

$f^*_{sqsp}(bx)$ = 5

$f^*_{sqsp}(xy)$ = 13

$f^*_{sqsp}(a\,v_i)$ = $(i+3)^2$, $1 \le i \le n$.

$f^*_{sqsp}(b\,u_i)$ = $(i+3)^2 + 1$, $1 \le i \le n$.

Clearly f^*_{sqsp} is an injection.

gcin of (a) = 1

gcin of (b) = 1

gcin of (x) = 1

gcin of (y) = 1

gcin of (v_i) = gcd of $\{f^*_{sqsp}(av_i), f^*_{sqsp}(bv_i)\}$

= gcd of $\{(i+3)^2, (i+3)^2 + 1\}$= 1, $1 \le i \le n$.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence J_n, admits square sum prime labeling. ■

Theorem 2.2 Splitting graph of star $K_{1,n}$ admits square sum prime labeling.

Proof: Let G = Spl($K_{1,n}$) and let a, b, v_1, v_2,----------------,v_n, u_1, u_2,----------------,u_n are the vertices of G

Here |V(G)| =2n+2 and |E(G)| = 3n

Define a function f : V → {0,1,2,3,----------------,2n+1 } by

$f(v_i) = i+1$, i = 1,2,------,n

$f(u_i) = n+i+1$, i = 1,2,------,n

f(a) = 0, f(b) = 1

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{sqsp} is defined as follows

$f^*_{sqsp}(a\,v_i)$ = $(i+1)^2$, $1 \le i \le n$.

$f^*_{sqsp}(b\,v_i)$ = $(i+1)^2 + 1$ $1 \le i \le n$.

$f^*_{sqsp}(b\,u_i)$ = $(n+i+1)^2+1$, $1 \le i \le n$.

Clearly f^*_{sqsp} is an injection.

gcin of (a) = 1

gcin of (b) = 1

gcin of (v_i) = gcd of $\{f^*_{sqsp}(av_i), f^*_{sqsp}(bv_i)\}$

= gcd of $\{(i+1)^2, (i+1)^2+1\}$= 1, $1 \le i \le n$.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence Spl($K_{1,n}$), admits square sum prime labeling. ■

Theorem 2.3 Jelly fish graph JF_n admits square sum prime labeling.

Proof: Let G = JF_n and let a, b, x, y,v_1,v_2,----------------,v_n, u_1,u_2,----------------,u_n are the vertices of G

Here |V(G)| = 2n+4 and |E(G)| = 2n+5

Define a function f : V → {0,1,2,3,----------------,2n+3 } by

$f(v_i)$ = n+ i+3 , i = 1,2,------,n

$f(u_i)$ = i+3 , i = 1,2,------,n

f(a) = 0, f(b) = 1, f(x) = 2, f(y) = 3

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{sqsp} is defined as follows

$f^*_{sqsp}(ay)$ = 9

$f^*_{sqsp}(by)$ = 10

$f^*_{sqsp}(ax)$ = 4

$f^*_{sqsp}(bx)$ = 5

$f^*_{sqsp}(xy)$ = 13

$f^*_{sqsp}(a\,u_i)$ = $(i+3)^2$, $1 \le i \le n$.

$f^*_{sqsp}(b\,v_i)$ = $(n+i+3)^2+1$, $1 \le i \le n$.

Clearly f^*_{sqsp} is an injection.

gcin of (a) = 1

gcin of (b) = 1

gcin of (x) = 1

gcin of (y) = 1

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence JF_n, admits square sum prime labeling. ■

Theorem 2.4 Two copies of cycle C_n sharing a common edge admits square sum prime labeling, when n >4.

Proof : Let G = 2(C_n) – e and let v_1,v_2,----------------,v_{2n-2} are the vertices of G.

Here |V(G)| = 2n-2 and |E(G)| = 2n-1

Define a function f : V → {0,1,2,3,----------------,2n-3 } by

$f(v_i)$ = i-1 , i = 1,2,------,2n-2

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{sqsp} is defined as follows

$f^*_{sqsp}(v_i\,v_{i+1})$ = $2i^2-2i+1$, $1 \le i \le 2n-3$.

