International Journal of Engineering, Science and Mathematics
Vol6. xIssue 7, Month 2017,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, Email: ijesmj@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

TO FIND THE COMPARISON OF DOMINATION NUMBER, RADIUS AND DIAMETER OF CIRCULAR ARC-GRAPH G

Dr.A.Sudhakaraiah*
T.Visalakshi**

Abstract

Few subjects in mathematics have as specified an origin as graph theory. Graph theory originated with the Konigsberg Bridge Problem, which Leonhard Euler solved in 1736. Over the past sixty years, there has been a great deal of exploration in the area of graph theory. Its popularity has increased due to its many modern day applications in Circular arc-graphs corresponding to circular arc family and it has become the source of interest to many researchers. Circular graphs are intersection graphs of arcs on a circle. These graphs are reported to have been studied since 1964, and they have been receiving considerable attention since a series of papers by Tucker in the 1970s. In the present paper we presented the Comparison of Domination Number, Radius and Diameter of Circular Arc-Graph G.

Keywords:

Circular Arc family;
Circular Arc Graph;
Domination Number;
Eccentricity;
Radius;
Diameter.

Author correspondence:

Dr.A.Sudhakaraiah,
Asst.Professor, Department of Mathematics, S.V.University,
Tirupati, Andhra Pradesh,India-517502

1 INTRODUCTION

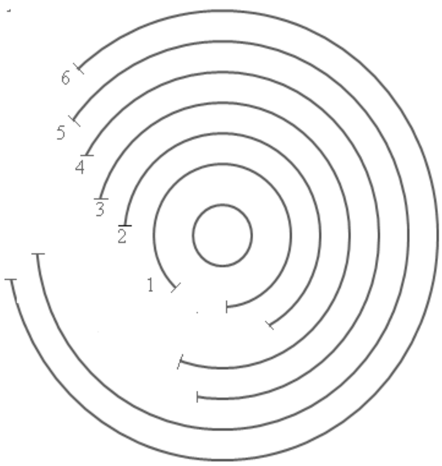
Graph theory is rapidly moving into the mainstream of mathematics mainly due to its applications in diverse fields which include biochemistry (genomics), electrical engineering (communications networks and coding theory), computer science (algorithms and computations) and operations research (scheduling). Although graph theory is one of the younger branches of mathematics, it is fundamental to a number of applied fields.Circular-arc graphs are a new class of intersection graphs, defined for a set of arcs on a circle. A graph is a circular - arc graph, if it is the intersection graph of a finite set of arcs on a circle. That is, there exists one arc for each vertex of G and two vertices in G are adjacent in G, if and only if the corresponding arcs intersect. A vertex is said to dominate another vertex if there is an edge between the two vertices. If we bend the arc into a line, then the family of arcs is transformed into a family of intervals. Therefore, every interval

T.Visalakshi,
Research Scholor, Department of Mathematics, S.V.University,
Tirupati, Andhra Pradesh, India. -517502.

graph is a CAG, where the opposite is always not true. However, these days CAG as well as interval graphs are being patronized very much. The combinatorial structures in CAG are varied and extensive, where it finds an application in many other fields such as biology, genetics, traffic control, and computer science and particularly useful in cyclic scheduling and computer storage allocation problems etc.

The year 1850, for the first time witnessed a study of domination in graphs, where the problem is to place the minimum number of queens on an nxn chess board so that every square gets covered or dominated. It actually took over a decade, i.e., in 1960 researchers took up a serious and comprehensive study of the subject. For the first time in 1962, the concepts were entitled "dominating set" and "domination number" by Ore[1]. In 1977, Cockayne and Hedetniemi[3] conducted a commandable and broad survey on the outcomes of the existing concepts of dominating sets in graphs at that time. The notation γ(G) for the domination number of a graph, for the first time was applied by the pair and was accepted widely since then. The domination theory of graphs put forth by Ore and Berge[2] has been the new area for the researchers, recently.

