
Amplification in Optical Fibers by Rare Earth Doping

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Abstract

An overview of the fundamental physical process responsible for optical amplification in Raman fiber amplifiers and in rare earth doped (thulium-doped and erbium- doped) fiber amplifiers, is discussed. We formulate basic equations governing optical amplification for each of the two types of amplifiers and define the important parameters used for performance characterization of these amplifiers. This paper outlines the method used in doped fiber amplifiers.

Keywords: Raman amplifier, Fiber, Spectral variation, Rare earth doping.

1. INTRODUCTION

Raman amplification takes place by the process of stimulated Raman scattering (SRS) which involves frequency shifting of an incident optical beam because of interaction having higher frequency vibration modes of a material. In process of Raman scattering, a photon of frequency ν_p is reflected by a molecule and excites single quanta of vibrational energy (a phonon at frequency Ω) and in the process releasing a scattered photon frequency $\nu_s = \nu_p - \Omega$. If the molecule possesses vibrational energy, so that the incident photon, that is being made incident, can take a quanta of vibrational energy to release a photon of shifted upward of frequency $\nu_q = \nu_p + \Omega$.

The photons shifted upward and downward are called Stokes and anti-Stokes photons respectively. Anti-Stokes scattering requires the molecule to be in the excited state; that is, it requires the presence of a phonon frequency Ω . The phonon population; n_Ω , which is a function of phonon frequency and temperature and it obeys Bose – Einstein distribution law;

$$n_\Omega = \frac{1}{e^{h\Omega/k_B T} - 1} \quad (1.1)$$

Where k_B is the Boltzmann constant; h is Planck's constant and T is the absolute temperature in degree Kelvin. Therefore at room temperature; the probability of anti – Stokes emission is much lower than that of stokes emission. Another important aspect of Raman scattering is that since it

involves optical phonons with frequencies close to the maxima in the dispersion curve, the momentum conservation condition gets automatically satisfied [1]. Therefore; unlike Brillouin scattering, Raman scattering is described in terms of energy levels where an incident photon at frequency ν_p excites a molecule up to a virtual level (non-resonant state) and the molecule decays almost instantaneously (\sim femtoseconds) to a lower vibrational/rotational energy level, emitting a photon at frequency $\nu_s = \nu_p - \Omega$.

The difference between energy of incident and Stokes photons is dissipated by the molecular vibrations of the host material. The presence of signal at Stokes-shifted frequency stimulates the conversion of pump photons into Stokes photons, and the process is termed as stimulated Raman scattering (SRS). SRS is a coherent process and it leads to the amplification of the signal. Raman gain exists in every molecular medium and the vibrational energy levels of the material determine the frequency shift $[\Delta\Omega = \nu_p - \nu_s \text{ (Hz)} = \frac{1}{\lambda_p} - \frac{1}{\lambda_s} \text{ (m}^{-1}\text{)}]$ and shape of the Raman gain curve. To achieve Raman amplification in optical fibers, a high power pump wave is co-launched with the signal (having frequency corresponding to Stokes shift), and energy transfer from the pump wave to the signal Stokes wave, through Raman scattering, amplifies the signal.

2. RAMAN AMPLIFIERS IN OPTICAL FIBER COMMUNICATION

Raman amplifiers can be typically used in two of configurations-distributed Raman amplifier, where the transmission fiber itself is used as a gain medium and discrete Raman amplifier, which employs specialty fiber (DCF or specially designed high Raman gain fiber) for amplification. The characteristics make DRAs more suitable for higher bit rates and solution transmissions.

Raman amplifiers are used in conjunction with EDFA (hybrid Raman-EDFA Configuration) to improve noise performance or to increase optical gain band width of EDFA_s. A typical DAR link consists of around 100 km standard transmission fiber pumped by a total power of about 1 W. Raman gain spectrum is highly non-flat, and the conventional technique to flatten by using multiple pumps (with proper wavelengths / power), so that the overall gain spectrum (a superposition of the gain spectrum corresponding to each of the pump) is flat. This is a tedious technique and requires rigorous numerical computation to find out the optimum distribution of pump. Figure 1 shows the spectrum for a DRA based on 50 km long G.652 fiber link carrying simultaneously launched 1 nm spaced signal channel (1520-1600 nm) with input power of -3 dbm per channel and signal-pumped by 700 mW power at 1480 nm. Here, we have taken typical spectral change in background attenuation [6] for standard transmission fiber.