$f^*_{sqsp}(v_1\,v_{2n-2})$ = $(2n-3)^2$.

Case(i) n is even

$f^*_{sqsp}\{v_{(\frac{n}{2})}v_{(\frac{3n-2}{2})}\}$ = $\frac{5n^2-14n+10}{2}$

Case(ii) n is odd

$f^*_{sqsp}\{v_{(\frac{n+1}{2})}v_{(\frac{3n-1}{2})}\}$ = $\frac{5n^2-10n+5}{2}$

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Clearly f^*_{sqsp} is an injection.

gcin of (v_i) = 1, $1 \le i \le 2n-4$.

gcin of (v_1) = 1

gcin of (v_{2n-2}) = gcd of $\{f^*_{sqsp}(v_1 v_{2n-2}), f^*_{sqsp}(v_{2n-3} v_{2n-2})\}$

= gcd of $\{(2n-3)^2, 8n^2-28n+25\}$,

= gcd of $\{(2n-3), (2n-3)(4n-8)+1\} = 1$ $1 \le i \le n$.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence $2(C_n) - e$, admits square sum prime labeling. ■

Theorem 2.5 Triangular belt TB(α) admits square sum prime labeling where $\alpha = (\uparrow\uparrow - - - - n-1\ times)$ and n is not a multiple of 5.

Proof : Let G = TB(α) and let v_1, v_2,----------------,v_{2n} are the vertices of G.

Here |V(G)| = 2n and |E(G)| = 4n-3

Define a function f : V → {0,1,2,3,----------------,2n-1 } by

$f(v_i) = i-1$, i = 1,2,------,2n

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{sqsp} is defined as follows

$f^*_{sqsp}(v_i v_{i+1})$ = $2i^2-2i+1$, $1 \le i \le 2n-1$.

$f^*_{sqsp}(v_{2i-1} v_{2i+1})$ = $8i^2-8i+4$, $1 \le i \le n-1$.

$f^*_{sqsp}(v_{2i} v_{2i+2})$ = $8i^2+2$, $1 \le i \le n-1$.

Clearly f^*_{sqsp} is an injection.

gcin of (v_i) = 1, $1 \le i \le 2n-2$.

gcin of (v_1) = 1

gcin of (v_{2n}) = gcd of $\{f^*_{sqsp}(v_{2n} v_{2n-1}), f^*_{sqsp}(v_{2n} v_{2n-2})\}$

= gcd of $\{8n^2-12n+5, 8n^2-16n+10\}$,

= gcd of $\{4n^2-8n+5, 8n^2-12n+5\}$,

= gcd of $\{(4n-5), 4n^2-8n+5\}$

= gcd of $\{(4n-5), n\}$,

= gcd of $\{(n-5), n\} = 1$, $1 \le i \le n$.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence TB(α), admits square sum prime labeling. ■

Theorem 2.6 The graph $P_2 (+) N_m$ admits square sum prime labeling.

Proof : Let G = $P_2 (+) N_m$ and let v_1, v_2,----------------,v_{m+2} are the vertices of G.

Here |V(G)| = m+2 and |E(G)| = 2m+1

Define a function f : V → {0,1,2,3,----------------,m+1 } by

$f(v_i) = i-1$, i = 1,2,------,m+2

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{sqsp} is defined as follows

$f^*_{sqsp}(v_1 v_2)$ = 1,

$f^*_{sqsp}(v_1 v_{i+2})$ = $(i+1)^2$, $1 \le i \le m$.

$f^*_{sqsp}(v_2 v_{i+2})$ = $(i+1)^2+1$ $1 \le i \le m$.