Let $A=\{A_1, A_2, \ldots, A_n\}$ be a family of arcs on a circle C. Each endpoint of the arc A_i is assigned a positive integer called a co-ordinate. The endpoints are located at the circumference of C in ascending order of the values of the co-ordinates in the clockwise direction. Without loss of generality assumes that the endpoints of all arcs are distinct and no arc covers the entire circle. Suppose an arc i begins at endpoint c and ends at endpoint d in the clockwise direction. Then we denote such an arc by (c,d) and the points c and d arc called respectively the head point and tail point of the arc. Thus $i=(h(i),t(i))$. The arcs are given labels in the increasing order of their head points.We use "arc" to refer to a member of A and "segment" to refer to a part of the circle between two endpoints. A point on the circle is said to be an arc (c,d) if it is contained in segment (c,d). An arc $i=(c,d)$ is said to be contained in another arc $j=(a,b)$ if segment (c,d) is contained in the segment (a,b). An arc family A is said to be proper if no arc in A is contained in another arc. Tucker has studied some subclasses of circular-arc graphs and proposed a characterization of proper circular-arc graphs.



Let $A=\{A_1, A_2,\ldots, A_n\}$ be a circular-arc family. Then the graph $G=(V,E)$ is called a circular-arc graph if there is a one-to-one correspondence between V and A such that two vertices in V are adjacent if and only if their corresponding arcs in A intersect. We denote this circular-arc graph by $G[A]$. If circular-arc family is proper then the corresponding graph is called a proper circular-arc graph.

Generally we deal with intervals/arcs instead of vertices. Further if there are n intervals/arcs in the existing interval/arc family, then we denote its corresponding vertex set by $\{1,2,\ldots,n\}$. So alternatively, depending on the convenience, we use intervals/arcs as vertices and vice versa.

A subset D of V is said to be a dominating set of G if every vertex in V\D is adjacent to a vertex in D. A dominating set with minimum cardinality is said to be a minimum dominating set. The domination number $\gamma(G)$ of the graph G is the minimum cardinality of the dominating set in G.

A dominating set D of a graph G is a non - split dominating set [4] if the vertex induced subgraph < V – D > is connected. The non - split domination number $\gamma_{ns}(G)$ of the graph G is the minimum cardinality of the non - split dominating set. A dominating set D of a graph G is called a split dominating set if the vertex induced subgraph < V – D > is disconnected. The split domination number $\gamma_s(G)$ of the graph G is the minimum cardinality of the split dominating set. The distance [5,6,7,8,9] between two vertices u and v of a graph is the length of the shortest path (path of minimum length) between them and is denoted by d_G of u,v.

The maximum distance from a vertex u to any vertex of G is called eccentricity of the vertex v and is denoted by e(v). i.e; ecc (v) = max {d (u,v): u ЄV (G)}.

The radius of a graph G is the minimum of eccentricity of all its vertices and is denoted by 'r'. i.e; Rad (G) = min {ecc (v): v ЄV (G)}.

The diameter of a graph G is the maximum of eccentricity of its vertices and is denoted by D. i.e;Diam (G) = max {ecc (v): v ЄV (G)}.Among all the vertices of a graph, the one which is having minimum eccentricity is called a central vertex of G. (or).

A vertex of a graph having eccentricity equal to the radius of the graph is called the central vertex.The set of all central vertices of a graph is called the center of the graph G and is denoted by C. C = {v ЄV (G)/E (v) = r, the radius of G}

2. MAIN THEOREM

2.1. Theorem : Let DS be a dominating set of the given circular-arc graph. If a_i and a_j are any two arcs in A such that a_j is contained in a_i, and a_iЄDS and if P is any arc in A, which is to the left of a_j in clock wise direction, such that $P<a_j$ and P intersects a_j and if there is at least one $a_k>a_j$, such that a_k intersects a_J , then non-split domination occurs in G or also DS (G) ≤ Rad (G) ≤ Dia (G) ≤2 [Rad (G)].

