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Quasi Umbilical Manifold of Almost Hyperbolic Hermite and KH-Structure

By

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Abstract. The purpose of the present paper is to study the Quasi Umbilical manifold of Almost Hyperbolic Hermite and KH-Structure Manifold with n-dimension. Hyper-surfaces immersed in an almost hyperbolic Hermite manifolds studied by Dube [4]. GF-structure manifold have been studied by Mishra, R.S. & Singh, S.D. [10]. we have obtained the conditions for this manifold to be W-Quasi umbilical. The paper is organized as follows quasi umbilical. In section one, introductory part of almost hyperbolic Hermite manifold and KH-structure is defined some useful equations. In section two, we prove that the some theorems, definitions and corollaries in Quasi Umbilical manifolds by using Almost Hyperbolic Hermite and KH-Structure Manifolds as well as Nijenhuis tensor. In the end, we have discussed about almost Hyperbolic Hermite and KH-structure in quasi umbilical manifold.

Key Words: Riemannian connexion, KH-structure, Almost Hyperbolic Hermite manifold, Riemannian metric, Nijenhuis tensor, W- quasi umbilical, generalized structure, vector field etc.

1. **Introduction.** Let us consider an n-dimensional differentiable manifold M^n of class C^∞ in which there exists a vector valued linear function F and a Riemannian metric g satisfying

(1.1) $F^2X = X$, where

(1.2) $\overline{X} = FX$

for an arbitrary vector field $X\ Y$, in M^n, then M^n is called an almost hyperbolic Hermite manifold.

Let this structure be endowed with Riemannian metric g such that

(1.3) $g(\overline{X},\overline{Y}) = -g(X,Y)$ or $g(\overline{X},\overline{Y}) + g(X,Y) = 0$,

Let M^n be given with H-structure, such that

(1.4) a $(D_X F)Y = 0$,

(1.4)b $(D_X F)\bar{Y} = 0$

where D be the Riemannian connexion is satisfied. Then the manifold M^n is said to have a KH-structure manifold. Let M^n be a differentiable manifold of almost hyperbolic Hermite manifold.

Such that

(1.5) $FBX = BfX + u \otimes P + v \otimes Q,$

(1.6) $FP = BU - \lambda Q,$

(1.7) $FQ = BV - \lambda P,$

Where P, Q are two unit normal vector fields to M^n, f is a tensor field of type (1,1) U, V are vector fields, u, v 1- forms and λ is a C^∞ function.

Operating equations (1.5),(1.6), and (1.7) by F and using equations (1.1),(1.2),(1.5),(1.6),(1.7) and taking the tangential and normal parts separately, we get

(1.8)a $F^2X = f^2X + u(X)U + v(X)V$

(1.8)b $u(fX) = (1-\lambda^2)V(X),\qquad v(fX) = (1-\lambda^2)U(X)$

(1.8)c $fU = (1-\lambda^2)V,\qquad fV = (1-\lambda^2)U$

(1.8)d $u(U) = -(1+\lambda^2),\qquad u(V) = 0$

(1.8)e $v(V) = -(1+\lambda^2),\qquad v(U) = 0.$

Let g be the induced Riemannian metric in M^n defined by

(1.9)a $g(X,Y) = -g(X,Y)$

Also, we have

(1.9)b $g(BX,P) = (BfX, BV) - g(BfX, \lambda Q) + g(uXf, BU)$
$\qquad - g(uXF, \lambda Q) + g(VXF, BV) - g(VXQ, \lambda Q),$

(1.9)c $g(P,P) = -(1+\lambda) = g(Q,Q).$

Then using equations (1.5) and (1.9) in equation (1.2), we get

(1.10) $g(fX, fY) = g(X,Y) - u(X)u(Y) - v(X)v(Y).$

(1.11) $g(U,X)=) = -u(X),\qquad g(V,X) = -v(X).$

Thus the manifold M^n is a generalized $\{f, g, u, v, \lambda\}$ structure manifold of almost hyperbolic Hermite manifold. Let D be an affine connection in M^n induced by the Riemannian connection of almost hyperbolic Hermite manifold M^n then Gauss and Weingarten equations are given by

