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A CLASS OF SEPERATE REGRESSION TYPE ESTIMATOR UNDER STRATIFIED RANDOM SAMPLING

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Abstract

A class of regression type estimators using the auxiliary information on population mean and population coefficient of variation is proposed under stratified random sampling. The expressions of its bias and mean square error under are obtained Further the expression of minimum mean square error under the optimum value of the characterizing scalar is also given in this section. An optimum allocation with the proposed class is obtained and its efficiency is compared with that of Neyman optimum allocation.

1. Introduction of The Proposed Estimator

Let a population of size $'N'$ be stratified in to $'L'$ non- overlapping strata, the h^{th} stratum size being $N_h\,(h = 1, 2, \dots, L)$ and $\sum_{h=1}^{L} N_h = N$. Suppose $'y'$ be characteristic under study and $'x'$ be the auxiliary variable. We denote by Y_{hj}: The observation on the j^{th} unit of the population for the characteristic $'y'$ under study $(j = 1, 2, \dots, N_h)$. X_{hj}: The observation on the j^{th} unit of the population for the characteristic $'x'$ under study $(j = 1, 2, \dots, N_h)$.

$$\bar{Y}_h = \frac{1}{N_h}\sum_{j=1}^{N_h} Y_{hj}\ ;$$

$$\bar{X}_h = \frac{1}{N_h}\sum_{j=1}^{N_h} X_{hj}\ ;$$

$$S_{yh}^2 = \frac{1}{(N_h - 1)}\sum_{j=1}^{N_h}\left(y_{hj} - \bar{Y}_h\right)^2 ;$$

$$S_{xh}^2 = \frac{1}{(N_h - 1)}\sum_{j=1}^{N_h}\left(x_{hj} - \bar{X}_h\right)^2 ;$$

$$S_{xyh} = \frac{1}{(N_h - 1)}\sum_{j=1}^{N_h}\left(X_{hj} - \bar{X}_h\right)\left(Y_{hj} - \bar{Y}_h\right) = \rho_h S_{xh} S_{hj}\ ;$$

where ρ_h is the population correlation coefficient between $'x'$ and $'y'$ for the h^{th} stratum $(h = 1, 2, \dots, L)$

$$\lambda_{yxh} = \frac{\mu_{21h}}{\bar{Y}_h S_{xh}^2}$$

$$R_h = \frac{\bar{X}_h}{\bar{Y}_h}$$

$$C_{yh}^2 = \frac{S_{yh}^2}{\bar{Y}_h^2} = \frac{\mu_{02h}}{\bar{Y}_h^2},$$

$$C_{xh}^2 = \frac{S_{yh}^2}{\bar{X}_h^2} = \frac{\mu_{02h}}{\bar{X}_h^2},$$

$$\mu_{pqh} = \frac{1}{N_h}\sum_{j=1}^{L}\left(X_{hj} - \bar{X}_h\right)^p\left(Y_{hj} - \bar{Y}_h\right)^q : \ the\ (p,q)^{th}$$

Product moment about mean between $'x'$ and $'y'$ for the h^{th} stratum $(h = 1, 2, \dots, L)$.

$$\beta_{1h} = \frac{\mu_{30h}^2}{\mu_{20h}^2}$$

$$\beta_{2h} = \frac{\mu_{40h}^2}{\mu_{20h}^2}$$

$$\beta_h = \frac{S_{xyh}}{S_{xh}^2} = \rho_h \frac{S_{yh}}{S_{xh}}$$

be the population regression coefficient of y on x for the h^{th} stratum $(h = 1, 2, \dots, L)$. Let a simple random sample of size n_h be selected from the h^{th} stratum without replacement and we denote by:

y_{hj}: The observation on the j^{th} unit of the sample for the characteristic $'y'$ under study $(j = 1, 2, \dots, n_h)$. x_{hj}: The observation on the j^{th} unit of the sample for the characteristic $'x'$ under study $(j = 1, 2, \dots, n_h)$.