Proof: Let G be a circular-arc graph corresponding to a circular-arc family A= {a_i, a_j ,a_k,……….a_n } be a circular-arc family on a circle, where each a_i is an arc.

Without loss of generality assumes that the end points of all arcs are distinct and no-arcs cover the entire circle.

First we will prove that G is connected towards the non-split dominating set of G.

In these a dominating DS of G is called a non-split dominating set if <V-DS> is connected.

Now we want show that G is connected.

Suppose there is at least one arc $a_k \neq a_i$, $a_k > a_j$ such that a_k intersects a_j . Then it is obvious that in <V-DS> , a_k is adjacent to a_j .

Further by hypotheses there is at least one $P<a_j$ in <V-DS> such that P intersects a_j .

Hence a_j is connected to its left as well as to its right, so that there will not be any disconnection in <V-DS>. Therefore G is connected.

In this connection any connected graph G that is dominating set of G is less than or equal to radius of G.That is DS (G) ≤ Rad (G)……………………………………………………….. (1)

Again the radius of G is less than or equal to diameter of G.

That is Rad (G) ≤ Dia (G)………………………………………………………………. (2)

And also the diameter of G is less than or equal to two radius of G.

That is Dia (G) ≤2 [Rad (G)]……………………………………………………………. (3)

Finally we will prove that, the dominating set of G is less than or equal to two radius of G

That is DS (G) ≤ 2[Rad (G)]……………………………………………………………. (4)

From (1),(2),(3) and (4) we have

=> DS (G) ≤ Rad (G),

=> DS (G) ≤ Dia (G),

=> DS (G) ≤ 2[Rad (G)]

Therefore DS (G) ≤Rad (G) ≤ Dia (G) ≤2 [Rad (G)].