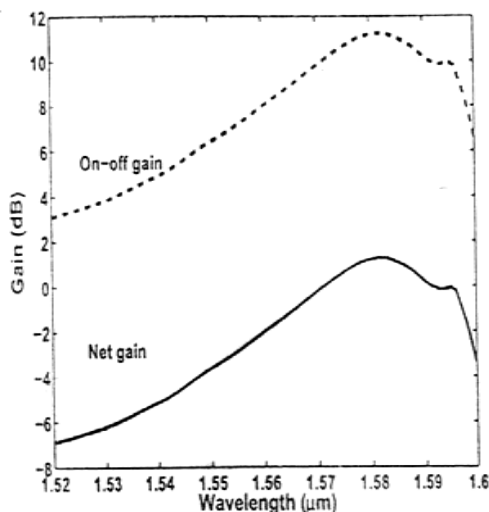


Fig. 1: Gain spectrum in a single pump DRA

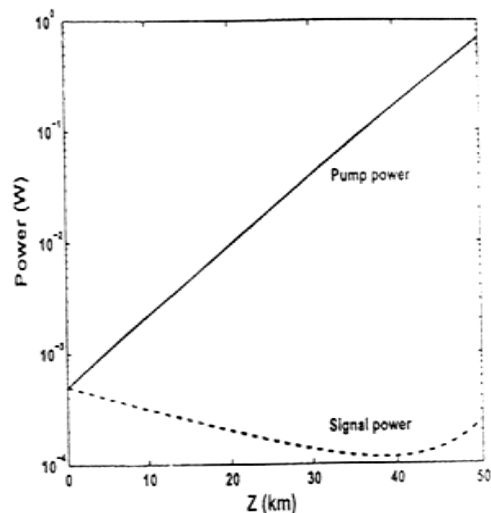


Fig. 2: Pump and signal power in a backward pumped DRA

Attenuation at pump wavelength is taken to be 0.3 dB/km . Figure 2 shows the pump and signal evolution along the fiber length z , for 1550 nm signal wavelength. The OSNR for all wavelengths is found to be greater than 40 dB with negligible effect of DRS noise. The gain ripple in the presented case is about $\pm 4 \text{ dB}$. It has been shown in literature [6] that for such a system a net gain of $(0 \pm 0.08) \text{ dB}$ can be obtained by 16 pumps at suitable wavelength and pump power. Here, one must note that the number of pump used is directly related to the gain flatness, and longer is the span length (i.e. more is the required on-off gain), lesser the flat-gain bandwidth, or equivalently, more is the number of pumps required for obtaining same gain – bandwidth [7]. We have verified the accuracy of our simulations by comparing the results with those in standard literature. The model is capable of handling bi-directional multiple pump, multiple signals, ASE and Rayleigh noise depending on the complexity of the problem, 5-20 iteration are generally required to obtain a convergent solution with an accuracy better than $\pm 0.001 \text{ dB}$ in signal gain.

3. ATOMIC RATE EQUATIONS FOR THREE LEVEL SYSTEM

The energy levels of the doped fiber amplification schemes-EDFA and L- band TDFA can be represented by a three level system. The various transitions in a three level system are shown in Fig. 3. R_{13} and R_{31} represent the stimulated absorption and stimulated emission rates for the pump between levels 1 and 3.