For the sake of simplicity we assume that N_h is so large that $1 - f_h = 1$. We define

$$\bar{y}_h = \frac{1}{n_h}\sum_{j=1}^{n_h} y_{hj}\ ;$$

$$\bar{x}_h = \frac{1}{n_h}\sum_{j=1}^{n_h} x_{hj}\ ;$$

$$s_{xh}^2 = \frac{1}{n_h - 1}\sum_{j=1}^{n_h}(x_{hj} - \bar{x}_h)^2 ;$$

$$s_{yh}^2 = \frac{1}{n_h - 1}\sum_{j=1}^{n_h}\left(y_{hj} - \bar{y}_h\right)^2 ;$$

$$S_{xyh} = \frac{1}{n_h - 1}\sum_{j=1}^{n_h}(x_{hj} - \bar{x}_h)\left(y_{hj} - \bar{y}_h\right);$$

$$b_h = \frac{S_{xyh}}{S_{xh}^2};$$

$$\hat{C}_{xh} = \frac{S_{xh}}{\bar{x}_h}$$

Assuming that $\bar{X}_h$ is known $\forall h = 1.2 \ldots, L$. In order to estimate the population mean of the study variable, an estimator $\hat{\bar{Y}}_{\theta S}$ is given by

$$\hat{\bar{Y}}_{\theta S} = \sum_{h=1}^{L} W_h \left\{\bar{y}_h \left[1 + \frac{\theta_h(\hat{\sigma}_{xh}^2 - \sigma_{xh}^2)}{\sigma_{xh}^2}\right] + b_h(\bar{X}_h - \bar{x}_h)\right\}$$

$$= \sum_{h=1}^{L} W_h \left\{\bar{y}_h + \theta_h \bar{y}_h \left(\frac{\hat{\sigma}_{xh}^2}{\sigma_{xh}^2} - 1\right) + b_h(\bar{X}_h - \bar{x}_h)\right\} \quad (1.1)$$

where θ_h are the characterizing scalars to be chosen suitably, strata means $\bar{X}_h$ and strata variances σ_{xh}^2 of the auxiliary variable $'x'$ are assumed to be known. We propose to use the following separate regression type estimator

$$\hat{\bar{Y}}_{\omega S} = \sum_{h=1}^{L} W_h \left\{\bar{y}_h \left[1 + \omega_h \frac{(\hat{C}_{xh} - C_{xh})}{C_{xh}}\right] + b_h(\bar{X}_h - \bar{x}_h)\right\}$$

$$= \sum_{h=1}^{L} W_h \left\{\bar{y}_h + \omega_h \bar{y}_h \left(\frac{\hat{C}_{xh}}{C_{xh}} - 1\right) + b_h(\bar{X}_h - \bar{x}_h)\right\} \quad (1.2)$$

where ω_h are the characterizing scalars to be chosen suitably, strata means $\bar{X}_h$ and strata coefficient of variations C_{xh} of the auxiliary variable $'x'$ are assumed to be known. It should be noted that for $\omega_h = 0$; $\forall h = 1.2 \ldots, L$, the proposed separate regression type estimator reduces to the separate linear regression estimator given by

$$\bar{Y}_{LRS} = \sum_{j=1}^{L} W_h\{\bar{y}_h + b_h(\bar{X}_h - \bar{x}_h)\} \quad (1.3)$$

2. Bias of The Proposed Estimator $\hat{\hat{\bar{Y}}}_{\omega S}$

Let

$$\bar{y}_h = \bar{Y}_h(1+e_{0h})$$
$$\bar{x}_h = \bar{X}_h(1+e_{1h})$$
$$s_{xyh} = S_{xyh}(1+e_{2h})$$
$$s_{xh}^2 = S_{xh}^2(1+e_{3h})$$
$$E(e_{0h}) = E(e_{1h}) = E(e_{2h}) = E(e_{3h}) = 0$$
$$\forall h = 1,2,\dots,L \qquad (2.1)$$