Therefore the theorem is proved

3. PRACTICAL PROBLEMS

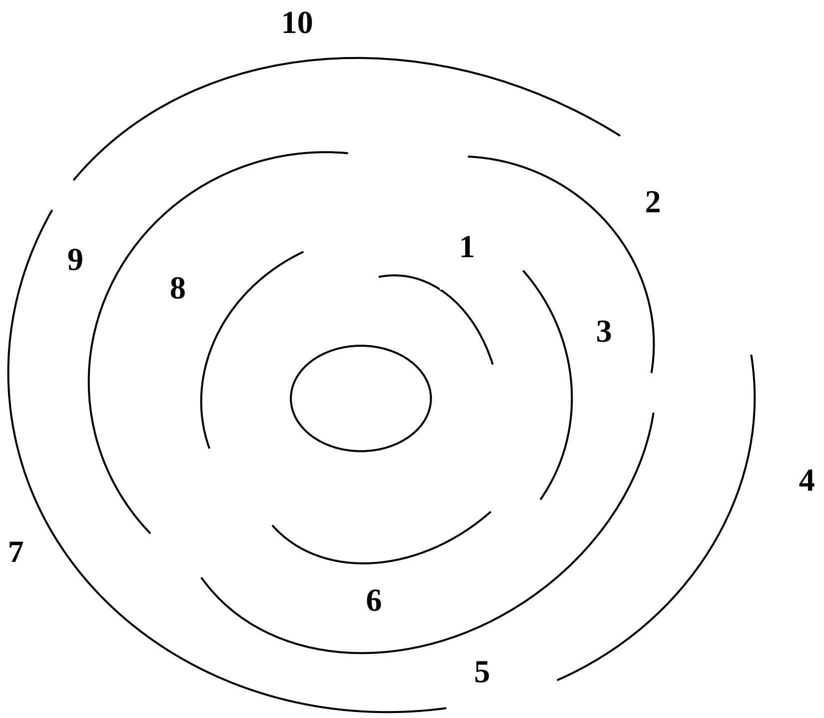


Fig. 1: Circular-Arc family

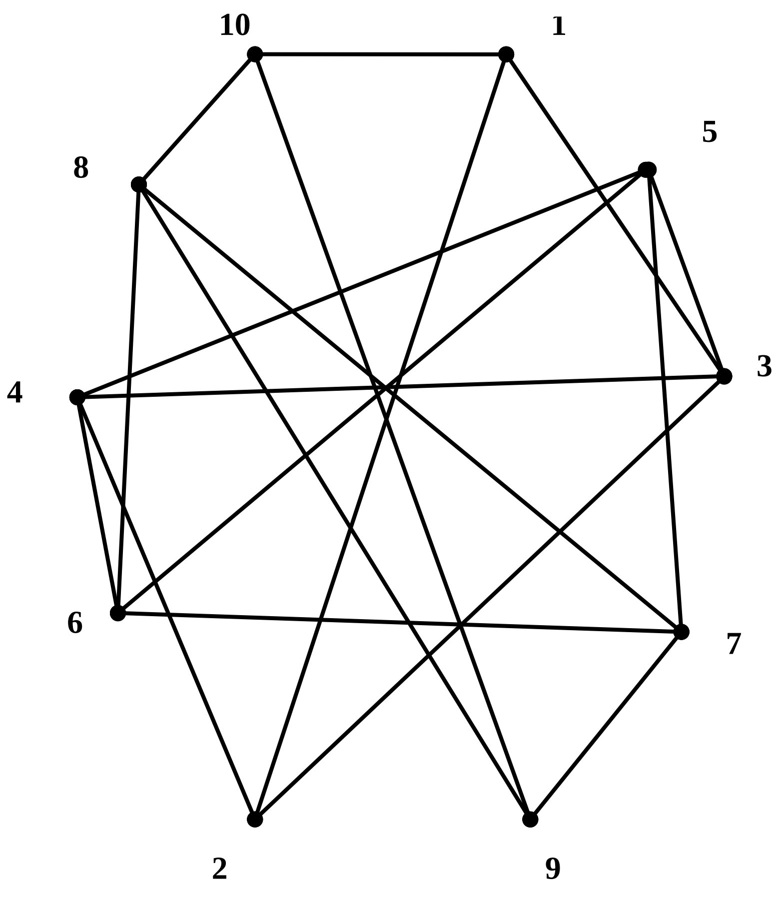


Fig. 2: Circular-arc graph G

4. TO FIND DISTANCE, ECCENTRACITY, RADIUS AND DIAMETER FROM G

4.1. TO FIND THE DISTANCES FROM G

d (1,1)= 0	d (2,1)= 1	d (3,1)= 1	d (4,1)= 2	d (5,1)= 2
d (1,2)= 1	d (2,2)= 0	d (3,2)= 1	d (4,2)= 1	d (5,2)= 2
d (1,3)= 1	d (2,3)= 1	d (3,3)= 0	d (4,3)= 1	d (5,3)= 1
d (1,4)= 2	d (2,4)= 1	d (3,4)= 1	d (4,4)= 0	d (5,4)= 1
d (1,5)= 2	d (2,5)= 2	d (3,5)= 1	d (4,5)= 1	d (5,5)= 0
d (1,6)= 3	d (2,6)= 2	d (3,6)= 2	d (4,6)= 1	d (5,6)= 1
d (1,7)= 3	d (2,7)= 3	d (3,7)= 2	d (4,7)= 2	d (5,7)= 1
d (1,8)= 2	d (2,8)= 3	d (3,8)= 3	d (4,8)= 3	d (5,8)= 2
d (1,9)= 2	d (2,9)= 3	d (3,9)= 3	d (4,9)= 3	d (5,9)= 2
d(1,10)=1	d(2,10)=2	d(3,10)=2	d(4,10)=3	d(5,10)=3
d (6,1)= 3	d (7,1)= 3	d (8,1)= 2	d (9,1)= 2	d (10,1)= 1
d (6,2)= 2	d (7,2)= 3	d (8,2)= 3	d (9,2)= 3	d (10,2)= 2
d (6,3)= 2	d (7,3)= 2	d (8,3)= 3	d (9,3)= 3	d (10,3)= 2
d (6,4)= 1	d (7,4)= 2	d (8,4)= 3	d (9,4)= 3	d (10,4)= 3
d (6,5)= 1	d (7,5)= 1	d (8,5)= 2	d (9,5)= 2	d (10,5)= 3
d (6,6)= 0	d (7,6)= 1	d (8,6)= 2	d (9,6)= 2	d (10,6)= 3
d (6,7)= 1	d (7,7)= 0	d (8,7)= 1	d (9,7)= 1	d (10,7)= 2
d (6,8)= 2	d (7,8)= 1	d (8,8)= 0	d (9,8)= 1	d (10,8)= 1
d (6,9)= 2	d (7,9)= 1	d (8,9)= 1	d (9,9)= 0	d (10,9)= 1
d(6,10)=3	d(7,10)=2	d(8,10)=1	d(9,10)=1	d(10,10)=0