(1.12) $\bar{D}_{BX} BY = BD_X Y + H(X,Y)P + K(X,Y)Q,$

(1.13) $\overline{D}_{BX} P = BHX + \ell(X)Q,$

(1.14) $\overline{D}_{BX} Q = BKX + \ell(X)P,$

where h & k are second fundamental forms and ℓ is the third fundamental form defined by

(1.15) $g(HX,Y) \stackrel{def}{=} h(X,Y),$

(1.16) $g(KX,Y) \stackrel{def}{=} k(X,Y).$

H and K being the tensors of type (1,1). Differentiating equations (1.4), (1.5), (1.6) covariant and using equations (1.1), (1.7), (1.12), (1.13) and (1.14), we get

(1.17) $(D_X f)Y = -h(X,Y)U - k(X,Y)V + u(Y)H(X) + v(Y)K(X),$

(1.18) $(D_X u)Y = h(X, fY)U + \lambda K(X,Y) + V(Y)\ell(X),$

(1.19) $(D_X v)Y = \lambda h(X,Y)U + K(X, fY) + v(Y)\ell(X),$

(1.20) $(D_X U)Y = -fHX - \lambda KX + \ell(X)V,$

(1.21) $(D_X V) = -fKX - \lambda HX + \ell(X)V,$

(1.22) $h(X,U) = u(H,X),$

(1.23) $K(X,V) = v(K,X).$

2. Quasi Umbilical manifold of n-dimension

Definition 2.1. Let in the quasi umbilical manifold in which fixed in the particular space but whose can measure the n-dimensional with the mathematical form, we have

(2.1)a $h(X,Y) = \alpha g(X,Y) + \beta W(X).W(Y),$

$h(X) = \alpha(X) + \beta W(X)W,$

and

(2.1)b $k(X,Y) = \alpha' g(X,Y) + \beta' W(X)W(Y),$

$k(X) = \alpha'(X) + \beta' W(X)W.$

Now in the above equations be satisfied, where α, β and α', β' are scalar function and W is 1-form. Then almost hyperbolic Hermite and KH-structure manifold M^n is called Quasi Umbilical manifold of n-dimension.

Definition 2.2. If in addition

(2.2) $W(X) = g(W, X)$,

where W is the vector field and g is Riemannian metric then M^n is called a W - Quasi manifold of class C^∞ [1].

Definition 2.3. Further if $\alpha = 0, \alpha' = 0, \beta \neq 0, \beta' \neq 0$ and $\ell = 0$, then the W - Quasi-Umbilical manifold is called a cylindrical manifold [1].

Theorem 2.1. If the almost Hyperbolic Hermite and KH-structure manifold M^n is quasi umbilical then,

(2.3)a $$(D_X f)Y = -\alpha g(X,Y)U - \beta W(X)W(Y)U - \alpha' g(X,Y)V - \beta' W(X)W(Y)V + u(Y)\{\alpha(X) + \beta W(X)W\} + v(Y)\{\alpha'(X) + \beta' W(X)W\},$$

(2.3)b $$(D_X u)Y = \alpha g(X, fY)U + \beta W(X)W(fY) + \lambda\alpha' g(X,Y) + \lambda\beta' W(X)W(Y) + V(Y)\ell(X),$$

(2.3)c $$(D_X v)Y = \lambda\alpha g(X,Y) + \lambda\beta W(X)W(Y) + \alpha' g(X, fY) + \beta' W(X)W(Y) + V(Y)\ell(X),$$

(2.3)d $$(D_X U) = -\alpha(fX) - \beta W(fX)W - \lambda\alpha'(X) - \lambda\beta' W(X)W + \ell(X)V,$$

(2.3)e $$(D_X V) = -\alpha'(fX) - \beta' W(fX)W - \lambda\alpha(X) - \beta W(X)W + \ell(X)V,$$

(2.3)f $$h(X,U) = \alpha u(X) + \beta W(X)u(W),$$

(2.3)g $$k(X,V) = \alpha' v(X) + \beta' W(X)u(W).$$

Proof. In view of equation (2.1) *a, b* and equations (1.16), (1.17), (1.18), (1.19), (1.20), (1.21), (2.1) and (1.23), we get the required results.