Now from (1.2), we have

$$\hat{\hat{\bar{Y}}}_{\omega S} = \sum_{h=1}^{L} W_h \left[\bar{Y}_h(1+e_{0h}) + \omega_h \bar{Y}_h(1+e_{0h})\left\{\frac{(1+e_{3h})^{1/2}}{(1+e_{3h})} - 1\right\} + \beta_h \frac{(1+e_{2h})}{(1+e_{3h})}(-\bar{X}_h e_{1h})\right]$$

$$= \sum_{h=1}^{L} W_h [\bar{Y}_h(1+e_{0h}) + \{1-\omega_h + \omega_h(1+e_{3h})^{1/2}(1+e_{1h})^{-1}\} + \beta_h(1+ \quad e_{2h})(1+$$
e3h−1−Xe1

$$= \sum_{h=1}^{L} W_h \left[\bar{Y}_h(1+e_{0h})\left[1-\omega_h + \omega_h\left\{\left(1+\frac{1}{2}e_{3h} - \frac{1}{8}e_{3h}^2 + \cdots\right)(1-e_{1h}+ \quad e_{1h}^2 -\right.\right.\right.$$
…+βhXh1+e2h1−e3h+e3h2e1h

$$= \sum_{h=1}^{L} W_h \bar{Y}_h \left[\left\{1 + e_{0h} + \omega_h\left(-e_{1h} + \frac{1}{2}e_{3h} + e_{1h}^2 - \frac{1}{8}e_{3h}^2 - e_{0h}e_{1h} + \frac{1}{2}e_{0h}e_{3h} -\right.\right.\right.$$
1 2e1he3h+…) +βhXh−e1h−e1he2h+e1he3h+… (2.2)

Let the sample size be so large that $|e_{ih}|, i = 0,1,2,3;\ \forall h = 1.2 \dots, L$; become so small that terms of $e_i^{'}s$ having power greater than two may be neglected.

$$E(\hat{\hat{\bar{Y}}}_{\omega S}) = \sum_{h=1}^{L} W_h \bar{Y}_h \left[1 + \omega_h\left\{E(e_{1h}^2) - \frac{1}{8}E(e_{3h}^2) - E(e_{0h}e_{1h}) + \frac{1}{2}E(e_{0h}e_{3h}) -\right.\right.$$
12Ee1he3h+…+βhXhYhEe1he3h−Ee1he2h

Using the following results

$$E(e_{0h}^2) = \frac{C_{yh}^2}{n_h}$$
$$E(e_{1h}^2) = \frac{C_{xh}^2}{n_h}$$
$$E(e_{0h}e_{1h}) = \frac{\rho_{xyh}C_{xh}C_{yh}}{n_h}$$
$$E(e_{3h}^2) = \frac{(\beta_{2xh}-1)}{n_h}$$

$$E(e_{0h}e_{3h}) = \frac{\lambda_{yx_h}}{n_h}$$
$$E(e_{1h}e_{2h}) = \frac{1}{n_h}\frac{\mu_{21h}}{\bar{X}_h S_{xyh}}$$
$$E(e_{1h}e_{3h}) = \frac{1}{n_h}\sqrt{\beta_{1xh}}C_{xh}\ ;\ \ \forall h = 1, 2, \dots, L \qquad (2.3)$$

We have

$$= \bar{Y} + \sum_{h=1}^{L} W_h \bar{Y}_h \left[\frac{\omega_h}{n_h}\left\{C_{xh}^2 - \frac{1}{8}(\beta_{2xh} - 1) - \rho_{xyh}C_{xh}C_{yh} + \frac{1}{2}\lambda_{yxh} - \frac{1}{2}\sqrt{\beta_{1xh}}C_{xh} + \quad \dots\right\} + \right.$$
$\beta hXhYh\beta 1xhCxh-\mu 21hXhSxyh$ (2.4)

Showing that $\hat{\hat{Y}}_{\omega S}$ is a biased estimator of population mean $\bar{Y}$ and its bias is given by

$$B\left(\hat{\hat{Y}}_{\omega S}\right) = E\left(\hat{\hat{Y}}_{\omega S}\right) = \bar{Y}$$
$$= \sum_{h=1}^{L} W_h \bar{Y}_h \left[\frac{\omega_h}{n_h}\left\{C_{xh}^2 - \frac{1}{8}(\beta_{2xh} - 1) - \rho_{xyh}C_{xh}C_{yh} + \frac{1}{2}\lambda_{yxh} - \frac{1}{2}\sqrt{\beta_{1xh}}C_{xh} + \right.\right.$$
$\dots + \beta hXhYh\beta 1xhCxh-\mu 21hXhSxyh$