4.2. TO [illegible] G

[illegible] V (G)] = max {d (u,v): u ∈V(G)}

e(1)= max{d (1,1),d (1,2),d (1,3),d (1,4),d (1,5),d (1,6),d (1,7),d (1,8),d (1,9),d (1,10)}
= max {0, 1, 1, 2, 2, 3, 3, 2, 2, 1} = 3

e(2)= max{d (2,1),d (2,2),d (2,3),d (2,4),d (2,5),d (2,6),d (2,7),d (2,8),d (2,9),d (2,10)}
= max {1, 0, 1, 1, 2, 2, 3, 3, 3, 2} = 3

e(3)= max{d (3,1),d (3,2),d (3,3),d (3,4),d (3,5),d (3,6),d (3,7),d (3,8),d (3,9),d (3,10)}
= max {1, 1, 0, 1, 1, 2, 2, 3, 3, 2} = 3

e(4)= max{d (4,1),d (4,2),d (4,3),d (4,4),d (4,5),d (4,6),d (4,7),d (4,8),d (4,9),d (4,10)}
= max {2, 1, 1, 0, 1, 1, 2, 3, 3, 3} = 3

e(5)= max{d (5,1),d (5,2),d (5,3),d (5,4),d (5,5),d (5,6),d (5,7),d (5,8),d (5,9),d (5,10)}
= max {2, 2, 1, 1, 0, 1, 1, 2, 2, 3} = 3

e(6)= max{d (6,1),d (6,2),d (6,3),d (6,4),d (6,5),d (6,6),d (6,7),d (6,8),d (6,9),d (6,10)}
= max {3, 2, 2, 1, 1, 0, 1, 2, 2, 3} = 3

e(7)= max{d (7,1),d (7,2),d (7,3),d (7,4),d (7,5),d (7,6),d (7,7),d (7,8),d (7,9),d (7,10)}
= max {3, 3, 2, 2, 1, 1, 0, 1, 1, 2} = 3

e(8)= max{d (8,1),d (8,2),d (8,3),d (8,4),d (8,5),d (8,6),d (8,7),d (8,8),d (8,9),d (8,10)}
= max {2, 3, 3, 3, 2, 2, 1, 0, 1, 1} = 3

e(9)= max{d (9,1),d (9,2),d (9,3),d (9,4),d (9,5),d (9,6),d (9,7),d (9,8),d (9,9),d (9,10)}
= max {2, 3, 3, 3, 2, 2, 1, 1, 0, 1} = 3

e(10)= max{d (10,1),d (10,2),d (10,3),d (10,4),d (10,5),d (10,6),d (10,7),d (10,8), d (10,9),d (10,10)}
= max {1, 2, 2, 3, 3, 3, 2, 1, 1, 0} = 3

ecc (V)= max {e(1), e(2), e(3), e(4), e(5), e(6), e(7), e(8), e(9), e(10)}
= max {3, 3, 3, 3, 3, 3, 3, 3, 3, 3} = 3