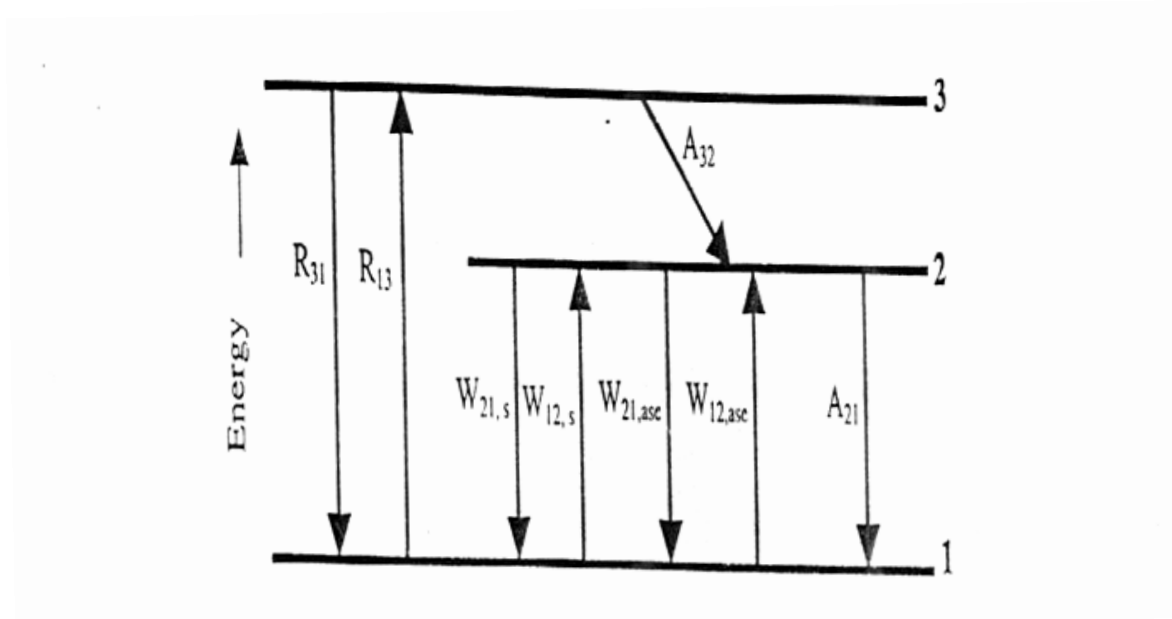


Fig. 3: Various transition processes in a three level system

In efficient amplification schemes, level 3 is usually characterized by a short lifetime (~ few micro-seconds), so the atoms rapidly decay (decay rate A_{32}) to level 2 causing build up of a population inversion between levels 1 and 2. The stimulated emission and absorption rates between level 1 and 2 are denoted by W_{12i} and W_{21i} respectively where the subscript $i = s,$ are stands for signal or ASE respectively. Spontaneous emission rate from level 2 to 1 is denoted by $A_{21} = 1/T$, where T is the fluorescence time interval of level of 2. Let p_d be the total doped ion density and N_1, N_2 and N_3 are the population densities of level 1, 2 and 3 respectively, so that $p_d = N_1 + N_2 + N_3$. The atomic rate equations for the three levels can them be written as:

$$\frac{dN_1}{dt} = -R_{13}N_1 + R_{31}N_3 - W_{12,s}N_1 + W_{21,s}N_2 + A_{21}N_2 - W_{12,ase}N_1 + W_{21,ase}N_2 \quad (3.1)$$

$$\frac{dN_2}{dt} = W_{12,s}N_1 - W_{21,s}N_2 - A_{21}N_2 + A_{32}N_3 + W_{12,ase}N_1 - W_{21,ase}N_2 \quad (3.2)$$

$$\frac{dN_3}{dt} = R_{13}N_1 - R_{31}N_3 - A_{32}N_3 \quad (3.3)$$

In steady state, the populations of various levels are time independent, That is $dN_i/dt = 0$ ($i=1,2,3$) where it is assumed that the non-radiative decay rate A_{32} dominates over the pumping rates R_{13} and R_{31} , that is, $A_{32} \gg R_{13,31}$, equations 3.1- 3.3 can be solved for N_1 and N_2 , to yield:

$$N_1 = P_d \frac{W_{21,s} + A_{21} + W_{21,ase}}{R_{13} + W_{12,s} + W_{12,ase} + A_{21} + W_{21,s} + W_{21,ase}} \quad (3.4)$$

$$N_2 = P_d \frac{R_{13} + W_{12,s} + W_{12,ase}}{R_{13} + W_{12,s} + W_{12,ase} + A_{21} + W_{21,s} + W_{21,ase}} \quad (3.5)$$

If the pumping rate R_{13} is high enough such that $N_2 > N_1$, then population inversion is said to be achieved and the gets amplified.