Therefore, mean square error of $\hat{\hat{Y}}_{\omega S}$ is given by

$$MSE\left(\hat{\hat{Y}}_{\omega S}\right) = E\left(\hat{\hat{Y}}_{\omega S} - \bar{Y}\right)^2$$
$$= E\left\{\sum_{h=1}^{L} W_h \bar{Y}_h \left(e_{0h} + \omega_h\left(-e_{1h} + \frac{1}{2}e_{3h}\right) - \beta_h R_h e_{1h}\right)\right\}^2$$
$$= \sum_{h=1}^{L} W_h^2 \bar{Y}_h^2\, E\left\{e_{0h} + \omega_h\left(-e_{1h} + \frac{1}{2}e_{3h}\right) - \beta_h R_h e_{1h}\right\}^2$$

Using (2.2) upto first order of approximation

$$= \sum_{h=1}^{L} W_h^2 \bar{Y}_h^2 \left[E(e_{0h}^2) + \beta_h^2 R_h^2 E(e_{1h}^2) - 2\beta_h R_h E(e_{0h}e_{1h}) + \omega_h^2\left\{E(e_{1h}^2) + \frac{1}{4}E(e_{3h}^2) - \right.\right.$$
$E(e1he3h)-2whEe0he1h-12Ee0he3h-\beta hRhEe12+\beta hRh2E(e1he3h)$

Using the results given in (2.3), we have

$$= \sum_{h=1}^{L} W_h^2 \frac{\bar{Y}_h^2}{n_h}\left[(1-\rho_{xyh}^2)C_{yh}^2 + \omega_h^2\left\{C_{xh}^2 + \frac{1}{4}(\beta_{2xh} - 1) - \sqrt{\beta_{1xh}}C_{xh}\right\} - 2\omega_h\left\{\rho_{xyh}C_{xh}C_{yh} - \frac{1}{2}\lambda_{xh} - \beta_h R_h C_{xh}^2 + \frac{\beta_h R_h}{2}\sqrt{\beta_{1xh}}C_{xh}\right\}\right]$$

$$= \sum_{h=1}^{L} W_h^2 \frac{\bar{Y}_h^2}{n_h}\left[\left(1-\rho_{xyh}^2\right)C_{yh}^2+\omega_h^2\left\{C_{xh}^2+\frac{1}{4}(\beta_{2xh}-1)-\sqrt{\beta_{1xh}}C_{xh}\right\}-2\omega h\beta hRh2\beta 1xhCxh-12\lambda xh\right. \quad (2.6)$$

(2.6) is minimum when

$$\omega_{hopt} = \frac{\left\{\frac{\beta_h R_h}{2}\sqrt{\beta_{1xh}}C_{xh}-\frac{1}{2}\lambda_{yxh}\right\}}{\left\{C_{xh}^2+\frac{1}{4}(\beta_{2xh}-1)-\sqrt{\beta_{1xh}}C_{xh}\right\}}\ ;\forall h=1,2,\ldots,L \quad (2.7)$$

And the minimum mean square error of $\hat{\hat{\bar{Y}}}_{\omega S}$ is given by

$$MSE(\hat{\hat{\bar{Y}}}_{\omega S})_{min} = \sum_{h=1}^{L} W_h^2 \frac{\bar{Y}_h^2}{n_h}\left[\left(1-\rho_{xyh}^2\right)C_{yh}^2-\frac{\left\{\frac{\beta_h R_h}{2}\sqrt{\beta_{1xh}}C_{xh}-\frac{1}{2}\lambda_{yxh}\right\}^2}{\left\{C_{xh}^2+\frac{1}{4}(\beta_{2xh}-1)-\sqrt{\beta_{1xh}}C_{xh}\right\}}\right]$$

$$= \sum_{h=1}^{L} W_h^2 \frac{\bar{Y}_h^2}{n_h}\left[\left(1-\rho_{xyh}^2\right)C_{yh}^2-\frac{\left\{\frac{\beta_h R_h}{2}\sqrt{\beta_{1xh}}C_{xh}-\frac{1}{2}\lambda_{yxh}\right\}^2}{\left\{\frac{1}{4}(\beta_{2xh}-\beta_{1xh}-1)+(\sqrt{\beta_{1xh}}-C_{xh})^2\right\}}\right] \quad (2.8)$$