Clearly f^*_{sqsp} is an injection.

gcin of (v_1) = 1

gcin of (v_2) = 1

gcin of (v_{i+2}) = gcd of $\{f^*_{sqsp}(v_1 v_{i+2}), f^*_{sqsp}(v_2 v_{i+2})\}$

= gcd of $\{(i+1)^2, (i+1)^2+1\}$

= 1, $1 \le i \le m$.

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So, ***gcin*** of each vertex of degree greater than one is 1.
Hence P_2 (+) N_m admits square sum prime labeling. ■

Theorem 2.7 Double graph of star $K_{1,n}$ admits square sum prime labeling.

Proof: Let G = D($K_{1,n}$) and let a, b,v_1,v_2,------------,v_n,u_1,u_2,------------,u_n are the vertices of G
Here |V(G)| =2n+2 and |E(G)| = 4n
Define a function f : V → {0,1,2,3,----------------,2n+1 } by

$f(v_i) = i+1$, i = 1,2,------,n
$f(u_i) = n+i+1$, i = 1,2,------,n
f(a) = 0, f(b) = 1

Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f^*_{sqsp} is defined as follows

$f^*_{sqsp}(a\ v_i) = (i+1)^2$, $1 \le i \le n$.
$f^*_{sqsp}(b\ v_i) = (i+1)^2 + 1$ $1 \le i \le n$.
$f^*_{sqsp}(b\ u_i) = (n+i+1)^2 + 1$, $1 \le i \le n$.
$f^*_{sqsp}(a\ u_i) = (n+i+1)^2$, $1 \le i \le n$.

Clearly f^*_{sqsp} is an injection.

gcin of (a) = 1
gcin of (b) = 1
gcin of (v_i) = gcd of $\{f^*_{sqsp}(av_i), f^*_{sqsp}(bv_i)\}$
= gcd of { $(i+1)^2$, $(i+1)^2$ +1}= 1, $1 \le i \le n$.
gcin of (u_i) = gcd of $\{f^*_{sqsp}(au_i), f^*_{sqsp}(bu_i)\}$
= gcd of { $(n+i+1)^2$, $(n+i+1)^2$ +1}= 1, $1 \le i \le n$.

So, ***gcin*** of each vertex of degree greater than one is 1.
Hence D($K_{1,n}$), admits square sum prime labeling. ■

Theorem 2.8 Graph obtained by adding a pendant edge to the central vertex of a gear graph admits square sum prime labeling .

Proof : Let G = G_n(+) e and let v_1,v_2,----------------,v_{2n+2} are the vertices of G.
Here |V(G)| = 2n+2 and |E(G)| = 3n+1
Define a function f : V → {0,1,2,3,----------------,2n+1 } by

$f(v_i) = i-1$, i = 1,2,------,2n+2

Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f^*_{sqsp} is defined as follows

$f^*_{sqsp}(v_i\ v_{i+1}) = 2i^2-2i+1$, $1 \le i \le 2n-1$.
$f^*_{sqsp}(v_{2n+1}\ v_{2i-1}) = 4n^2+(2i-2)^2$, $1 \le i \le n$.
$f^*_{sqsp}(v_1\ v_{2n}) = (2n-1)^2$
$f^*_{sqsp}(v_{2n+1}\ v_{2n+2}) = 4n^2+(2n+1)^2$

Clearly f^*_{sqsp} is an injection.

gcin of (v_i) = 1, $1 \le i \le 2n-2$.
gcin of (v_1) = 1
gcin of (v_{2n+1}) = gcd of $\{f^*_{sqsp}(v_1 v_{2n+1}), f^*_{sqsp}(v_{2n+1} v_{2n+2})\}$
= gcd of { $4n^2$, $4n^2 + (2n+1)^2$} = 1.
gcin of (v_{2n}) = gcd of $\{f^*_{sqsp}(v_1 v_{2n}), f^*_{sqsp}(v_{2n} v_{2n-1})\}$
= gcd of { $(2n-1)^2$, $(2n-1)^2 + (2n-2)^2$} = 1.

So, ***gcin*** of each vertex of degree greater than one is 1.
Hence G_n(+) e , admits square sum prime labeling. ■

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