We have described the doped fiber in a polar cylindrical co-ordinate system (r, ϕ, z) . We denote the emission and absorption cross-section at frequency ν_k by σ_{ek} and σ_{ak} respectively. The subscript k can take values s, p corresponding to the signal and pump respectively. The emission and absorption rates at signal frequency ν_s , at any point (r, ϕ, z) in the fiber are proportional to the signal intensity $I_s(r, \phi, z)$ as follow:

$$W_{21,s} = \frac{\sigma_{es} I_s}{h\nu_s} \quad \text{and} \quad W_{12,s} = \frac{\sigma_{es} I_s}{h\nu_s} \quad (3.6)$$

The pumping rate R_{13} at pump frequency ν_p and intensity $I_p(r, \phi, z)$ is given by:

$$R_{13} = \frac{\sigma_{ap} I_p}{h\nu_p} \quad (3.7)$$

The emission and absorption rates for the ASE are given by:

$$W_{12,ase} = \left[\int_0^\infty \frac{\sigma_{ak}}{h\nu_k} S_{ase}(\nu_k, z) \right] f_s^{lm}(r, \phi) \quad (3.8)$$

$$W_{21,ase} = \left[\int_0^\infty \frac{\sigma_{ak}}{h\nu_k} S_{ase}(\nu_k, z) \right] f_s^{lm}(r, \phi) \quad (3.9)$$

Where, $S_{ase}(\nu_k, z)$ is the ASE spectral power density, since the ASE power in a unit frequency interval around frequency ν_k at position z . ASE power is generated over a continuum of frequencies spanning the entire gain spectrum and therefore the contribution to emission and absorption rates involve an integral over frequency. $f_s^{lm}(r, \phi)$ represents the square of the transverse mode field profile in a fiber and has units of area^{-1} ; lm stands for the $L P_{lm}$ mode in the fiber.

$$\int_0^\infty \int_0^{2\pi} f_k^{lm}(r, \phi) r dr = 1 \quad (3.10)$$

The effective indices and mode field profile for any arbitrary refractive index profile have been obtained using matrix method.

4. PROPAGATION EQUATIONS

When a light beam of intensity I at frequency ν_k traverses an amplifier of infinitesimal thickness dz , the change in the intensity is given by:

$$dI(z) = [\sigma_{ek} N_2 - \sigma_{ak} N_1] I(z) dz \quad (4.1)$$

We assume a homogeneously broadened amplifying medium where the ions are characterized by identical cross-sections at any point in the medium. The optical power $P_k(z)$ at frequency ν_k can be related to the intensity I_k through:

$$I_k(r, \phi, z) = p_k(z) f_k^{lm}(r, \phi) \quad (4.2)$$

Integrating Eqn. 4.2 over r and ϕ , we obtain the rate of change of optical power with propagation distance z :

$$\frac{dp_k z}{dz} = p_k(z) \int_0^\infty \int_0^{2\pi} [\sigma_{ek} N_2 - \sigma_{ak} N_1] f_k^{lm}(r, \phi) r dr d\phi \quad (4.3)$$

Here $k = p, s$ refer to pump and signal respectively. This equation can be rewritten as:

$$\frac{dp_k}{dz} = [\gamma_e(vk, z) - \gamma_a(vk, z)]p_k(z) \tag{4.4}$$

$$\gamma_e(vk, z) = \sigma_{ek} \int_a^b \int_0^{2\pi} N_2 f_k^{lm}(r, \phi) r dr d\phi \tag{4.5}$$

$$\gamma_a(vk, z) = \sigma_{ak} \int_a^b \int_0^{2\pi} N_1 f_k^{lm}(r, \phi) r dr d\phi \tag{4.6}$$

with a and b being the inner and outer radii of the doped region. The rate equation governing propagation of forward and backward ASE is given by:

$$\frac{dS_{ase}^{\pm}}{dz} = \pm 2hv\Delta\nu \gamma_e(v, z) \pm [\gamma_e(vk, z) - \gamma_a(vk, z)]S_{ase}(v, z) \tag{4.7}$$

where the first term on the RHS corresponds to the amplification of spontaneous noise equivalent to one fictitious input photon per mode in bandwidth $\Delta\nu$. The factor of 2 signifies generation of spontaneous noise in both polarization modes.