3. Optimum Allocation with The Proposed Class

Considering the cost function $C = C_0 + \sum_{h=1}^{L} c_h n_h$, where C_0 and c_h are the cost per unit within h^{th} stratum respectively minimizing the approximate minimum variance.

$$MSE(\hat{\hat{\bar{Y}}}_{\omega S})_{min} = \sum_{h=1}^{L} W_h^2 \frac{\bar{Y}_h^2}{n_h}\left[\left(1-\rho_{xyh}^2\right)C_{yh}^2-\frac{\left\{\frac{\beta_h R_h}{2}\sqrt{\beta_{1xh}}C_{xh}-\frac{1}{2}\lambda_{yxh}\right\}^2}{\left\{\frac{1}{4}(\beta_{2xh}-\beta_{1xh}-1)+(\sqrt{\beta_{1xh}}-C_{xh})^2\right\}}\right]$$

By Lagrange's method of multipliers subject to the cost restriction $C - C_0 = \sum_{h=1}^{L} c_h n_h$, on the lines of Cochran (1977), n_h and the multiplies λ are found so as to minimize

$$\emptyset = MSE\left(\hat{\hat{\bar{Y}}}_{\omega S}\right)_{min} + \lambda \sum_{h=1}^{L} c_h n_h - C + C_0$$

$$= \sum_{h=1}^{L} W_h^2 \frac{\bar{Y}_h^2}{n_h}\left[\left(1-\rho_{xyh}^2\right)C_{yh}^2-\frac{\left\{\frac{\beta_h R_h}{2}\sqrt{\beta_{1xh}}C_{xh}-\frac{1}{2}\lambda_{yxh}\right\}^2}{\left\{\frac{1}{4}(\beta_{2xh}-\beta_{1xh}-1)+(\sqrt{\beta_{1xh}}-C_{xh})^2\right\}}\right] + \lambda\left(\sum_{h=1}^{L} c_h n_h - C + C0\right.$$

differentiating (3.2) with respect to n_h and equating to zero, we get

$$= \frac{W_h^2 \bar{Y}_h^2}{n_h^2}\left[\left(1-\rho_{xyh}^2\right)C_{yh}^2-\frac{\left\{\frac{\beta_h R_h}{2}\sqrt{\beta_{1xh}}C_{xh}-\frac{1}{2}\lambda_{yxh}\right\}^2}{\left\{\frac{1}{4}(\beta_{2xh}-\beta_{1xh}-1)+\left(\sqrt{\beta_{1xh}}-C_{xh}\right)^2\right\}}\right] + \lambda c_h = 0$$

$$n_h = \frac{1}{\sqrt{\lambda}}\frac{W_h}{\sqrt{c_h}}\left\{\left(1-\rho_{xyh}^2\right)C_{yh}^2-\frac{\left\{\frac{\beta_h R_h}{2}\sqrt{\beta_{1xh}}C_{xh}-\frac{1}{2}\lambda_{yxh}\right\}^2}{\left\{\frac{1}{4}(\beta_{2xh}-\beta_{1xh}-1)+\left(\sqrt{\beta_{1xh}}-C_{xh}\right)^2\right\}}\right\}^{\frac{1}{2}} \forall h = 1,2,\ldots,L \quad (3.3)$$

Summing over all strata we have

$$n_h = \frac{1}{\sqrt{\lambda}}\frac{W_h}{\sqrt{c_h}}\left\{\left(1-\rho_{xy\,h}^2\right)C_{yh}^2 - \frac{\left\{\frac{\beta_h R_h}{2}\sqrt{\beta_{1xh}}C_{xh}-\frac{1}{2}\lambda_{yx\,h}\right\}^2}{\left\{\frac{1}{4}(\beta_{2xh}-\beta_{1xh}-1)+\left(\sqrt{\beta_{1xh}}-C_{xh}\right)^2\right\}}\right\}^{\frac{1}{2}} \tag{3.4}$$