Corollary 2.1. If $\alpha \neq 0, \alpha' \neq 0, \beta = \beta' = 0$ and $\ell = 0$, then 1- forms u, v are projectively killing in M^n i.e.,

(2.4)a $$(D_X u)(Y) + (D_Y u)(X) = 2\lambda\alpha' g(X,Y)$$

and

(2.4)b $$(D_X v)(Y) + (D_Y v)(X) = 2\lambda\alpha g(X,Y).$$

Proof. By using the condition in equations (2.3) b and (2.3) c, we get equations (2.4) a and (2.4) b.

Corollary 2.2. If $\alpha = \alpha' = 0, \beta \neq 0, \beta' \neq 0$ and $\ell = 0$ be satisfied in the manifold M^n, then

(2.5)a $$(D_X f)Y = -\beta W(X)W(Y)U - \beta' W(X)W(Y)V + \beta u(Y)W(X)W$$

$$+\beta' v(Y)W(X)W,$$

(2.5)b $\quad (D_X u)Y = \beta W(X)W(fY) + \lambda\beta' W(X)W(Y),$

(2.5)c $\quad (D_X V)Y = \lambda\beta W(X)W(Y) + \beta' W(X)W(fY).$

Proof. Using the conditions in equations (2.3) a, b, c, we get equations (2.5) a,b,c.

Corollary 2.3. If the manifold M^n, being W- Quasi manifold of satisfies $\alpha = \alpha' = 0, \beta = \beta' = 0$ and $\ell = 0$. Then the 1-form u, v in M^n are killing and the vector field U , V are parallel fields in M^{2n}.

(2.5)d $\quad (D_X f)Y = 0,$

(2.5)e $\quad (D_X u)Y = 0,$

(2.5)f $\quad (D_X v)Y = 0,$

(5.2.5)g $\quad (D_X U) = 0,$

(2.5)h $\quad (D_X V) = 0,$

(2.5)i $\quad h(X,U) = 0,$

(2.5)j $\quad k(X,V) = 0.$

Proof. By using $\alpha = \alpha' = 0, \beta = \beta' = 0$ and $\ell = 0$ in equations (2.3) a, b, c, d, e, f, g, we get (2.5) d, e, f, g, h, i, j.

Theorem 2.2. In a generalized cylindrical manifold M^n of n-dimensional, we have

(2.6)(a) $\quad (D_X u)Y - (D_Y u)X = \beta[W(fX)W(Y) - W(X)W(fX)]$ and

(2.6)(b) $\quad (D_X v)Y - (D_Y v)X = \beta'[W(fX)W(Y) - W(X)W(fY)].$

Proof. From equation (2.5)b, we have

$$(D_X u)Y = \beta W(X)W(fY) + \lambda\beta' W(X)W(Y),$$

$$(D_X u)X = a^4\beta W(Y)W(fX) + a^4\lambda\beta' W(Y)W(X).$$

Now subtracting the above two equations, we get equation (2.6)a. The proof of equation (2.6)b follows in the same manner.