We have verified the validity of our simulations by comparing the result for a standard EDFA with those published in literature [8]. We consider a step-index fiber with numerical aperture of 0.2 and cutoff wavelength as 800 nm, doped with erbium ions (uniform concentration of $1.0 \times 10^{24} \text{ m}^{-3}$) in the core region. The absorption and emission scattering cross-sections refer to a Ge/Al/P-silica fiber. The radial variation of population density and erbium concentration as well as the transition rates and the modal intensities are considered at 40 points within the erbium-doping radius, and the spatial overlap integrals are solved numerically using these 40 points.

Figure 4 shows the variation of gain at 1532 and 1552 nm for different fiber lengths. A 50 mW pump at 980 nm wavelength and a single signal channel with 0.1 μW of power is considered to be co-directionally launched into the fiber.

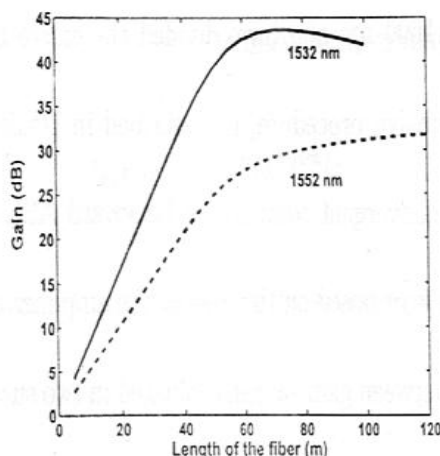


Fig. 4: Variation of EDFA gain

These results are in close matching with those published in literature [8] and thus verify our simulations. Figures 5 and 6 show the spectral variation of small signal gain and ASE

spectral density for a 70 m long EDF pumped with 50 mW power at 980nm. simultaneously launched signal channels from 1520 to 1570 nm, with 0.1 μ W of power per channel, have been considered in obtaining these spectra.

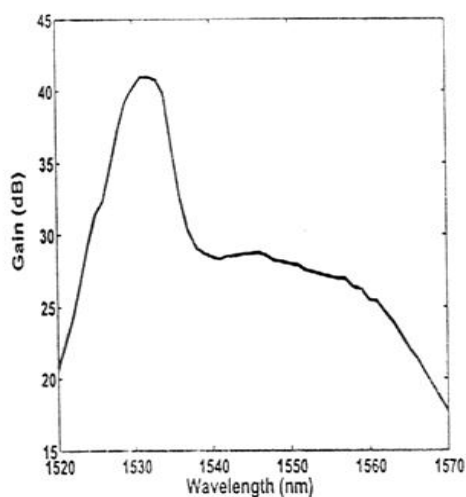


Fig. 5: Spectral variation of EDFA gain

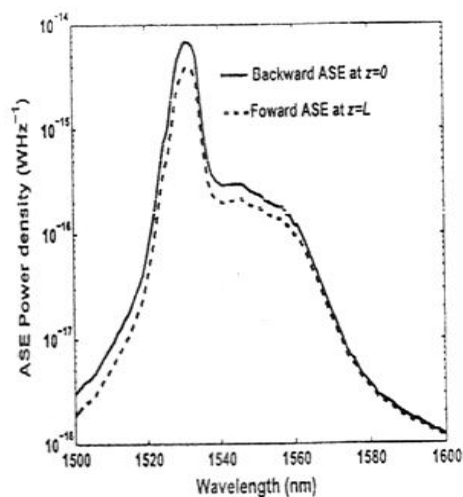


Fig. 6: Spectral variation of forward and backward ASE spectral density in an

EDFA

The program has to undergo 18 forward-backward iterations to obtain the result.

5. CONCLUSION

In this paper, we have presented the theoretical background for Raman amplification and amplification by rare earth doping, in optical fibers. We have formulated the basic equations governing the evolution of pump, signal and noise power for both Raman fiber amplifiers and three-level system based doped fiber amplifiers. The analysis presented here forms the basis of modeling and designs of Raman Fiber Amplifiers and three-level system based on doped fiber amplifiers.

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