The eccentricity of vertices is ecc [v(G)] =3

4.3. TO FIND THE RADIUS FROM G

Rad (G) = min {ecc (v): v ЄV (G)}

Rad (G) = min {e(1), e(2), e(3), e(4), e(5), e(6), e(7), e(8), e(9), e(10)}
= min {3, 3, 3, 3, 3, 3, 3, 3, 3, 3} = 3
Therefore Radius = 3

4.4. TO FIND THE DIAMETER FROM G:

Diam (G) = max {e (v): v ЄV (G)}

Diam(G) = max {e(1), e(2), e(3), e(4), e(5), e(6), e(7), e(8), e(9), e(10)}
= max {3, 3, 3, 3, 3, 3, 3, 3, 3, 3} = 3
Therefore diameter = 3

In this connection we get an eccentricity equal to the Diameter ecc [v (G)] = Diam (G)
And also we can easily to prove that Rad (G) ≤ Diam (G) ≤ 2Rad (G)

5. MAIN THEOREM

5.1. Theorem: For any connected induced sub graph G^{l} of G,
DS (<V-DS>) (G^{l}) ≤Rad (G^{l}) ≤ Dia (G^{l}) ≤ 2 [Rad (G^{l})].

Proof: Let G be a graph and the dominating set of G. Any circular-arc connected graph is DS (G) ≤ Rad (G) ≤ Dia (G) ≤2 [Rad (G)].

Already proved these in theorem (1). We will to prove that G^{l} be a connected induced sub graph of G.

Now we have to show that for any connected induced sub graph G^{l} of G and the dominating set of connected induced sub graph of G^{l} of G is

DS (<V-DS>) (G^{l}) ≤Rad (G^{l}) ≤ Dia (G^{l}) ≤ 2 [Rad (G^{l})].

The inequality radius of G^{l} is less than or equal to diameter of G^{l} is direct consequences of the definitions. Since from G^{l} the smallest eccentricity or minimum eccentricity can't exceed the largest or maximum eccentricity from G^{l} .

There fore the dominating set of induced sub graph of G^{l} is denoted by DS (<V-DS>) (G^{l}) and the radius of induced sub graph of G^{l} is denoted by Rad (G^{l}). From theorem (1)

DS (<V-DS>) (G^{l}) ≤ Rad (G^{l}) ……………………………….…. (1)

Again the radius of in G^{l} and the induced sub graph of diameter is denoted by Dia (G^{l}). That is Rad (G^{l}) ≤ Dia (G^{l}) ……………………………………………. (2)

Now the diameter of G^{l} and two times radius of the induced sub graph G^{l} is denoted by 2 [Rad (G^{l})].

That is Dia (G^{l}) ≤ 2 [Rad (G^{l})] ………………………………………. (3)

Also we will prove that the dominating set of induced sub graph G^{l} is less than or equal to two time radius of induced sub graph G^{l}.

That is DS (<V-DS>) ($G^{|}$) ≤ 2 [Rad ($G^{|}$)] ……………………………. (4)

Finally from (1),(2),(3) and (4) we get

=> DS (<V-DS>) ($G^{|}$) ≤ Rad ($G^{|}$).

=> DS (<V-DS>) ($G^{|}$) ≤ Dia ($G^{|}$).

=> DS (<V-DS>) ($G^{|}$) ≤ 2 [Rad ($G^{|}$)].

There fore the theorem is proved for any connected induced sub graph dominating set of $G^{|}$ is the circular-arc graph of G is always

DS (<V-DS>) ($G^{|}$) ≤ Rad ($G^{|}$) ≤ Dia ($G^{|}$) ≤ 2 [Rad ($G^{|}$)]

There fore the theorem is proved.