Taking ratio of (3.3) and (3.4) we obtain

$$n_h = n\frac{\frac{1}{\sqrt{\lambda}}\frac{W_h}{\sqrt{c_h}}\left\{\left(1-\rho_{xy\,h}^2\right)C_{yh}^2 - \frac{\left\{\frac{\beta_h R_h}{2}\sqrt{\beta_{1xh}}C_{xh}-\frac{1}{2}\lambda_{yx\,h}\right\}^2}{\left\{\frac{1}{4}(\beta_{2xh}-\beta_{1xh}-1)+\left(\sqrt{\beta_{1xh}}-C_{xh}\right)^2\right\}}\right\}^{\frac{1}{2}}}{\frac{1}{\sqrt{\lambda}}\sum_{h=1}^{L}\frac{W_h\bar{Y}_h}{\sqrt{c_h}}\left\{\left(1-\rho_{xy\,h}^2\right)C_{yh}^2 - \frac{\left\{\frac{\beta_h R_h}{2}\sqrt{\beta_{1xh}}C_{xh}-\frac{1}{2}\lambda_{yx\,h}\right\}^2}{\left\{\frac{1}{4}(\beta_{2xh}-\beta_{1xh}-1)+\left(\sqrt{\beta_{1xh}}-C_{xh}\right)^2\right\}}\right\}^{\frac{1}{2}}}\ \forall h = 1, 2, \dots, L \tag{3.5}$$

As a particular case for $c_h = c_1\,; \forall h = 1, 2, \dots, L$ i.e., the given cost function $c_1\sum_{h=1}^{L} n_h + C_0 = C{=}C0{+}c1n$

The optimum allocation (3.5) reduces to

$$n_h = n\frac{W_h\bar{Y}_h\left\{\left(1-\rho_{xy\,h}^2\right)C_{yh}^2 - \frac{\left\{\frac{\beta_h R_h}{2}\sqrt{\beta_{1xh}}C_{xh}-\frac{1}{2}\lambda_{yx\,h}\right\}^2}{\left\{\frac{1}{4}(\beta_{2xh}-\beta_{1xh}-1)+\left(\sqrt{\beta_{1xh}}-C_{xh}\right)^2\right\}}\right\}^{\frac{1}{2}}}{\sum_{h=1}^{L}W_h\bar{Y}_h\left\{\left(1-\rho_{xy\,h}^2\right)C_{yh}^2 - \frac{\left\{\frac{\beta_h R_h}{2}\sqrt{\beta_{1xh}}C_{xh}-\frac{1}{2}\lambda_{yx\,h}\right\}^2}{\left\{\frac{1}{4}(\beta_{2xh}-\beta_{1xh}-1)+\left(\sqrt{\beta_{1xh}}-C_{xh}\right)^2\right\}}\right\}^{\frac{1}{2}}}\ \forall h = 1, 2, \dots, L \tag{3.6}$$

Substituting the value from (3.6) in (3.1) we have

$$MSE\left(\hat{\hat{Y}}_{\omega S}\right)_{OPT} = \frac{1}{n}\sum_{h=1}^{L}\left[W_h\bar{Y}_h\left\{\left(1-\rho_{xy\,h}^2\right)C_{yh}^2 - \frac{\left\{\frac{\beta_h R_h}{2}\sqrt{\beta_{1xh}}C_{xh}-\frac{1}{2}\lambda_{yx\,h}\right\}^2}{\left\{\frac{1}{4}(\beta_{2xh}-\beta_{1xh}-1)+\left(\sqrt{\beta_{1xh}}-C_{xh}\right)^2\right\}}\right\}^{\frac{1}{2}}\right]^2$$

$$= M_{OPT}\,(say) \tag{3.7}$$

4. Concluding Remarks

The mean square error of the separate linear regression estimator is given by

$$MSE(\bar{y}_{LRS}) = \sum_{h=1}^{L} W_h^2\left(\frac{1}{n_h}-\frac{1}{N_h}\right)(1-\rho_h^2)S_{yh}^2 \tag{4.1}$$