Theorem 2.3. The Nijenhuis tensor of W - Quasi umbilical manifold M^n of a KH-structure manifold with generalized $\{f, g, u, v, \lambda\}$ structure is given by

(2.7) $$N(X,Y) = -\alpha g(fX,Y)U - \beta W(fX)W(Y)U - \alpha' g(fX,Y)V$$
$$-\beta' W(fX)W(Y)V + u(Y)\{\alpha(fX) + \beta W(fX)W\}$$
$$+v(Y)\{\alpha'(fX) + \beta' W(fX)W\} + a\alpha g(fY,X)U$$
$$+\beta W(fY)W(X)U + \alpha' g(fY,X)V + \beta' W(fY)W(X)V$$
$$-u(X)\{\alpha(fY) + \beta W(fY)W\} - V(X)\{\alpha'(fY)$$
$$+\beta' W(fY)W\} - f[-\alpha g(X,Y)U$$

$$-\beta W(X)W(Y)U-\alpha' g(X,Y)V-\beta' W(X)W(Y)V$$
$$+V(Y)\{\alpha(X)+\beta(X)W\}+V(Y)\{\alpha'(X)$$
$$+\beta' W(X)W\}]+f[-\alpha g(X,Y)U-\beta W(Y)W(X)U-\alpha' g(Y,X)V$$
$$-\beta' W(Y)W(X)V+u(X)\{\alpha(Y)+\beta W(Y)W\}+V(X)\{\alpha'(Y)+\beta' W(Y)W\}].$$

Proof. Let N be the Nijenhuis tensor corresponding to the tensor field f in M^n, given by

(2.8) $$N(X,Y)=(D_{fX}f)(Y)-(D_{fY}f)(X)-f(D_X f)(Y)+f(D_Y f)(X).$$

In view of equation (2.3) a and (2.8), we get the required result.

Theorem 2.4. The Nijenhuis tensor of a generalized cylindrical manifold M^n of n-dimension of a KH-structure manifold M^n is given by

(2.9)a $$N(X,Y)=\beta[W(fu)W(X)U-W(fX)W(Y)U+W(X)W(Y)(fU)$$
$$-W(Y)W(X)(fU)]+\beta'[W(fu)W(X)V-W(fX)W(Y)V$$
$$-W(X)W(Y)V-W(Y)W(X)V]+\beta[W(fX)Wu(Y)$$
$$-u(X)W(fu)W+u(Y)W(X)W+u(X)W(Y)W]$$
$$+\beta'[V(Y)W(fX)W-V(X)W(fu)W+V(Y)W(X)W$$
$$+V(X)W(Y)W].$$

(2.9)b $$N(X,Y)=\alpha[-g(fX,Y)U+g(fY,X)U+g(X,Y)(fU)-g(Y,X)(fu)]$$
$$+\alpha'[-g(fX,Y)V+g(fY,X)V+g(X,Y)(fV)+g(Y,X)(fV)]$$
$$+[u(Y)\alpha(fX)+v(Y)\alpha'(fX)-u(X)\alpha(fY)-v(X)\alpha'(fY)$$
$$+\alpha g(X,Y)(fV)]+f[u(Y)\alpha(X)+V(Y)+\alpha'(X)-u(X)\alpha f(Y)$$
$$-V(X)\alpha'(Y)].$$

(2.9)c $$N(X,Y)=0.$$

Proof. By putting $\alpha=\alpha'=0,\beta\neq 0,\beta'\neq 0$ and $\alpha\neq 0,\alpha'\neq 0,\beta=\beta'=0$ and also $\alpha=\alpha'=\beta=\beta'=0$ in equation (2.8) we get the required results (2.9) a,b.

Discussion. The key point of this paper is the concept of Quasi umbilical which is the backbone of manifold and also observed that we worked out on quasi umbilical manifold of n-dimension by using the almost Hyperbolic Hermite and KH-structure manifolds, we have also discussed n-dimensional cases in the manifolds. That means quasi umbilical is the neighborhood point of the center of manifolds in which all lines of curvature indeterminate (not fixed). Quasi Umbilical manifold are important role of dealing the extended of n-dimensional space heavenly body in the neighborhood point because it is measured the higher dimensional space and we measure the more complicated structures. Whose shape and size are not fixed but major some particular areas that is, clouds, moon, earth, brain, nervous system, respiratory system, snowflakes, mountains ranges, lighting, river and much, much more. Our purpose is to study easily calculate n-dimensional spaces of manifolds with the help of Quasi umbilical manifolds.

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