6. PRACTICAL PROBLEM G^1 FROM G:

From the illustration 1, the Dominating set of the circular-arc graph is DS = {1, 4, 8} and the cardinality **|DS|=3**

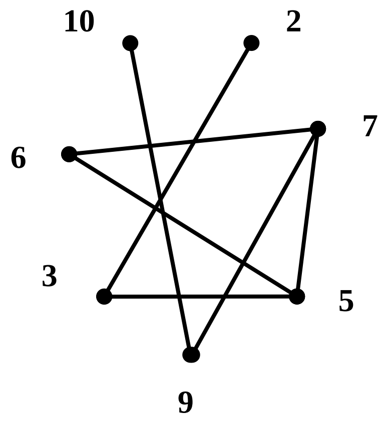


Fig. 3: vertex circular-arc <V-DS>-connected graph

From an induced sub-graph DS = {1, 4, 8} in this the cardinality **| <V-DS> |=3**

7. TO FIND DISTANCE, ECCENTRACITY, RADIUS AND DIAMETER FROM G^1

7.1. TO FIND THE DISTANCES FROM AN INDUCED CONNECTED SUBGRAPH $G^{|}$

d (2,2)= 0	d (3,2)= 1	d (5,2)= 2	d (6,2)= 3
d (2,3)= 1	d (3,3)= 0	d (5,3)= 1	d (6,3)= 2
d (2,5)= 2	d (3,5)= 1	d (5,5)= 0	d (6,5)= 1
d (2,6)= 3	d (3,6)= 2	d (5,6)= 1	d (6,6)= 0
d (2,7)= 3	d (3,7)= 2	d (5,7)= 1	d (6,7)= 1
d (2,9)= 4	d (3,9)= 3	d (5,9)= 2	d (6,9)= 2
d(2.10)=5	d(3,10)=4	d(5.10)=3	d(6,10)=3

d (7,2)= 3	d (9,2)= 4	d (10,2)= 5
d (7,3)= 2	d (9,3)= 3	d (10,3)= 4
d (7,5)= 1	d (9,5)= 2	d (10,5)= 3
d (7,6)= 1	d (9,6)= 2	d (10,6)= 3
d (7,7)= 0	d (9,7)= 1	d (10,7)= 2
d (7,9)= 1	d (9,9)= 0	d (10,9)= 1
d(7,10)=2	d(9,10)=1	d(10,10)=0

7.2. TO FIND AN ECCENTRICITY FROM INDUCED CONNECTED SUBGRAPH G^{l}

ecc [v(G^{l})] = max {d (u,v): u ЄV($\mathbf{G}^{l}$)}

e(2)= max{d (2,2),d (2,3),d (2,5),d (2,6),d (2,7),d (2,9),d (2,10)}
= max {0, 1, 2, 3, 3, 4, 5} = 5

e(3)= max{d (3,2),d (3,3),d (3,5),d (3,6),d (3,7),d (3,9),d (3,10)}
= max {1, 0, 1, 2, 2, 3, 4} = 4

e(5)= max{d (5,2),d (5,3),d (5,5),d (5,6),d (5,7),d (5,9),d (5,10)}
= max {2, 1, 0, 1, 1, 2, 3} = 3

e(6)= max{d (6,2),d (6,3),d (6,5),d (6,6),d (6,7),d (6,9),d (6,10)}
= max {3, 2, 1, 0, 1, 2, 3} = 3

e(7)= max{d (7,2),d (7,3),d (7,5),d (7,6),d (7,7),d (7,9),d (7,10)}
= max {3, 2, 1, 1, 0, 1, 2} = 3

e(9)= max{d (9,2),d (9,3),d (9,5),d (9,6),d (9,7),d (9,9),d (9,10)}
= max {4, 3, 2, 2, 1, 0, 1} = 4

e(10)= max{d (10,2),d (10,3),d (10,5),d (10,6),d (10,7),d (10,9),d (10,10)}
= max {5, 4, 3, 3, 2, 1, 0} = 5

ecc [v(G^{l})] = max {e(2), e(3), e(5), e(6), e(7), e(9), e(10)}
= max {5, 4, 3, 3, 3, 4, 5} = 5