Also the minimum mean square error of the proposed generalized regression type estimator $\hat{\hat{Y}}_{\omega S}$ is given by

$$MSE(\hat{\hat{Y}}_{\omega S})_{min} = \sum_{h=1}^{L} W_h^2\frac{\bar{Y}_h^2}{n_h}\left[\left(1-\rho_{xy\,h}^2\right)C_{yh}^2 - \frac{\left\{\frac{\beta_h R_h}{2}\sqrt{\beta_{1xh}}C_{xh}-\frac{1}{2}\lambda_{yx\,h}\right\}^2}{\left\{\frac{1}{4}(\beta_{2xh}-\beta_{1xh}-1)+(\sqrt{\beta_{1xh}}-C_{xh})^2\right\}}\right] \tag{4.2}$$

Therefore the proposed generalized class of estimators $\hat{\hat{Y}}_{\omega S}$ may be preferred to the separate linear regression estimator, separate ratio estimator, separate product estimator and the usual stratified sample mean in the sense of smaller mean square error. Further the parameter involved ω_h may be estimated by the corresponding sample value in order to get a class of estimators depending upon estimated optimum value. Also the variance of stratified sample mean $\bar{y}_{st}$ under Neyman optimum allocation $n_h = n\frac{W_h S_{yh}}{\sum_{h=1}^{L} W_h S_{yh}}$ is

$$V(\bar{y}_{st})_{Ney} = \frac{1}{n}\left(\sum_{h=1}^{L} W_h S_{yh}\right)^2 \tag{4.3}$$

Also from (4.3) and (3.6), we have

$$n_h = n\frac{W_h \bar{Y}_h\left\{\left(1-\rho_{xy\,h}^2\right)C_{yh}^2 - \frac{\left\{\frac{\beta_h R_h}{2}\sqrt{\beta_{1xh}}C_{xh}-\frac{1}{2}\lambda_{yx\,h}\right\}^2}{\left\{\frac{1}{4}(\beta_{2xh}-\beta_{1xh}-1)+\left(\sqrt{\beta_{1xh}}-C_{xh}\right)^2\right\}}\right\}^{\frac{1}{2}}}{\sum_{h=1}^{L} W_h \bar{Y}_h\left\{\left(1-\rho_{xy\,h}^2\right)C_{yh}^2 - \frac{\left\{\frac{\beta_h R_h}{2}\sqrt{\beta_{1xh}}C_{xh}-\frac{1}{2}\lambda_{yx\,h}\right\}^2}{\left\{\frac{1}{4}(\beta_{2xh}-\beta_{1xh}-1)+\left(\sqrt{\beta_{1xh}}-C_{xh}\right)^2\right\}}\right\}^{\frac{1}{2}}}\ \forall h = 1, 2, \dots, L \tag{4.4}$$

$$MSE\left(\hat{\hat{Y}}_{\omega S}\right)_{OPT} = \frac{1}{n}\sum_{h=1}^{L}\left[W_h \bar{Y}_h\left\{\left(1-\rho_{xyh}^2\right)C_{yh}^2 - \frac{\left\{\frac{\beta_h R_h}{2}\sqrt{\beta_{1xh}}C_{xh}-\frac{1}{2}\lambda_{yx\,h}\right\}^2}{\left\{\frac{1}{4}(\beta_{2xh}-\beta_{1xh}-1)+\left(\sqrt{\beta_{1xh}}-C_{xh}\right)^2\right\}}\right\}^{\frac{1}{2}}\right]^2 \tag{4.5}$$

From (4.3) and (4.5), M_{OPT} is always smaller than $V(\bar{y}_{st})_{Ney}$ except for the case when $\rho_h = 0$ and $\frac{\beta_h R_h}{2}\sqrt{\beta_{1xh}}C_{xh} = \frac{1}{2}\lambda_{yxh}$ $\forall h = 1, 2, \dots, L$ simultaneously.

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