The eccentricity of vertices is ecc [v(G^{l})] = 5

7.3. TO FIND THE RADIUS FROM AN INDUCED CONNECTED SUBGRAPH G^{l}

Rad (G) = min {ecc (v): v ЄV (G^{l})}

Rad (G^{l}) = min {e(2), e(3), e(5), e(6), e(7), e(9), e(10)}
= min {5, 4, 3, 3, 3, 4, 5} = 3

Therefore Radius = 3

7.4. TO FIND THE DIAMETER FROM AN INDUCED CONNECTED SUBGRAPH G^{l}

Diam (G^{l}) = max {ecc (v): v ЄV (G^{l})}

Diam (G^{l}) = max {e(2), e(3), e(5), e(6), e(7), e(9), e(10)}
= max {5, 4, 3, 3, 3, 4, 5} = 5

Therefore diameter = 5

In this we have an eccentricity is equal to the Diameter ecc [v(G^{l})] = Diam (G^{l})

And also we prove that easily Rad (G^{l}) ≤ Diam (G^{l}) ≤ 2Rad (G^{l})

From the theorem any connected graph G, Rad (G) ≤ Diam (G) ≤ 2(Rad (G)) and an induced connected subgraph G^{l}, the Diam |V-DS| (G^{l}) > Diam (G^{l}).

There fore the theorem is proved from G and G^{l}.

8. Conclusion

Resolving the Comparison of Domination Number, Radius and Diameter of Circular Arc-Graph of some special classes of circular- arc graphs has been the main focus of the paper. Nature of the arcs of the graph and the relation between the Distances, Radius and Diameter paved the way for the present progressive revelations. Especially, the nature of the arcs played a major role in determining the Domination Number, Radius and Diameter of the circular –arc graphs with amazing case. Some categorized graphs have been chosen in the process of exploration. In future, efforts will be put to identify the circular-arc graphs with Domination Number, Radius and Diameter.

References

[1] O.Ore, *Theory of Graph*, Amer, Math.Soc.Colloq.Publ.38, Providence (1962), P.206.

[2] C.Berge, *Graphs and Hyperactive graphs*,North Holland, Amsterdram in graphs, Networks,Vol.10(1980), 211- 215.

[3] E.J.Cockayne, S.T.Hedetniemi, *Towards a theory of domination in graphs*, Networks, Vol.7(1977), 247-261.

[4] V.R.Kulli, B.Janakiram, *The Non-split domination number of a graph*, Indian J.Pure.Applied Mathematics, Vol.31(5), 545-550, May 2000.

[5] Dr.A.Sudhakaraiah, K. Ramakrishna, M. Reddappa, *To Find The Comparison Of The Non Split dominating, induced Non Split Dominating Sets And The Distance, Radius, Eccentricity And Diameter Towards An Interval Graph G and G from An Interval family*, International Journal of New Technologies in Science and Engineering, Vol.2, Issue. 1, 2015, ISSN 2349-0780, 2015.

[6] Dr. A. Sudhakaraiah, B. Radhamma, M. Reddappa, K. Ramakrishna, *The Comparison of Bondage Number and the Diameter of an Interval Graph*, IJARCSSE, Volume 5, Issue 12, December 2015.

[7] Bienstock, D., and Gyon, E., *Average distance in graphs with removed elements*, J. Combin. Theoryser.B, 48(1998) 140-142.

[8] Anita Saha, *Computation of Average distance, radius and centre of a circular-arc graph in parallel*, Journal of physical sciences, Physical sciences, Vol. 10,2006,178-187.

[9] Dr.A.Sudhakaraiah*,B.Radhamma,K.Ramakrishna,M.Reddappa, To Find the Comparison of Non-Split Domination Number and the Average Distance of an Interval Graph, IJERMET, Volume-4, Issue-12, Dec-2015.